Absolute Value Toolkit

\[ y = |x| \]

**Solutions to Absolute Value**

- **Two Solutions if the output is a positive value**

  Absolute value bars can have more than one answer. What could be the values of \( x \) that would make the sentence below true?

  \[ |x| = 7 \]
  \[ x = +7 \text{ or } -7 \]
  Therefore \( x = \{7, -7\} \)

- **No Solution or Empty Set when the output is negative because the absolute value is always positive distance from zero.**

  \[ |x| = -5 \text{ or } |x - 4| = -6 \]
  Ever heard of a negative distance?

  \[ |x + 3| + 12 = 5 \]
  \[ |x + 3| = -7 \]
  \[ x = \emptyset \]

  **no solution**

- **One solution if the output is zero. Zero is neither negative or positive.**

  \[ |4x - 12| = 0 \]
  \[ x = 3 \]

**CCA 3.3.1 Solve Absolute Value Equations MATH NOTE**

One way to solve absolute value equations is to think about “looking inside” the absolute value. The “inside” must be positive or negative, so you should solve the equation both ways. For example, you could record your steps as shown below.

\[
|5 - 2x| = 19 < \text{ positive means two solutions}
\]

-19 is the distance from zero

DO NOT rewrite \( 5 - 2x \) \( \neq \)

**CCA 10.2.6 Solving Absolute Value Equations MATH NOTE**

- **Solve the equation. Be sure to find all possible answers and check your solutions**

  a. \[ 8|x - 3| + 1 = 33 \]

  \[ \frac{x - 3}{8} = \frac{32}{8} \]
  \[ x - 3 = 4 \]
  \[ x = 7 \]

  b. \[ 2|x + 1| = -4 \]

  \[ \frac{1}{x + 1} = -2 \]
  \[ \text{no solution} \]

  Note that distributing over an absolute value is **not** allowed.

  For example, \[ -2|3 + 1| \neq -2(3) + -2(1) \]

  \[ \frac{5(2x + 3)}{5} \neq |10x + 15| \]
  \[ \text{Nope} \]
Absolute Value Inequalities

Basic Absolute Value Inequalities (k > 0)

<table>
<thead>
<tr>
<th>Absolute Value Inequality</th>
<th>Equivalent Inequality</th>
<th>Solution Set</th>
<th>Graph of Solution Set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(-∞, -k) U (k, ∞)</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>(-∞, -k] U [k, ∞)</td>
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<td></td>
<td></td>
<td>[k, ∞)</td>
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Absolute Value Inequality with No Solution

- How can you tell immediately that the following inequality has no solution?
  \[ |5x - 7| < -2 \]
  - less than

- It says that absolute value (or distance) is negative — contrary to the definition of absolute value

- Absolute value inequalities of this form always have no solution:
  \[ |x| < -n \] (where \(-n\) represents a negative number)
  \( \phi \) composed to \[ 5x - 7 > -2 \] greater than

Solve the absolute value inequality and graph on a number line.

\[ |x - 3| + 1 > 2 \]
\[ |2x + 5| + 1 < 4 \]

Graphing and Transforming \( y = |x| \)

**Transformations (the whole enchilada)**

\[ y = a |x - h| + k \]
- \( a = \text{slope} \)
- \#1: vertical stretch (\(|a| > 1\)) or shrink (\(0 < |a| < 1\))
- *negative: vertical reflection (opposite)*
- \#2: horizontal translation
- \#3: vertical translation

Parent function \( y = |x| \)
Key points (-1, 1), (0, 0), (1, 1)

Vertex \((h, k)\) is minimum point on graph

Graph the parent transformation then
\[ y < -2|x + 3| + 5 \]

- \( a = \text{slope} = -2 \)
- sketch dashed line
- vertex = \((-3, 5)\)
- less than

Graph the parent transformation then
\[ y > 1/3|x - 3| - 4 \]

- \( a = \text{slope} = 1/3 \)
- compression dashed line
- vertex = \((3, -4)\)
- greater than