### Quadratic Factoring, Zero Product Property, Vertex Form

#### Forms of a Quadratic Function

There are three main forms of a quadratic function. Assume that \(a \neq 0\) and that the meaning of \(a, b,\) and \(c\) are different for each form below.

**Standard form:** \(f(x) = ax^2 + bx + c.\)

The \(y\)-intercept is \((0, c)\).

**Factored form:** \(f(x) = a(x + b)(x + c).\)

The roots/\(x\)-intercepts are \((-b, 0)\) and \((-c, 0)\).

**Graphing form** (vertex form): \(f(x) = a(x - h)^2 + k.\)

The vertex is \((h, k)\).

If the expression is a polynomial (see Math Notes box in Lesson 3.2.3) the number of terms can help you name the polynomial.

**Examples of monomials:** \(15x^2\) and \(-2m\)

**Examples of binomials:** \(16m^2 - 25\) and \(7h^9 + \frac{1}{2}h\)

**Examples of trinomials:** \(12 - 3k^2 + 5k\) and \(x^2 - 15x + 26\)

#### 8.1.4 Can it still be factored?

**Factoring Completely**

Factor: \(3n^2 + 9n + 6\) \(\frac{2}{3} \div \frac{2}{3} \div \frac{2}{3}\)

\(3(n^2 + 3n + 2) = 3(n + 1)(n + 2)\)

**Factored form**

#### 8.1.5 Is there a shortcut?

**Factoring Shortcuts**

Factoring when the "a" term is equal to 1. Write your answer as a product.

Factor: \(x^2 - 8x + 7\) and \(x^2 - 5x - 36\)

**Difference of two perfect squares** ("b" term is missing creates a plus and minus pattern in product):

Factor:

\[
\begin{align*}
9x^2 - 4 & \quad \frac{25x^2 - 1}{(5x + 1)(5x - 1)} \\
(3x + 2)(3x - 2) & \quad (x + 8)(x - 8)
\end{align*}
\]

#### Perfect square trinomial pattern

\[
\begin{align*}
9x^2 - 12x + 4 & \quad \frac{144x^2 - 24x + 144}{-12x + 12} \\
(3x + 2)^2 & \quad (x - 12)^2
\end{align*}
\]

Not Factorable: \(x^2 + 4\) and \(x^2 + 6x + 7\)

**True or False** \((x + 8)^2 = x^2 + 64 + 16x + 6y\)

**Perfect square Trinomial**

**Not a Equation but an expression:**
8.2.2 How are quadratic rules and graphs connected?

Find the exact x-intercepts/roots using the Zero Product Property (ZPP) \( y = 0 \).

<table>
<thead>
<tr>
<th>a. ( 0 = x^2 - 2x - 8 )</th>
<th>b. ( x^2 + 7x + 12 = 0 )</th>
<th>c. ( x^2 - 4x + 21 = 0 )</th>
<th>d. ( 0 = x^2 + 6x + 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((x-4)(x+2) = 0)</td>
<td>((x+3)(x+4) = 0)</td>
<td>((x-7)(x+3) = 0)</td>
<td>(x(x+6) = 0)</td>
</tr>
<tr>
<td>(x = -2, 0)</td>
<td>(x = -3)</td>
<td>(x = -7)</td>
<td>(x = 0)</td>
</tr>
<tr>
<td>((2,0) \notin (-2,0))</td>
<td></td>
<td>((-7,0) \notin (-3,0))</td>
<td></td>
</tr>
</tbody>
</table>

Now graph \( y = x^2 - 2x - 8 \) by using four points.

Step 1: Identify the \( y \)-intercept.

Step 2: Calculate the x-intercepts/roots by using ZPP. (above "a" work)

Step 3: Draw the line of symmetry or calculate the x-value of the vertex - use \( x = -\frac{b}{2a} \), then substitute \( x \) back into the quadratic rule to find \( y \).

Step 4: Graph all four points with labels.
\[
\begin{align*}
x = 1 \text{ w.t. } y & = (1)^2 - 2(1) - 8 = -9 \\
y = -2 \text{ w.t. } y & = 1 - 2 - 8 = -9 \\
\text{vertex of } x & = \frac{y}{2} = \frac{-2}{2} = -1 \\
y & = -9 \text{ vertex } (1, -9)
\end{align*}
\]

8.2.3 How else can I find the roots?

More Ways to Find the x-Intercepts

A Quadratic function in \( y = a(x - h)^2 + k \) form is called **vertex, perfect square form**, or **graphing form** where \((h, k)\) is the vertex of the parabola.

8.2.4 What is the connection?

Completing the Quadratic Web

Table to Rule:

<table>
<thead>
<tr>
<th>( x )</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>6</td>
<td>0</td>
<td>-4</td>
<td>-6</td>
<td>0</td>
<td>-6</td>
<td>-4</td>
<td>0</td>
<td>6</td>
<td>14</td>
</tr>
</tbody>
</table>

1. Identify the \( y \)-intercept/"c" term (\( 0, -6 \)) \( x = 0 \)
2. Identify the roots/x-intercepts (\( -3, 0 \)) \& (\( 2, 0 \)), \( y = 0 \)
3. Use the Zero Product Property in reverse to find the rule in \( y = ax^2 + bx + c \)

8.2.5 How can I write it in graphing form?

Completing the Square

Use **Completing the Square (CTS)** method to change standard form, \( f(x) = ax^2 + bx + c \), into graphing form \( f(x) = a(x - h)^2 + k \).

Use \( \left( \frac{b}{2} \right)^2 \) to determine the perfect square value. Find \( h \) & \( k \).