Chapter 7 Exponential Functions Toolkit

7.1.1 What do exponential graphs look like?

Investigating $y = b^x$

Sketch the shape of the graph of the function $y = b^x$ given each of the following values of $b$.

- $b > 1$ is a number larger than 1: **increasing**
  
- $0 < b < 1$ is a number between 0 and 1: **decreasing**
  
- $b = 1$ is equal to 1: **horizontal line**

Graphs with Asymptotes

Asymptotes can be diagonal lines or even curves. The graph of a function has a **horizontal asymptote** if, as you trace along the graph out to the left or right, the distance between the graph of the function and the asymptote gets closer to **zero**.

A graph has a **vertical asymptote** if, as you choose x-coordinates closer and closer to a certain value, from either the left or right (or both), the y-coordinate gets farther away from zero, either toward

7.1.2 What is the connection?

Multiple Representations of Exponential Functions

\[ y = ab^x \]

- $a =$ starting value
- $b =$ multiplier

Situation to an Exponential Equation and solve

Ex#1 A powerful computer is purchased for $1500 but on the average loses 20% of its value each year. How much will it be worth 4 years from now?

\[ y = ab^x \]

\[ a = 1500 \]
\[ b = 0.8 \]
\[ y = 1500(0.8)^4 \]

Ex#2 Dinner at your grandfather's favorite restaurant now costs $25.25 and has been increasing steadily at 4% per year. How much did it cost 35 years ago when he was courting your grandmother?

\[ y = ab^x \]

\[ a = 25.25 \]
\[ b = 1.04 \]
\[ y = 25.25(1.04)^{-35} \]

Equation to a table and graph

\[ y = 4(0.5)^x \]
\[ a = 4 \]
\[ b = 0.5 \]
\[ \text{decreasing} \]

\[ f(x) = 5(1.2)^x \]
\[ a = 5 \]
\[ b = 1.2 \]
\[ \text{increasing} \]

Rule to a situation

Write a possible context based on the equation.

\[ y = 32,500(0.85)^x \]

I bought a Toyota Truck for $32,500 and each year $x$ it depreciated 15%.

Table to an equation and graph

\[ y = 5(0.8)^x \]
\[ a = 5 \]
\[ b = 0.8 \]
\[ \text{decreasing} \]

\[ y = 50 \]
\[ b = 0.8 \]
\[ a = \]
7.1.3 How does it grow?
More Applications of Exponential Growth

Write the multiplier for each increase or decrease described below.

A 35% increase = \(100\% + 35\% = 135\% = 1.35\) 
A decrease of 16% = \(100\% - 16\% = 84\% = 0.84\) 
A depreciation of 19% = \(100\% - 19\% = 81\% = 0.81\) 
A appreciation of 83% = \(100\% + 83\% = 183\% = 1.83\)

**Bonds with Simple Interest:**
The relationship grows **linearly** (like an arithmetic sequence). For example, 5% simple interest compounded annually on an initial investment of $2500 would grow in a sequence with a common difference: \(\frac{15}{2} \times 2.5 = 75\). The equation and table follow:

\[ t(n) = 75n + 2500 \]

<table>
<thead>
<tr>
<th>Number of Years, (n)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount in Bank, (t(n))</td>
<td>2500</td>
<td>2575</td>
<td>2650</td>
<td>2725</td>
<td>...</td>
<td>3250</td>
</tr>
</tbody>
</table>

**Accounts with Compound Interest:**
The relationship is **exponential** (like a geometric sequence). For example, 3% annual interest, compounded annually on an initial investment of $2500 would grow in a sequence with a multiplier of 1.03 every year. The example and table using the example above are:

\[ t(n) = 2500(1.03)^n \]

<table>
<thead>
<tr>
<th>Number of Years, (n)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount in Bank, (t(n))</td>
<td>2500</td>
<td>2575</td>
<td>2652.25</td>
<td>2731.81</td>
<td>...</td>
<td>3355.79</td>
</tr>
</tbody>
</table>

7.1.5 Step Functions

A **Step Function** is a special kind of function. A step function has a graph that is a series of **line segments**. Step functions are used to model real-world situations where there are abrupt changes in the output of the function.

**Open circles** (indicating this point is not part of the segment) or **filled-in circles** (indicating this point is part of the segment).

The graph at right models a situation in which a tour bus company has busses that can each hold up to 20 passengers with 5 available busses.

7.2.1 How can I find the equation?
Curve Fitting and Fractional Exponents

Find the equation of the line in \(y = mx + b\) form with slope 3 that passes through the point (5, 19).

\[
\begin{align*}
\text{m} &= 3 \\
19 &= 3(5) + b \\
10 &= 15 + b \\
-5 &= b \\
\text{EQ: } y &= 3x + 4
\end{align*}
\]

Find the equation of an exponential function in \(y = ab^x\) with an asymptote at \(y = 0\) that passes through the points (0, 5) and (3, 320).

\[
\begin{align*}
\frac{320}{5} &= (b)^3 \\
64 &= (b)^3 \\
b &= 4
\end{align*}
\]

\[
\text{EQ: } y = 5(4)^x
\]

**Zero, Negative and Fractional Exponents**

For all \(x\) not equal to zero: \(x^0 = 1\)

Examples:
\[
\begin{align*}
2^0 &= 1 \\
(-3)^0 &= 1 \\
(1/4)^0 &= 1
\end{align*}
\]

For non-negative (positive) values of \(x\): \(x^{-n} = \frac{1}{x^n}\)

Examples:
\[
\begin{align*}
x^{-3} &= \frac{1}{x^3} \\
y^{-4} &= \frac{1}{y^4} \\
4^{-2} &= \frac{1}{4^2} = \frac{1}{16}
\end{align*}
\]

For non-negative (positive) values of \(n\): \(\frac{1}{x^{-n}} = x^n\)

Examples:
\[
\begin{align*}
\frac{1}{x^{-5}} &= x^5 \\
\frac{1}{x^{-2}} &= x^2 \\
\frac{1}{3^{-2}} &= 3^2 = 9
\end{align*}
\]

**Exponential Fractions and root form**

\(x^{a/b} = (x^a)^{1/b} = \sqrt[b]{x^a}\) or \(x^{a/b} = (x^{1/b})^a = \sqrt[a]{x}\) **power radical**

Examples:
\[
\begin{align*}
5^{1/2} &= \sqrt{5} \\
3^{2/3} &= \sqrt[3]{3^2} = 3\sqrt[3]{3} \\
10^{2/3} &= \sqrt[3]{10^2} = 3\sqrt[3]{100} \\
16^{3/4} &= (\sqrt[4]{16})^3 = \frac{2^4}{2^{3/4}} = 2^1 = 2
\end{align*}
\]