5.3.3 Arithmetic Sequences \( t(n) \) vs Linear Function \( f(x) \)

\[
t(n) = d(n) + \text{initial value}
\]

\[
f(x) = mx + b
\]

Is a sequence a function? Yes, it has one output for every input. However, the domain is only positive integer values of \( n \).

An Arithmetic Sequence is modeled after a linear function because it has an \( \text{addition} \) or \( \text{subtraction} \) generator. The number added (or subtracted) to each term \( t(n) \) to get the next term \( t(n+1) = \) is called the \( \text{common difference} \). Use \( d \) for the symbol in the rule.

Create multiple representations for both the sequence and function:

\[
\begin{array}{c|c|c}
\text{Domain} & \text{Common difference} & \text{Function} \\
\hline
\text{only positive whole integers} & +3 & \text{Domain of } x \\
0, 1, 2, 5, 3, 8, 4, 11, 5, 14, 0, 17 & & \text{is all real #s}
\end{array}
\]

**Sequence**

<table>
<thead>
<tr>
<th>( n )</th>
<th>( t(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
</tr>
</tbody>
</table>

**Explicit equation**

\[ t(n) = 3n - 1 \]

**Recursive equation**

\[ t(n+1) = t(n) + 3 \]

**Discrete Graph**

**Linear model**

\[ f(x) = mx + b \]

\[ \text{slope } = m = 3 \]

\[ \text{y-intercept } = b = -1 \]

**Continuous Graph**

Is 200 a term \( t(n) \) in the sequence?

No, because decimals are not part of the domain of whole numbers.

Write the next four terms:

a) \( 33, -25, -18, -11, -4, 3, 10, 17, \ldots \)

\[ t(n) = 7n - 32 \]

\[ t(n+1) = t(n) + 7 \]

b) \( -2, 17, 12, 7, 2, -3, -8, -13, -18, \ldots \)

\[ t(n) = -5n + 22 \]

\[ t(n+1) = t(n) - 5 \]

c) \( 7, -2, -11, -20, -29, -36, -45, \ldots \)

\[ t(n) = -9n + 16 \]

\[ t(n+1) = t(n) - 9 \]

Is it possible for \( f(x) \) to equal 200?

Yes, since functions are continuous the domain allows decimals.

\[ n = 66.3 \]
### 5.3.2 Geometric Sequences \( t(n) \)

A geometric sequence \( t(n) \) equation models after exponential functions \( f(x) \)

\[
t(n) = (\text{start value})(\text{multiplier})^n
\]

- **Write the next four terms:**
  - a) \( \left( \frac{5}{3} \right) 4, 12, 36, \ldots \)
  - b) \( 9, 18, 36, 72, 144, \ldots \)
  - c) \( \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \)

- **Explicit equation**
  - \( t(n) = \left( \frac{5}{3} \right)^n \)
  - \( t(n) = \left( \frac{1}{2} \right)^n \)
  - \( t(n) = \frac{a}{2^n} \)

- **Recursive equation**
  - \( t(n+1) = 3(t(n)) \)
  - \( t(n+1) = \frac{1}{2}(t(n)) \)
  - \( t(n+1) = -2(t(n)) \)

Write an explicit rule for the \( n \)th term of the geometric sequence below:

\[
y = a \cdot b^x
\]

- \( a = 16 \)
- \( b = \frac{1}{2} \)
- \( t(n) = 16 \left( \frac{1}{2} \right)^n \)

### 5.3.2 PERCENTS AS MULTIPLIERS in discounts and sales tax

A **multiplier** (a generator/common ratio) can also be used to increase or decrease by a given percentage, but is converted to a decimal. For example:

- a) the multiplier for a tax increase of 7% is \( 100 \% + 7 \% = 107 \% \) converted to \( 1.07 \)

- b) The multiplier for a sales discount of 7% is \( 100 \% - 7 \% = 93 \% \) converted to \( 0.93 \)

List the next three terms and the equation of a sequence that starts:

- c) at 10 and appreciates by 8%: **multiplier** = 1.08
  - \( 10, 10.8, 11.66, 12.5928, \ldots \)
  - \( t(n) = 10 \left( 1.08 \right)^n \)

- d) at 100 and shrinks by 15%: **multiplier** = 0.85
  - \( 100, 85, 72.25, 61.4125, \ldots \)
  - \( t(n) = 100 \left( 0.85 \right)^n \)

- e) at 175 and grows by 12%: **multiplier** = 1.12
  - \( 175, 196.25, 219.5, \ldots \)
  - \( t(n) = 175 \left( 1.12 \right)^n \)

Identify the following sequence generators. Then label each as either arithmetic, geometric or neither.

- \( (kn)^2 + 1 \)
  - \( \text{Neither} \)

- \( \left( \frac{1}{4} \right)^n \not+ 4 \)
  - \( \text{Geometric} \)

- \( n^3 \)
  - \( \text{Neither} \)

- \( -64, -16, -4, -1, \ldots \)
  - \( \text{Arithmetic} \)

- \( 1, 6, 36, 216, \ldots \)
  - \( \text{Geometric} \)