Probability & Statistics

These curriculum maps are designed to address Common Core State Standards (CCSS) Mathematics and Literacy outcomes. The overarching focus for all curriculum maps is building students' content knowledge focusing on their math practice abilities and literacy skills. Each unit provides several weeks of instruction. Each unit also includes various assessments. Taken as a whole, this curriculum map is designed to give teachers recommendations and some concrete strategies to address the shifts required by CCSS.

Instructional Shifts in Mathematics

Focus: Focus strongly where the Standards focus	**Focus requires that we significantly narrow and deepen the scope of content in each grade so that students experience concepts at a deeper level.** • Instruction engages students through cross-curricular concepts and application. Each unit focuses on implementation of the Math Practices in conjunction with math content. • Effective instruction is framed by performance tasks that engage students and promote inquiry. The tasks are sequenced around a topic leading to the big idea and essential questions in order to provide a clear and explicit purpose for instruction.
Coherence: Think across grades, and link to major topics within grades	**Coherence in our instruction supports students to make connections within and across grade levels.** • Problems and activities connect clusters and domains through the art of questioning. • A purposeful sequence of lessons build meaning by moving from concrete to abstract, with new learning built upon prior knowledge and connections made to previous learning. • Coherence promotes mathematical sense making. It is critical to think across grades and examine the progressions in the standards to ensure the development of major topics over time. The emphasis on problem solving, reasoning and proof, communication, representation, and connections require students to build comprehension of mathematical concepts, procedural fluency, and productive disposition.
Rigor: In major topics, pursue conceptual understanding, procedural skills and fluency, and application	**Rigor helps students to read various depths of knowledge by balancing conceptual understanding, procedural skills and fluency, and real-world applications with equal intensity.** • Conceptual understanding underpins fluency; fluency is practiced in contextual applications; and applications build conceptual understanding. • These elements may be explicitly addressed separately or at other times combined. Students demonstrate deep conceptual understanding of core math concepts by applying them in new situations, as well as writing and speaking about their understanding. Students will make meaning of content outside of math by applying math concepts to real-world situations. • Each unit contains a balance of challenging, multiple-step problems to teach new mathematics, and exercises to practice mathematical skills

SAUSD Curriculum Map 2015-2016: Probability & Statistics

8 Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. They describe how students should learn the content standards, helping them to build agency in math and become college and career ready. **The Standards for Mathematical Practice are interwoven into every unit. Individual lessons may focus on one or more of the Math Practices, but every unit must include all eight.**

1. Make sense of problems and persevere in solving them	Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.
2. Reason Abstractly and quantitatively	Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
3. Construct viable arguments and critique the reasoning of others	Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense,

	and ask useful questions to clarify or improve the arguments.
4. Model with mathematics	Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.
5. Use appropriate tools strategically	Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.
6. Attend to precision	Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.
7. Look for and make use of structure	Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a

	positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.
8. Look for and express regularity in repeated reasoning	Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

English Language Development Standards

The California English Language Development Standards (CA ELD Standards) describe the key knowledge, skills, and abilities in core areas of English language development that students learning English as a new language need in order to access, engage with, and achieve in grade-level academic content, with particular alignment to the key knowledge, skills, and abilities for achieving college- and career-readiness. English Learners must have full access to high quality English language arts, mathematics, science, and social studies content, as well as other subjects, at the same time as they are progressing through the ELD level continuum. The CA ELD Standards are intended to support this dual endeavor by providing fewer, clearer, and higher standards. **The ELD Standards are interwoven into every unit.**

Interacting in Meaningful Ways
A. Collaborative (engagement in dialogue with others)
1. Exchanging information/ideas via oral communication and conversations
B. Interpretive (comprehension and analysis of written and spoken texts)
5. Listening actively and asking/answering questions about what was heard
8. Analyzing how writers use vocabulary and other language resources
C. Productive (creation of oral presentations and written texts)
9. Expressing information and ideas in oral presentations
11. Supporting opinions or justifying arguments and evaluating others' opinions or arguments

How to Read this Document

- The purpose of this document is to provide an overview of the progression of units of study within a particular grade level and subject describing what students will achieve by the end of the year. The work of **Big Ideas and Essential Questions** is to provide an overarching understanding of the mathematics structure that builds a foundation to support the rigor of subsequent grade levels. The **Performance Task** will assess student learning via complex mathematical situations. Each unit incorporates components of the **SAUSD Theoretical Framework** and the philosophy **of Quality Teaching for English Learners (QTEL).** Each of the math units of study highlights the Common Core instructional shifts for mathematics of focus, coherence, and rigor.

- The **8 Standards for Mathematical Practice** are the key shifts in the pedagogy of the classroom. These 8 practices are to be interwoven throughout every lesson and taken into consideration during planning. These, along with the **ELD Standards,** are to be foundational to daily practice.

- First, read the **Framework Description/Rationale** paragraph, as well as the **Common Core State Standards**. This describes the purpose for the unit and the connections with previous and subsequent units.

- The units show the progression of units drawn from various domains.

- The timeline tells the length of each unit and when each unit should begin and end.

Scope and Sequence for Probability & Statistics

Unit 1	Unit 2	Unit 3	Unit 4
9/2-9/18 3 Weeks	9/21-10/30 6 Weeks	11/2-12/4 4 Weeks	12/7-1/22 5 Weeks
Introduction to Statistics	Descriptive Statistics	Conditional Probability and the Multiplication Rule	Discrete Probability Distributions

******SEMESTER******

Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
2/1-3/4 5 Weeks	3/7-4/15 5 Weeks	4/18-5/6 3 Weeks	5/9-5/27 3 Weeks	5/30-6/16 3 Weeks
Normal Probability Distributions	Correlation and Regression	Confidence Intervals	Hypothesis Testing with One Sample	Enrichment

SAUSD Curriculum Map 2015-2016: Probability & Statistics

Probability & Statistics Overview:

This Probability & Statistics course contains all of the standards from CCSS High School Statistics and Probability, and select standards from previous grade levels from the Statistics and Probability domain. It begins with a unit containing standards to prepare the learner, to provide students with necessary supports so they can be successful with the content of the year. Since many students taking this course will also be taking Placement Exams this school year, the units also include information for review and links for the Practice Placement Exam implementation. Students are encouraged to take the online Practice Placement Exam prior to taking the actual test.

From the Probability & Statistics Framework:

The Probability and Statistics course offers an alternative fourth course to Precalculus. In Probability and Statistics students continue to develop a more formal and precise understanding of statistical inference, which requires a deeper understanding of probability. Students learn that formal inference procedures are designed for studies in which the sampling or assignment of treatments was random, and these procedures may be less applicable to nonrandomized observational studies. Probability is still viewed as long-run relative frequency but the emphasis now shifts to conditional probability and independence, and basic rules for calculating probabilities of compound events. In the plus (+) standards are the Multiplication Rule, probability distributions, and their expected values. Probability is presented as an essential tool for decision making in a world of uncertainty.

Students extend their work in probability and statistics by applying statistics ideas to real-world situations. They link classroom mathematics and statistics to everyday life, work, and decision-making, by applying these standards in modeling situations. They choose and use appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Students in Probability and Statistics take their understanding of probability further by studying expected values, interpreting them as long-term relative means of a random variable. They use this understanding to make decisions about both probability games and real-life examples using empirical probabilities. The fact that numerous standards are repeated from previous courses does not imply that those standards should not be covered in those courses. In keeping with the CA CCSSM theme that mathematics instruction should strive for depth rather than breadth, teachers should view this course as an opportunity to delve deeper into those repeated Probability and Statistics standards while addressing new ones.

(From the CA Mathematics Framework for Probability and Statistics)

Unit 1: Introduction to Statistics *(4 weeks 9/2-9/18)*

Big Idea	Numbers, expressions, and measures can be compared by their relative values. Some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it.

Essential Questions	Performance Task	Problem of the Month
• How can the study of Statistics be used in real life scenarios? • What are the benefits of interpreting data? • How do we study data?	• Taxi Times C2 2003 p.3-4 • Media Surfing C1 2012 p.4-5 • Population C1 2004 p.19-20	Through the Grapevine and Teacher's Notes

Unit Topics/Concepts	Content Standards	Resources
Introduction to Statistics • Distinguish between population and sample, parameter and statistic, and descriptive and inferential statistics • Recognize purpose and difference of sample surveys, experiments, and observational studies • Understand differences of qualitative and quantitative data • Classify data with respect to levels of measurement: nominal, ordinal, interval and ratio • Determine the steps in data collection • Apply the design of a statistical study • Use data from a sample survey • Use simulations to decide if differences between parameters are significant • Understand various sampling methods: random, simple random, stratified, cluster, and systematic • Identify a biased sample **Practice Placement Exam-** *(See information on next page)*	**6.SP.A.1** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. **ID.A** Summarize, represent, and interpret data on a single count or measurement variable **IC.B.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. **IC.B.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. **IC.B.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. **IC.B.6** Evaluate reports based on data. Write a function that describes a relationship between two quantities.	**Essential Resource:** Textbook- *Elementary Statistics,* Chapter 1 **Additional Resources:** Chapter 1 Supplements- • Case Study: Rating Television Shows in the United States • Real Statistics –Real Decisions **Data Collection:** • Census At School: www.amstat.org/censusatschool/ • Discovery Education: High Stakes World of Statistics Series (Video Lessons)- www.discoveryeducation.com

Unit 1: Introduction to Statistics *(Support & Strategies)*

Framework Description/Rationale

Students have encountered standards S-IC.1-3 in previous courses, however in this unit students can build off of these standards, now using the data from sample surveys to estimate such attributes as the population mean or proportion. In this unit, with their understanding of the importance of random sampling, students learn that running a simulation and obtaining multiple sample means will yield a roughly normal distribution when plotted as a histogram. They use this to estimate the true mean of the population and can develop a margin of error. This unit also contains a review of basic skills that are necessary for the Level 1 Practice Placement Exam. This review and practice test will be helpful to establish and build student understanding in areas that are relevant to future placement as students matriculate into college.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner (number of days)
Key Terms • Ratio • Proportion • Percent • Population • Sample • Parameter • Statistic • Qualitative • Quantitative • Equivalent fractions	Strategy Examples • Thinking Maps • Venn Diagrams • Sentence Frames	

Placement Exam Review

The unit length includes 4-5 days to set up and take the online practice exam provided by Santa Ana College (SAC). Students should take the practice placement exam in Unit 3 and Unit 4 to show student growth and to gather more data on how the teacher can differentiate the algebraic reviews in the course. It is recommended that the review be administered in smaller chunks throughout the unit. One suggestion would be to use the review during 10-15 minute warm ups. Below is a recommendation of topics and concepts that would be covered during unit 1.

Topics/Concepts

- Divide whole numbers by fractions
- Divide fractions by fractions
- Divide multi digit numbers
- Use ratio and rate reasoning to solve unit price, constant speed, percent, proportions, and use conversion of measurements
- Use ratio and reasoning to solve real-world and mathematical problems
- Understand the difference between ratio, rate, and unit rate
- Add, subtract, multiply, and divide multi-digit decimals
- Add, subtract, and multiply fractions
- Find the GCF of two whole numbers
- Find the LCM of two whole numbers

Web Resources for spiral review (suggested):
Khan Academy

SAC Resources:
www.mymathtest.com (Level 1 or 2)
Level 1 Practice Placement Test Codes (SAC) --WSSMMT-GIBLI-BRAYS-BURAN-PIZZA-WISES
Level 1 Practice Test Testing Website (SAC)

Teacher Notes:

Unit 2: Descriptive Statistics *(6 weeks 9/21-10/30)*

Big Idea(s)	Data can be represented visually using tables, charts, and graphs. There are special numerical measures that describe the center and spread of numerical data sets.

Essential Questions	Performance Task	Problem of the Month
• How do we organize, display, and describe data? • How can frequency tables help us to find trends in real life scenarios? • How does mean, median, and mode describe data? • How can percentiles be used when comparing an individual to the norm? • How does exploratory data analysis help us to better understand our data?	• Snakes C1 2003 p.18-19 • Population C1 2004 p.19-20 • Media Surfing C1 2012 p.4-5 • Best Buy 8th Grade 2012 p.2-3	Pick a Pocket with Teacher Notes

Unit Topics/Concepts	Content Standards	Resources
• Compare center and spread of two or more data sets • Use knowledge of functions to fit models to quantitative data • Summarize, represent, and interpret data on a single count or measurement variable • Interpret differences in shape, center, and spread including effects of outliers • Use shape, center, and spread of comparable data to decide on appropriate statistical measures and justify choice through statistical reasoning • Closely examine the story that data and computed statistics are trying to tell • Construct a frequency distribution including limits, midpoints, relative and cumulative frequencies and boundaries • Construct frequency histograms, polygons, and relative frequency histograms • Graph and interpret quantitative data sets using a variety of graphs • Measures of central Tendency: Mean, Mode, Median and Range of Population and Sample • Variance and Standard deviation of population and sample. • Understand how to interpret Fractiles (percentiles) • Find z-score **Practice Placement Exam-** *(See information on next page)*	**ID.A.1** Represent data with plots on the real number line (dot plots, histograms, and box plots). **ID.A.2** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. **ID.A.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). **CP.A.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.	**Essential Resource:** Textbook- *Elementary Statistics,* Chapter 2 **Additional Resource:** Chapter 2 Supplements- • Case Study: Earning of Athletes • Activity 2.3 • Real Statistics-Real Decisions • Technology Activity: Monthly Milk Production **Data Collection:** • Census At School: www.amstat.org/censusatschool/ **Technology :** • TI-84 Mini Tab • Excel spreadsheets • Discovery Education: High Stakes World of Statistics Series (Video Lessons) www.discoveryeducation.com

Unit 2: Descriptive Statistics *(Instructional Support & Strategies)*

Framework Description/Rationale

The standards of the Probability and Statistics conceptual category are all considered modeling standards, providing a rich ground for studying the content of this course through real-world applications. In this unit, the first set of standards deals with interpreting data, and while students have already encountered standards ID. 1-3, they can be provided opportunities to refine their ability to represent data and apply their understanding to the world around them.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner
• Frequency • Frequency distribution • Classes • Intervals • Relative frequency • Cumulative frequency • Histogram • Ogive • Mean • Mode • Median • Range • IQR • Standard Deviation • Variance • Fractiles • Z-score	• Thinking Maps • Sentence frames • Gallery Walk • Technology • Rubrics • Co-operative groups • Presentations • Random number table	

Placement Exam Review

This review should be based around the needs of the students based off of the results on the first practice placement exam. However, a list of topics has been listed as a recommendation that would be covered during Unit 2. Students should retake the practice placement exam in Unit 3 to show student growth and to gather more data on how the teacher can differentiate the algebraic reviews in the course. It is recommended that the review be administered in smaller chunks throughout the unit. One suggestion would be to use the review during 10-15 minute warm ups.

Topics/Concepts

- Solving one variable equations
- Understand slope as a rate of change
- Graph linear equations
- Solve systems of equations graphically (parallel, intersecting, or neither)
- Solve systems of equations algebraically

Web Resources for spiral review (suggested):
Khan Academy

Teacher Notes:

Unit 3: Conditional Probability and the Multiplication Rule

(5 weeks 11/2-12/4)

Big Idea(s)	The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.

Essential Questions	Performance Task	Problem of the Month
• How can large numbers based on a pattern be efficiently calculated to form probabilities? • How can you model a simulation to represent a real life situation? • How does theoretical probability relate to empirical probability? • How do mutually exclusive events affect probability calculations?	• Discs C2 2002 p.1 • Math Team C3 2013 p.6-7 • Dropping Cups C2 2000 p.1 • A Random Choice C3 2012 p.4-5 • Marble Game G8 2009 p.68-69 • Flora, Freddie, Future • Playoff Party G7 2014 p.10-11	Fair Games and Teacher's Notes

Unit Topics/Concepts	Content Standards	Resources
• Know how to collect the data and analyze that data in order to make predictions based on the subject of probability of events • Describe Events and Sample Spaces • Understand what makes two events independent, dependent, or mutually exclusive • Summarize independent and conditional probability in the context and state how to use the data in everyday life • Construct and interpret two-way frequency tables of data and decide if events are independent and approximate conditional probabilities • Determine the appropriate tools, such as the tree diagram, to find the probability of an event • Apply the complement of an event to find probabilities • Apply the Fundamental Counting Principle • Apply the Addition or Multiplication Rule to find probabilities in a model • Understand the difference between permutations and combinations to use them to compute probabilities of compound events **Practice Placement Exam-** *(See information on next page)*	**CP.A.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **CP.A.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent. **CP.A.3** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*. **CP.A.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. **CP.A.5** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. **CP.B.6** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model. **CP.B.7** Apply the Addition Rule, P (A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model. **CP.B.8** (+) Apply the general Multiplication Rule in a uniform probability model, P (A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. **CP.B.9** (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	**Essential Resource:** Textbook- *Elementary Statistics,* Chapter 3 **Additional Resources:** Chapter 3 Supplements- • Simulating the Stock Market • Simulations • Uses and Abuses • Real Statistics- Real Decisions • Technology Activity • Case Study: Probability and Parking Lot Strategies **Data Collection:** • Census At School: www.amstat.org/censusatschool/ **Technology:** • TI-84 • Excel spreadsheets • Discovery Education: High Stakes World of Statistics Series www.discoveryeducation.com

Unit 3: Conditional Probability and the Multiplication Rule

(Instructional Support & Strategies)

Framework Description/Rationale

In this unit, students will deepen their understanding of the rules of probability, especially when finding probabilities of compound events in standards S-CP.7-9. Students can generalize from simpler events exhibiting independence (such as rolling number cubes) to understand that independence is often used as a simplifying assumption in constructing theoretical probability models that approximate real situations. For example, suppose a school laboratory has two smoke alarms as a built-in redundancy for safety. One has probability of 0.4 of going off when steam (not smoke) is produced by running hot water and the other has probability 0.3 for the same event. The probability that they both go off the next time someone runs hot water in the sink can be reasonably approximated as the product 0.4×0.3=0.12, even though there may be some dependence between the two systems in the same room.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner (number of days)

• Sample Space
• Theoretical and Empirical Probability
• Fundamental Counting Principle
• Independent, Dependent, Complementary, and Mutually Exclusive
• Conditional Probability
• Certain
• Impossible
• Permutations and Combinations
• AND – OR statements
• Compound statements
	• Venn Diagrams
	• Thinking Maps (Tree Diagrams)
	• Pie Charts
	• 2-way tables
	• Manipulatives:
	  ▪ Dice
	  ▪ Coins
	  ▪ Playing cards
	  ▪ Spinner
	  ▪ Marbles	

Placement Exam Review

Students should be retaking the practice placement exam during this unit. It is recommended that students retake the exam at the end of the unit to provide more exposure to the review topics to optimize student growth from the first practice placement exam taken in Unit 1. As a suggestion, the teacher should have students compare their first results to the second, focusing on growth in the various algebraic areas. It is recommended that the review during the unit be administered in smaller chunks throughout the unit. One suggestion would be to use the review during 10-15 minute warm ups. Below is a recommendation of topics and concepts that would be covered during Unit 3.

Topics/Concepts

- Discover the exponential rules
- Apply the exponential rules to simplify expressions and equations
- Solving using square roots
- Simplifying radicals
- Working with rational numbers
- Solving inequalities algebraically and graphically
- Graphing absolute value
- Solving equations with absolute value

Web Resources for spiral review (suggested):
Khan Academy

SAC Resources:
www.mymathtest.com (Level 1 or 2)
Level 1 Practice Placement Test Codes (SAC) --WSSMMT-GIBLI-BRAYS-BURAN-PIZZA-WISES
Level 1 Practice Test Testing Website (SAC)

Teacher Notes:

Unit 4: Discrete Probability Distributions *(6 weeks 12/7-1/22)*

Big Idea(s)	There are special numerical measures that describe the center and spread of numerical data sets. The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.

Essential Questions	Performance Task	Problem of the Month
• What probability distribution patterns occur in real life situations? • How do you distinguish when to use the three distributions (poison, binomial, geometric)? • How do you apply your understanding of probability distribution to determine examples of it?	• Will it Happen? [7th Grade 2008] • Flora, Freddy, and the Future • Duck Game [7th Grade 2001] p.5-6 • Dice Game [7th Grade 2002] p.2 • Fair Game? [7th Grade 2003] p.48-49 • Counters [7th Grade 2004] p.46-47	DataSense and Teacher's Notes

Unit Topics/Concepts	Content Standards	Resources
• Distinguish between discrete and continuous random variables • Construct a discrete probability distribution and its graph • Determine and apply the type of distribution (poison, binomial, or geometric) based on the given model • Find the mean, variance and standard deviation of various distribution models • Interpret and compare strategies on the basis of the expected value of a discrete probability distribution • Determine if an experiment is a binomial experiment • Find binomial probabilities: formula, technology, and table • Graph a binomial distribution • Use probabilities to make fair decisions and analyze decisions and strategies **Practice Placement Exam-** *(See information on next page)*	**MD.A.1** (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. **MD.A.2** (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. **MD.A.3** (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **MD.A.4** (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **MD.B.5** (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. **MD.B.5.a** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **MD.B.5.b** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* **MD.B.6** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator) **MD.B.7** (+) Analyze decisions and strategies using probability concepts.	**Essential Resource:** Textbook- *Elementary Statistics*, Chapter 4 **Additional Resources:** Chapter 4 Supplements- • Binomial Distribution Activity • Uses and Abuses • Real Statistics – Real Decisions • Airplane Accidents **Data Collection:** • Census At School: www.amstat.org/censusatschool/ **Technology :** • TI-84 • Mini Tab • Excel spreadsheets • Discovery Education: High Stakes World of Statistics Series

Unit 4: More Discrete Probability Distributions *(Support & Strategies)*

Framework Description/Rationale

In this unit, the standards of the S-MD domain allow students the opportunity to apply concepts of probability to real-world situations. For example, a political pollster will want to know how many people are likely to vote for a particular candidate while a student may want to know the effectiveness of guessing on a true-false quiz. They begin to see the outcomes in such situations as *random variables*, functions of the outcomes of a random process, with associated probabilities attached to their possible values.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner (number of days)
• Discrete • Random variable • Continuous • Discrete distribution • Mean • Variance • Standard Deviation • Expected Value • Success • Failure • Binomial experiment • Binomial distribution • Trials • Poisson • Most likely • Least likely	• Modeling word problems • Graphic organizers • Foldables • Real life examples • Manipulatives • Marbles	

Placement Exam Review

To help prepare students, this review should be based around the needs of the students based off of the results on the practice placement exam from Unit 3. However, a list of topics has been listed for your convenience. It is recommended that the review be administered in smaller chunks throughout the unit. One suggestion would be to use the review during 10-15 minute warm ups during Unit 4. The teacher may choose to have students take the practice placement exam an additional time to give them one more exposure before taking the actual exam.

Topics/Concepts

- Understand the behaviors of quadratic function graphs (parabolas)
- Understand how factoring, the quadratic formula, and zeroes can be used to find x-intercepts of parabolas
- Applying algebraic principles to real life situations

Web Resources for spiral review (suggested):
Khan Academy

SAC Resources:
www.mymathtest.com (Level 1 or 2)
Level 1 Practice Placement Test Codes (SAC) --WSSMMT-GIBLI-BRAYS-BURAN-PIZZA-WISES
Level 1 Practice Test Testing Website (SAC)

Teacher Notes:

Unit 5: Normal Probability Distributions *(5 weeks 2/1-3/4)*

Big Idea	There are special numerical measures that describe the center and spread of numerical data sets.

Essential Questions	Performance Task	Problem of the Month
• How do all Normal distributions relate to each other? • How can we find examples of normal distribution in real world scenarios? • How does the z-score relate to the standard normal distribution? • How does the positional z-score relate to the percentile of the data?	• T-Shirts [5th Grade 2002] p.5-6 • Wintry Showers [5th Grade 2005] p.10-11 • Life of an Umbrella [5th Grade 2009] p.64 • Pencils [6th Grade 2000] p.1 • Baseball Players [6th Grade 2003] p.4 • Speech Speeds [6th Grade 2011] p.3 • World Sports Leagues [6th Grade 2013] p.4-5 • Supermarket [7th Grade 2000] p.4-5 • Ducklings [7th Grade 2005] p.14-15	Diminishing Return and Teacher's Notes

Unit Topics/Concepts	Content Standards	Resources
• Interpret graphs of normal distributions • Use the areas under the standard normal curve to establish probability • Determine the probability of normal distributions • Determine z-scores given the area under the curve • Transform a z-score to a x-value and interpret its meaning in the given context • Discover a data value given the probability • Compute Sample distributions and interpret their properties • Understand the Central Limit Theorem • Apply the Central Limit Theorem to find the probability of a sample mean • Decide when a normal distribution can approximate a binomial distribution • Apply the correction for continuity when dealing with discrete probability • Use normal distribution to approximate binomial probabilities **Practice Placement Exam-** *(See information on next page)*	**MD.A.3** (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* **MD.A.4** (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* **MD.B.5.a** Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* **MD.B.5.b** Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* **MD.B.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	**Essential Resource:** Textbook- *Elementary Statistics*, Chapter 5 **Additional Resources:** Chapter 5 Supplements- • Sampling distributions • Uses and Abuses • Putting it all together • Technology Activity • Birth weights in America **Data Collection:** • Census At School: www.amstat.org/censusatschool/ **Technology :** • TI-84 • Mini Tab • Excel spreadsheets • Youtube • Discovery Education: High Stakes World of Statistics Series (Video Lessons) www.discoveryeducation.com

Unit 5: Normal Probability Distributions *(Support & Strategies)*

Framework Description/Rationale

In this unit, the standards of the S-MD domain allow students the opportunity to apply concepts of probability to real-world situations. For example, a political pollster will want to know how many people are likely to vote for a particular candidate while a student may want to know the effectiveness of guessing on a true-false quiz. They begin to see the outcomes in such situations as *random variables*, functions of the outcomes of a random process, with associated probabilities attached to their possible values.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner (number of days)
• Continuous Random variable • Continuous Probability distribution • Probability density • Standard normal • Z-score • X-values • Sampling distribution • Standard error of the mean • Central Limit theorem • Approximations • Point of inflection	• Distribution tables • Modeling with tables, graphs, calculator • Real world applications	

Placement Exam Review

Students will be participating in a pull-out day where they will be taking the Math Placement Exam at SAC. These dates are varied by school site. This review should be based around the needs of the students based off of the results on the practice placement exam. However, a list of topics has been listed for your convenience in Unit 5. It is recommended that the review be administered in smaller chunks throughout the unit. One suggestion would be to use the review during 10-15 minute warm ups.

Topics/Concepts

- Understand true and false logic statements
- Discover the Pythagorean Theorem
- Understand area and perimeter formulas for rectangles, triangles, and circles

Web Resources for spiral review (suggested):
Khan Academy

Teacher Notes:

Unit 6: Correlation and Regression *(3 weeks 3/7-4/15)*

Big Idea(s)	Data can be represented visually using tables, charts, and graphs. There are special numerical measures that describe the center and spread of numerical data sets.

Essential Questions	Performance Task	Problem of the Month
• How do you describe the relationship between two variables? • Can accurate predictions be made with extrapolated data? • How can correlations be used to make predictions? • Can a strong correlation imply causation?	• House Prices [Algebra 2007] p.46-47 • Scatter Diagrams C1 2005 p.14-15 • Bird's Egg C2 2004 p.28-29	

Unit Topics/Concepts	Content Standards	Resources
• Analyze bivariate data • Summarize and interpret data for two categories and recognize associations and trends in the data • Represent data on two quantitative variables and describe how the variables are related • Interpret the parameters of a linear model in the context of the data that it represents, including line of best fit, correlation coefficient, slope and intercept • Distinguish between correlation and causation • Understand linear correlation • Understand how the Independent and Dependent variables are represented in correlation graphs • Determine the Correlation coefficient • Establish if the correlation is strong or weak by examining the correlation coefficient • Determine the slope of the equation of a regression line • Predict y-values (interpolation and extrapolation) using the regression equation	**ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use the given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association. **ID.C.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. **ID.C.8** Compute (using technology) and interpret the correlation coefficient of a linear fit. **ID.C.9** Distinguish between correlation and causation. **IC.A.1**Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**Essential Resource:** Textbook- *Elementary Statistics*, Chapter 9 **Additional Resources:** Chapter 9 Supplements- • Regression by Eye • Case Study: Correlation of Body Measurements **Data Collection:** • Census At School: www.amstat.org/censusatschool/ **Technology :** • TI-84 • Mini Tab • Excel spreadsheets • Khan Academy • Discovery Education: High Stakes World of Statistics Series (Video Lessons) www.discoveryeducation.com

Unit 6: Correlation and Regression *(Instructional Support & Strategies)*

Framework Description/Rationale

In this unit, students understand that the process of fitting and interpreting models for discovering possible relationships between variables requires insight, good judgment and a careful look at a variety of options consistent with the questions being asked in the investigation. Students work more with the correlation coefficient, which measures the "tightness" data points about a line fitted to the data. Students understand that when the correlation coefficient is close to 1 or −1, the two variables are said to be highly correlated, and that high correlation does not imply causation.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner (number of days)
• Correlation coefficient • Dependent • Independent • Causation • Regression Line • Coefficient of Determination	• Modeling scatter plots • Modeling/graphing linear equations • Real world applications	• Linear equations • Plotting points on coordinate plane

Teacher Notes:

Unit 7: Confidence Intervals *(2 Weeks 4/18-5/6)*

Big Idea(s)	Data can be represented visually using tables, charts, and graphs. There are special numerical measures that describe the center and spread of numerical data sets.

Essential Questions	Performance Task	Problem of the Month
• How do you extend the idea of estimating a parameter to allow for uncertainty? • How does sample size raise the confidence level for the true mean?	Review: • Heart Beat [Algebra 1999] p.7-8 • Rope [Algebra 2000] p.7 • Airplanes [Algebra 2001] p.5-6	

Unit Topics/Concepts	Content Standards	Resources
• Find the point estimate and margin of error in a given scenario • Construct and interpret confidence intervals for the population mean • Determine minimum sample size requirements when estimating mean, μ • Interpret the t-distribution and use t-distribution table in real life scenarios • Construct confidence intervals when the sample size, n, is less than 30, population is normally distributed, and standard deviation, σ, is unknown	*The contents of this unit are not addressed in the new state standards. The purpose of covering these materials is to support students in future college courses.*	**Essential Resource:** Textbook- Elementary Statistics, Chapter 6 **Additional Resources:** Chapter 6 Supplements- • Shoulder Heights of Appalachian Black Bears • Confidence Intervals for a Mean **Data Collection:** • Census At School: www.amstat.org/censusatschool/ • Discovery Education: High Stakes World of Statistics Series (Video Lessons) www.discoveryeducation.com

Unit 7: Confidence Intervals *(Instructional Support & Strategies)*

Framework Description/Rationale

To support them in future math pathways, students will be getting further exposure to content that will be presented in the college level equivalent course for Probability and Statistics. Students will be determining confidence intervals to begin to hypothesize if a large enough sample size has been taken to closely reflect the true mean of the population.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner (number of days)
• Point Estimate • Interval Estimate • Level Of Confidence • Margin of Error • Confidence Interval • Minimum Sample size • T-distribution • Degrees of freedom	• Graphing • Modeling with technology	

Teacher Notes:

Unit 8: Hypothesis Testing with One Sample (*2 Weeks 5/9-5/27)*

Big Idea(s)	The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.

Essential Questions	Performance Task	Problem of the Month
• How do you use statistical ideas to test assumptions about data?	• Snakes [Algebra 2003] p.18-19 • Population [Algebra 2004]	

Unit Topics/Concepts	Content Standards	Resources
• Understand how to interpret a hypothesis tests • State a null hypothesis and an alternative hypothesis • Identify Type I and Type II errors and interpret the level of significance • Use One-tailed and two-tailed statistical tests to find p-value • Make and interpret decisions on comparing two hypotheses based on results of a statistical test • Write a claim for a hypothesis test	*The contents of this unit are not addressed in the new state standards. The purpose of covering these materials is to support students in future college courses.*	**Essential Resource:** Textbook- *Elementary Statistics*, Chapter 7 **Additional Resources:** Chapter 7 Supplements- • Use and Abuses • Exercise 7.1 **Data Collection:** • Census At School: www.amstat.org/censusatschool/ • Discovery Education: High Stakes World of Statistics Series (Video Lessons) www.discoveryeducation.com

Unit 8: Hypothesis Testing with One Sample *(Support & Strategies)*

Framework Description/Rationale

To support them in future math pathways, students will be getting further exposure to content that will be presented in the college level equivalent course for Probability and Statistics. Students will be determining confidence intervals to begin to hypothesize if a large enough sample size has been taken to closely reflect the true mean of the population. Students will be classifying the type of errors that can occur during experiments. Students will be able to make decisions on the hypothesis based on their own analysis of the data. Students will also be exposed to the level of significance and how this effects a decision to accept or deny a hypothesis.

(See CCSS for CA Mathematics Framework for Probability and Statistics for more details)

Academic Language Support	Instructional Tool/Strategy Examples	Pre-Unit: Preparing the Learner (number of days)
• Hypothesis test • Null and Alternate hypothesis • Type I error • Type II error • Level of Significance • Left tailed test • Right tailed test • Two tailed test	• Graphing • Tables • Modeling Hypothesis testing	

Teacher Notes: