Algebra 2

These curriculum maps are designed to address Common Core State Standards (CCSS) Mathematics and Literacy outcomes. The overarching focus for all curriculum maps is building students’ content knowledge focusing on their math practice abilities and literacy skills. Each unit provides several weeks of instruction. Each unit also includes various assessments. Taken as a whole, this curriculum map is designed to give teachers recommendations and some concrete strategies to address the shifts required by CCSS.

**Instructional Shifts in Mathematics**

<table>
<thead>
<tr>
<th>Focus</th>
<th>Focus requires that we significantly narrow and deepen the scope of content in each grade so that students experience concepts at a deeper level.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Instruction engages students through cross-curricular concepts and application. Each unit focuses on implementation of the Math Practices in conjunction with math content.</td>
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<td>• Effective instruction is framed by performance tasks that engage students and promote inquiry. The tasks are sequenced around a topic leading to the big idea and essential questions in order to provide a clear and explicit purpose for instruction.</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Coherence</th>
<th>Coherence in our instruction supports students to make connections within and across grade levels.</th>
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<tbody>
<tr>
<td></td>
<td>• Problems and activities connect clusters and domains through the art of questioning.</td>
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<td></td>
<td>• A purposeful sequence of lessons build meaning by moving from concrete to abstract, with new learning built upon prior knowledge and connections made to previous learning.</td>
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<td>• Coherence promotes mathematical sense making. It is critical to think across grades and examine the progressions in the standards to ensure the development of major topics over time. The emphasis on problem solving, reasoning and proof, communication, representation, and connections require students to build comprehension of mathematical concepts, procedural fluency, and productive disposition.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Rigor</th>
<th>Rigor helps students to read various depths of knowledge by balancing conceptual understanding, procedural skills and fluency, and real-world applications with equal intensity.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Conceptual understanding underpins fluency; fluency is practiced in contextual applications; and applications build conceptual understanding.</td>
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<td></td>
<td>• These elements may be explicitly addressed separately or at other times combined. Students demonstrate deep conceptual understanding of core math concepts by applying them in new situations, as well as writing and speaking about their understanding. Students will make meaning of content outside of math by applying math concepts to real-world situations.</td>
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<td>• Each unit contains a balance of challenging, multiple-step problems to teach new mathematics, and exercises to practice mathematical skills</td>
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</table>
## 8 Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. They describe how students should learn the content standards, helping them to build agency in math and become college and career ready. The Standards for Mathematical Practice are interwoven into every unit. Individual lessons may focus on one or more of the Math Practices, but every unit must include all eight.

<p>| 1. Make sense of problems and persevere in solving them | Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. |
| 2. Reason Abstractly and quantitatively | Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. |
| 3. Construct viable arguments and critique the reasoning of others | Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |</p>
<table>
<thead>
<tr>
<th>4. Model with mathematics</th>
<th>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</th>
</tr>
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<tbody>
<tr>
<td>5. Use appropriate tools strategically</td>
<td>Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</td>
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<tr>
<td>6. Attend to precision</td>
<td>Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.</td>
</tr>
<tr>
<td>7. Look for and make use of structure</td>
<td>Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.</td>
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<tr>
<td>8. Look for and express regularity in repeated reasoning</td>
<td>Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1, 2)$ with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.</td>
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</table>
English Language Development Standards

The California English Language Development Standards (CA ELD Standards) describe the key knowledge, skills, and abilities in core areas of English language development that students learning English as a new language need in order to access, engage with, and achieve in grade-level academic content, with particular alignment to the key knowledge, skills, and abilities for achieving college- and career-readiness. English Learners must have full access to high quality English language arts, mathematics, science, and social studies content, as well as other subjects, at the same time as they are progressing through the ELD level continuum. The CA ELD Standards are intended to support this dual endeavor by providing fewer, clearer, and higher standards. The ELD Standards are interwoven into every unit.

<table>
<thead>
<tr>
<th>Interacting in Meaningful Ways</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Collaborative (engagement in dialogue with others)</td>
</tr>
<tr>
<td>1. Exchanging information/ideas via oral communication and conversations</td>
</tr>
<tr>
<td>B. Interpretive (comprehension and analysis of written and spoken texts)</td>
</tr>
<tr>
<td>5. Listening actively and asking/answering questions about what was heard</td>
</tr>
<tr>
<td>8. Analyzing how writers use vocabulary and other language resources</td>
</tr>
<tr>
<td>C. Productive (creation of oral presentations and written texts)</td>
</tr>
<tr>
<td>9. Expressing information and ideas in oral presentations</td>
</tr>
<tr>
<td>11. Supporting opinions or justifying arguments and evaluating others’ opinions or arguments</td>
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</tbody>
</table>
How to Read this Document

- The purpose of this document is to provide an overview of the progression of units of study within a particular grade level and subject describing what students will achieve by the end of the year. The work of **Big Ideas and Essential Questions** is to provide an overarching understanding of the mathematics structure that builds a foundation to support the rigor of subsequent grade levels. The **Performance Task** will assess student learning via complex mathematical situations. Each unit incorporates components of the **SAUSD Theoretical Framework** and the philosophy of **Quality Teaching for English Learners (QTEL)**. Each of the math units of study highlights the Common Core instructional shifts for mathematics of focus, coherence, and rigor.

- The **8 Standards for Mathematical Practice** are the key shifts in the pedagogy of the classroom. These 8 practices are to be interwoven throughout every lesson and taken into consideration during planning. These, along with the **ELD Standards**, are to be foundational to daily practice.

- First, read the **Framework Description/Rationale** paragraph, as well as the **Common Core State Standards**. This describes the purpose for the unit and the connections with previous and subsequent units.

- The units show the progression of units drawn from various domains.

- The timeline tells the length of each unit and when each unit should begin and end.
# SAUSD Scope and Sequence for Algebra 2

### **Unit 1**
- **9/1/15-10/9/15**
- **6 Weeks**
- Linear, & Piecewise-Defined Functions

### **Unit 2**
- **10/12/15-11/20/15**
- **6 Weeks**
- Polynomial Functions Including Quadratic and Cubic Functions

### **Unit 3**
- **11/30/15-1/22/16**
- **6 Weeks**
- Exponential and Logarithmic Functions

**Semester 1**

### **Unit 4**
- **2/1/16-3/11/16**
- **6 Weeks**
- Radical and Rational Functions

### **Unit 5**
- **3/14/16-4/1/16**
- **3 Weeks**
- Modeling and Function Connections

### **Unit 6**
- **4/11/16-5/13/16**
- **5 Weeks**
- Trigonometric Functions
  - Note: and enrich materials
  - Transformation preparing learner

### **Unit 7**
- **5/16/16-6/3/16**
- **3 Weeks**
- Probability and Statistics

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**Semester 2**

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Algebra 2 Overview:

The purpose of this course is to extend students’ understanding of functions and the real numbers, and to increase the tools students have for modeling the real world. They extend their notion of number to include complex numbers and see how the introduction of this set of numbers yields the solutions of polynomial equations and the Fundamental Theorem of Algebra. Students deepen their understanding of the concept of function, and apply equation-solving and function concepts to many different types of functions. The system of polynomial functions, analogous to the integers, is extended to the field of rational functions, which is analogous to the rational numbers. Students explore the relationship between exponential functions and their inverses, the logarithmic functions. Trigonometric functions are extended to all real numbers, and their graphs and properties are studied. Finally, students’ statistics knowledge is extended to understanding the normal distribution, and they are challenged to make inferences based on sampling, experiments, and observational studies.

Building on their work with linear, quadratic, and exponential functions, in Algebra II students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Based on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students now use the coordinate plane to extend trigonometry to model periodic phenomena. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying function. They identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

(From the CA Mathematics Framework for Algebra 2)
### Unit 1: Linear & Piecewise-Defined Functions

#### (6 weeks 9/1 - 10/9)

**Big Idea**  
Linear functions can mathematically represent real-life situations and can be extended to create new functions.

<table>
<thead>
<tr>
<th>Essential Questions</th>
<th>Performance Task</th>
<th>Problem of the Month</th>
<th>Resources</th>
</tr>
</thead>
</table>
| - How can functions be represented in multiple ways?  
- How can they be useful in modeling given situations?  
- How do various functions compare to each other?  
- How can new functions be created from linear functions?  
- How can linear functions be used in real-life situations?  
- How do transformations affect the parent linear function? | **Patchwork Quilt** C1 2006 p.30-31  
**Swimming Race** C1 1999 p.1  
**Speed, Distance, Time** C1 2000 p.1  
**Toothpick Square** C1 2002 p.7-8  
**Number Machine** C1 2002 p.9-10 | • **On Balance** and Teacher's Notes |

**Unit Topics/Concepts**

<table>
<thead>
<tr>
<th>Linear Functions</th>
<th>Content Standards</th>
<th>Essential Resource:</th>
</tr>
</thead>
</table>
| - Determine the relationship between data and the corresponding function  
- Recognize key features of graphs and tables including: intercepts; intervals where the function is increasing/decreasing, positive/negative; relative maximum/minimum, symmetries; end behavior  
- Identify slope in terms of Rates of Change  
- Graph in terms of transformations; effect of $k$ on transforming a function  
- Solving equations, including rearranging formulas  
- Create equations  
- Model using real-life contexts  
- Identify domain  
- Use technology to graph and find intercepts  
- Represent constraints by equations or inequalities, and systems and interpret solutions | A-SSE.1a Interpret expressions that represent a quantity in terms of its context. Interpret parts of an expression, such as terms, factors, and coefficients.  
A-CED.1 Create equations and inequalities in one variable, including ones with absolute value, and use them to solve problems.  
A-CED.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.  
A-CED.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.  
A-CED.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. | • Algebra II Mathematics Framework  
• Textbook: 1.2, 1.3, 1.5, 2.1-2.6  
• Real World Application: Ch. 2 Project  
• **Geogebra** algebra/geometry modeling software  
• MARS – Building and Solving Equations  
• MARS – Algebraic Expressions  
• Dan Meyer 3-act videos (list and interactive link to Dan Meyer’s videos by standard) |

<table>
<thead>
<tr>
<th>Piecewise-Defined Functions</th>
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</thead>
</table>
| - Graph, including step and absolute value  
- Identify key features of graphs: intercepts, max and min, symmetry  
- Identify domain | | |

**Optional Review Topics:**

**Systems of Equations:**

Systems are now covered in Math 8 and Algebra 1. As we transition into the new standards, as an optional review, a list of topics has been included below to revisit this topic.

- Solve systems Exactly (Algebraically) and approximately (e.g. with graphs)
- Represent and understand systems of equations with graphs, tables, rules, and situations.
- Understand the meaning of a solution/point of intersection as it relates to a situation.
- Solve with Elimination/ Linear Combination
- Solve with Substitution

<table>
<thead>
<tr>
<th>F-IF.5</th>
<th>Relate the domain of a function to its graph.</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-IF.6</td>
<td>Calculate and interpret the average rate of change of a function over a specified interval. Estimate the rate of change from a graph.</td>
</tr>
<tr>
<td>F-IF.7b</td>
<td>Graph piecewise-defined functions, including step functions and absolute value functions.</td>
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</tbody>
</table>
Unit 1: Linear & Piecewise-Defined Functions  
(Support & Strategies)

Framework Description/Rationale
The purpose of this unit is to extend students’ understanding of functions and the real numbers, and to increase the tools students have for modeling the real world. Students deepen their understanding of the concept of function, and apply equation solving, and function concepts to linear functions. They explore the effects of transformations on graphs of linear functions in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of underlying function (See CCSS CA Algebra 2 Framework for more details).

<table>
<thead>
<tr>
<th>Academic Language Support</th>
<th>Instructional Tool/Strategy Examples</th>
<th>Preparing the Learner</th>
</tr>
</thead>
</table>
| Key Terms for Word Bank:  | Work to develop ways of thinking that are general and allow students to approach the work with any function. Have students first create a T-chart that will lead to a graph, to build on their understanding of input and output. Activity examples:  
- Silent board game: Given x and y values in a T-chart, students must fill in remaining values and as a final result, student determine the resultant functions  
- Matching equations to and expressions to a real-life context, and be able to connect the context to questions and answers |  
- Sets of numbers  
- Properties of Real Numbers  
- Simplify Algebraic Expressions  
- Relations and Functions  
- Function notation and evaluating functions |

Teacher Notes:
### Unit 2: Polynomial Functions Including Quadratic and Cubic Functions (6 weeks 10/12-11/20)

**Big Idea**
Relationships can be defined how one member is relates to another member by the rules in which it functions in a given situation.

<table>
<thead>
<tr>
<th>Essential Questions</th>
<th>Performance Task</th>
<th>Problem of the Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>• What key features of higher degree polynomial function distinguish it from those of linear functions?</td>
<td>• Patchwork C2 2000 p.3-4</td>
<td>• Miles of Tiles and Teacher’s Notes</td>
</tr>
<tr>
<td>• How transformations of polynomial functions are related their parent function?</td>
<td>• One less than a Square C2 2001 p.9-10</td>
<td></td>
</tr>
<tr>
<td>• How can you find a solution to a polynomial equation algebraically and graphically?</td>
<td>• Painted Cubes C2 2002 p.8-10</td>
<td></td>
</tr>
<tr>
<td>• How can features of polynomial functions such as the equation, solutions, axis of symmetry, vertex, etc. be represented in tables, equations, and in “real world” contexts?</td>
<td>• Number Patterns C2 2003 p.5-6</td>
<td></td>
</tr>
<tr>
<td>• How do zeros and imaginary numbers represent solutions to polynomial equations?</td>
<td>• Sum of Two Squares C2 2005 p.10-11</td>
<td></td>
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<tr>
<td>• Identify rates of change</td>
<td>• Functions C1 2008 p.77-78</td>
<td></td>
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</tbody>
</table>

**Unit Topics/Concepts**
- Connect key features of graphs and tables including: intercepts; intervals where the function is increasing/decreasing, positive/negative; relative maximum/minimum, symmetries; end behavior
- Identify domain in the appropriate context
- Identify arithmetic effects and transformations on graphs by specific values of k such as f(x)+k, etc.
- Identify rates of change
- Understand the relationships between zeros and factors and graphs of polynomials
- Graph quadratic functions in standard/vertex forms
- Solve quadratics by completing the square, using the quadratic formula, and using the zero product property
- Introduce the inverse and find the inverses of quadratics
- Perform arithmetic operations on polynomials
- Find zeros, factors, and imaginary solutions to polynomial functions

**Content Standards**

**A-APR.1** Understand that polynomials form a closed system under addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**A-APR.2** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).

**A-APR.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**A-APR.4** Use polynomial identities to solve problems. Prove and use polynomial identities to describe numerical relationships. E.g., \((x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2\) can be used to generate Pythagorean triples.

**N-CN.1** Know there is a complex number \(i\) such that \(i^2 = -1\), and every complex number has the form \(a + bi\) with \(a\) and \(b\) real.

**N-CN.2** Use the relation \(i^2 = -1\) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

**N-CN.7** Solve quadratic equations with real coefficients and complex solutions.

**A-SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity. E.g., interpret \(P(1 + r)^n\) as the product of \(P\) and a factor not depending on \(P\).

**A-SSE.2** Use the structure of an expression to identify ways to rewrite it. E.g., \(see x^2 - y^2 as (x^2 - y^2)^2 - (y^2)^2\), thus recognizing it as a difference of squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\).

**F-IF.4** Interpret key features of graphs and tables in terms

**Resources**
- **Essential Resources:**
  - Algebra II Mathematics Framework
  - Resource Packet
  - Textbook: 1.2, 1.3, 1.5, 2.1-2.6, 10.1-10.3
  - Engage NY: Polynomial, Rational, and Radical Relationships
  - Real World Application: Ch. 5 & Ch. 6 Project

- **Additional Resources:**
  - Geogebra algebra/geometry modeling software
  - Khan Academy Complex Numbers
  - MARS - Solving Quadratic Equations
  - MARS – Forming Quadratic
  - MARS –
<table>
<thead>
<tr>
<th>Understand Remainder Theorem</th>
<th>Representing Polynomials</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prove polynomial identities to describe numerical relationships</td>
<td>Mars- Manipulating Polynomials</td>
</tr>
<tr>
<td>Operations on complex in the form a + bi</td>
<td>Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard)</td>
</tr>
<tr>
<td>Interpret complicated expressions by viewing their parts</td>
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<tr>
<td>Use structure of known expressions of the quantities, and sketch graphs showing key features, such as intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior, given a verbal description of the relationship.</td>
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<tr>
<td>F-IF.5 Relate the domain of a function to its graph and to the quantitative relationship it describes. E.g., if the function h(n) gives the number of hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</td>
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<tr>
<td>F-IF.6 Calculate, interprets, and/or estimate the average rate of change of a function (presented symbolically or as a table) over a specified interval or from a graph.</td>
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<tr>
<td>F-IF.7c Analyze functions using different representations. Graph functions by factoring, identifying zeros, and showing end behavior.</td>
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<tr>
<td>F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</td>
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<tr>
<td>F-IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</td>
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<tr>
<td>F-BF.1b Build a function that models a relationship between 2 quantities. Combine standard function types using arithmetic operations. E.g., build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</td>
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<tr>
<td>F-BF.3 Build new functions from existing functions Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.</td>
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<tr>
<td>G-GPE.3.1 - Given a quadratic equation of the form ax^2 + by^2 + cx + dy + e = 0, use the method for completing the square to put the equation into standard form; identify whether the graph of the equation is a circle, ellipse, parabola, or hyperbola, and graph the equation.</td>
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</table>
Unit 2: Polynomial Functions Including Quadratic and Cubic Functions (Instructional Support & Strategies)

Framework Description/Rationale

The purpose of this unit is to extend students’ understanding of functions and the real numbers, and to increase the tools students have for modeling the real world. This unit is lengthier in order to include key features of quadratic and cubic functions as well as other polynomial functions. Students should understand that quadratic functions are inclusive in the set of polynomial functions and should not be taught in isolation of other degree functions.

Students extend their notion of number to include complex numbers and see how the introduction of this set of numbers yields the solutions of polynomial equations and the Fundamental Theorem of Algebra. Students will explore solutions to polynomial functions, cases where there is no real solution, and how they relate to complex numbers.

Students deepen their understanding of the concept of function, and apply equation-solving and function concepts to many different types of functions.

Graphs help us reason about rates of change of functions (F.IF.6). Students learned in Grade 8 that the rate of change of a linear function is equal to the slope of its graph. And because the slope of a line is constant, the phrase “rate of change” is clear for linear functions. For nonlinear functions, however, rates of change are not constant, and so we talk about average rates of change over an interval.

Students can make good use of graphing software to investigate the effects of replacing a function \( f(x) \) by \( f(x) + k, \ k f(x), \ f(kx) \), and \( f(x + k) \) for different types of functions (MP.5). For example, starting with the simple quadratic function \( (x) = x^2 \), students see the relationship between these transformed functions and the vertex-form of a general quadratic, \( (x) = (x - h)^2 + k \). They understand the notion of a family of functions, and characterize such function families based on their properties. These ideas will be explored further with trigonometric functions (F-TF.5).

In F-BF.4a, students learn that some functions have the property that an input can be recovered from a given output, i.e., the equation \( x = c \) can be solved for \( x \) given that \( c \) lies in the range of \( f \). They understand that this is an attempt to “undo” the function, or to “go backwards.” Tables and graphs should be used to support student understanding here. This standard dovetails nicely with standard F-LE.4 and should be taught in progression with it. Students will work more formally with inverse functions in advanced mathematics courses, and so this standard should be treated carefully as preparation for a deeper understanding. They will later discover the inverse relationship with polynomial/radical and logarithmic/exponential functions.

Students use what they learned about polynomial functions to examine curves represented by the equation \( ax^2 + by^2 + cx + dy + e = 0 \). They use complete the square to determine whether the equation represents a circle or parabola. They graph the shapes and relate them to their equation. They will explore ellipses and hyperbolas in later units.

(See CCSS CA Algebra 2 Framework for more details)

<table>
<thead>
<tr>
<th>Key Terms for Word Bank</th>
<th>Activity examples:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic</td>
<td>Pattern growth activity</td>
</tr>
<tr>
<td>Polynomial</td>
<td>Showing the change resulting from a linear or quadratic equation A group of images is presented that changes slightly from one image to another (either increasing or decreasing in size)</td>
</tr>
<tr>
<td>Parabola</td>
<td>Students create T-chart to represent the change and start to recognize that this change can also determine different types of functions. They also represent them graphically and as a function, and they begin making connections between them.</td>
</tr>
<tr>
<td>Maximum</td>
<td>Collect class data, graph, and try to fit models or functions to the data</td>
</tr>
<tr>
<td>Minimum</td>
<td>Use software or graphing calculators to model transformations with functions to help students make connections between the various representations</td>
</tr>
<tr>
<td>Vertex</td>
<td>Use quadratics to help introduce polynomial identities. To help students perform arithmetic operations with complex numbers, have them treat i like they would any other variable once they understand what it means</td>
</tr>
<tr>
<td>Axis/line of symmetry</td>
<td>Perform partner work to discuss similarities and differences in polynomials by reviewing the various properties and Euclidean Algorithm</td>
</tr>
<tr>
<td>Root</td>
<td>Use a sequence of diagrams to create a pattern of terms, then sum the areas to create a sequence.</td>
</tr>
<tr>
<td>Zero</td>
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<tr>
<td>Intercept</td>
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<tr>
<td>Solution</td>
<td></td>
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<tr>
<td>Imaginary number</td>
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<tr>
<td>Complex number</td>
<td></td>
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<tr>
<td>Factoring</td>
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<tr>
<td>Inverse</td>
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</table>

Instructional Tool/Strategy Examples

<table>
<thead>
<tr>
<th>Preparing the Learner</th>
</tr>
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<tbody>
<tr>
<td>Using of tables to graph functions. A basic understanding of factoring. Students should have already had experience with solving and graphing linear equations and quadratics. They can then learn to apply rules and functions to polynomials as a whole. Students should see that linear and quadratics belong to the set of polynomials and are just special, more simplified cases of polynomials.</td>
</tr>
</tbody>
</table>

Teacher Notes:

(Activity examples continued)

- Pattern growth activity
- Showing the change resulting from a linear or quadratic equation A group of images is presented that changes slightly from one image to another (either increasing or decreasing in size)
- Students create T-chart to represent the change and start to recognize that this change can also determine different types of functions. They also represent them graphically and as a function, and they begin making connections between them.
- Collect class data, graph, and try to fit models or functions to the data
- Use software or graphing calculators to model transformations with functions to help students make connections between the various representations
- Use quadratics to help introduce polynomial identities. To help students perform arithmetic operations with complex numbers, have them treat i like they would any other variable once they understand what it means
- Perform partner work to discuss similarities and differences in polynomials by reviewing the various properties and Euclidean Algorithm
- Use a sequence of diagrams to create a pattern of terms, then sum the areas to create a sequence.)
## Unit 3: Exponential Functions and Logarithms

**Big Idea(s)**

Relationships can be defined how one member is relates to another member by its function in a given situation. Some instances have situations where certain relationships are always true. They can be represented both “real life” as well in (mathematically) logarithmically.

### Essential Questions

- How do you evaluate exponential functions for given values?
- How do you use transformations to sketch graphs of exponential and logarithmic functions?
- How do you solve exponential and logarithmic equations?
- How do you use exponential models so solve real-world problems?
- How do you change bases in logarithmic expressions?
- How do you use properties of logarithms to evaluate or rewrite expressions?

### Performance Task

- **Multiply Cells** C2 2005 p.6-7
- **Growth Rates** C3 2012 p.2-3
- **Shooting Rubberbands** C3 2013 p.2-3
- **Height by Age** C3 2014 p.4-5
- **Shrinking Shapes** C3 2012 p.8-9

### Problem of the Month

- **Double Down** and Teacher’s Notes

### Unit Topics/Concepts

- Key features of graphs and tables including: intercepts; intervals where the function is increasing/decreasing, positive/negative; relative maximum/minimum, symmetries; end behavior
- Graph exponential and logarithmic functions and identify the key features
- Identify the rate of change of exponential and logarithmic functions
- Perform transformations of exponential and logarithmic functions
- Utilize the laws of logarithms to rewrite and evaluate
- Continue with idea of inverse from the previous unit to understand logarithmic functions as being inverse functions of exponential functions

### Content Standards

**F-IF.4** Interpret functions that arise in applications in terms of the context. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include:** intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums or minimums; symmetries; end behavior; and periodicity.

**F-IF.7e** Analyze functions using different representations. Graph exponential and logarithmic functions, showing intercepts and end behavior.

**F-LE.4** Construct and compare linear, quadratic and exponential models and solve problems. For exponential models, express as a logarithm the solution to \( ab^{c+d} \) where \( a, c, \) and \( d \) are numbers and the base \( b \) is 2, 10, or \( e \); evaluate the logarithm using technology.

**F-LE.4.1** Construct and compare linear, quadratic and exponential models and solve problems. Prove simple laws of logarithms.

**F-LE.4.2** Construct and compare linear, quadratic and exponential models and solve problems. Use the definition of logarithms to translate between logarithms of any base.

**F-LE.4.3** Construct and compare linear, quadratic and exponential models and solve problems. Understand and use the properties of logarithms to simplify logarithmic numeric expressions and to identify their approximate values.

### Resources

- **Essential Resources:**
  - Algebra II Mathematics Framework
  - Resource Packet
  - Textbook: 8.1-8.6
  - Real World Application: Ch. 8 Project

- **Additional Resources:**
  - Geogebra algebra/geometry modeling software
  - Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard)

### Optional Review Topic: Exponents Review

Exponents are covered in Math 8 and Algebra 1. As we transition into the new standards, as an optional review, a list of topics has been included below to revisit this topic.

- Understand, rewrite, use and explain negative and fractional exponential powers
Unit 3: Exponential Functions and Logarithms

(Instructional Support & Strategies)

Framework Description/Rationale

Students have worked with exponential models in Algebra I and further in Algebra II. Since the exponential function \( f(x) = b^x \) is always increasing or always decreasing for \( b \neq 0,1 \), we can deduce that this function has an inverse, called the logarithm to the base \( b \), denoted by \( g(x) = \log_b x \). The logarithm has the property that \( \log_b x = y \) if and only if \( b^y = x \), and arises in contexts where one wishes to solve an exponential equation. Students find logarithms with base \( b \) equal to 2, 10, or \( e \), by hand and using technology (MP.5). In F.LE.4.1-4.3, students explore the properties of logarithms, such as that \( \log_b xy = \log_b x + \log_b y \), and connect these properties to those of exponents (e.g., the previous property comes from the fact that the logarithm is representing an exponent, and that \( b^{n+m} = b^n \cdot b^m \)). Students solve problems involving exponential functions and logarithms and express their answers using logarithm notation (F-LE.4). In general, students understand logarithms as functions that undo their corresponding exponential functions; opportunities for instruction should emphasize this relationship.

(See CCSS CA Algebra 2 Framework for more details)

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Key Terms for Word Bank:</td>
<td>• Use real life examples to illustrate various types of growth</td>
<td></td>
</tr>
<tr>
<td>• Exponential</td>
<td>• Use multiple representations (graphs, tables, equations) to help students make connections to other functions</td>
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<tr>
<td>• Logarithmic</td>
<td>• Use software or calculators to illustrate inverses and logarithms to help students make connections</td>
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<tr>
<td>• Base</td>
<td>Inverses</td>
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<tr>
<td>• Exponent</td>
<td>Laws of exponents</td>
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<td>• Asymptote</td>
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<td>• Evaluate</td>
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<td>• Product</td>
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<td>• Quotient</td>
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<tr>
<td>• Power</td>
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</tbody>
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Teacher Notes:
# Unit 4: Radical and Rational Functions

**6 weeks 2/16-3/25**

## Big Idea
Situations can be represented in many ways whether they be mathematically or in "real life."

## Essential Questions
- What are the key features of the graphs of radical and rational functions?
- How can functions be manipulated to make new functions?

## Performance Task
- **Cubic Graph** C2 2011 p.6-7
- **Shooting a Jump Shot** C3 2014 p.10-11
- **Sorting Functions** C1 2008 p.69-70
- **Circles and Spheres** C1 2009 p.88-89

## Problem of the Month
- **Perfect Pair** and Teacher’s Notes

## Unit Topics/Concepts

### Key Features of Graphs: Intercepts
- Intervals (increasing, decreasing, positive, negative)
- Max and Min
- Symmetries
- End Behavior
- Domain and Range
- Discontinuities
- Calculate Average Rate of Change
- Solve Equations and identify extraneous solutions
- Rewrite Rational Expressions using Long Division
- Build New Functions from Old
- Graph in terms of transformations; effect of $k$ on transforming a parent function
- Representing a function in more than one way
- Graph square root and cube root functions
- Find inverse functions
- Explore the inverse relationship between radicals and exponential functions

### Content Standards
- **A-SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.
- **A-SSE.2** Use structures of an expression to identify ways to rewrite it.
- **A-CED.1** Create equations in one variable and use them to solve problems. Include simple rational functions
- **A-APR.6** Rewrite simple rational expressions in different forms: write $a(x) \div b(x)$ in the form $q(x) + \frac{r(x)}{b(x)}$, using inspection or long division
- **A-REL.2** Solve simple rational and radical equations in one variable, and give examples of how extraneous solutions may arise.
- **F-IF.4** Interpret key features of graphs and tables. Sketch graphs showing key features given a verbal description of the relationship between two quantities. Key features include: intercepts; intervals of increase; decrease, positive, or negative; max and min; symmetry; and end behavior.
- **F-IF.5** Relate the domain of a function to its graph.
- **F-IF.6** Calculate and interpret the average rate of change of a function over a specified interval.
- **F-BF.1b** Combine standard function types using arithmetic operations.
- **F-BF.3** Identify the effect on the graph by replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of $k$. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
- **F-BF.4** Find inverse functions
- **N-RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radical in terms of rational exponents.
- **N-RN.2** Rewrite expressions involving radicals and rational exponents using properties of exponents.

## Resources
- **Essential Resource:**
  - Algebra II Mathematics Framework
  - Unit of Study
  - Real World Application: Ch. 7 Project
  - Real World Application: Ch. 9 Project

- **Additional Resources:**
  - Refer to Engage NY from Unit 2
  - Geogebra algebra/geometry modeling software
  - Dan Meyer 3-act videos (list and interactive link to Dan Meyer’s videos by standard)
### Unit 4: Radical and Rational Functions

*(Instructional Support & Strategies)*

#### Framework Description/Rationale
Building on previous units and prior courses that explored linear equations and expressions, students will begin to explore radicals and rational functions. Student will learn the inverse relationship between radicals and exponential functions. This unit is a further exploration of polynomial functions with rational (fractional) as well as integer exponents. Students will be able to explore the features of radical and rational functions and compare their different functions by certain features such as end behavior, average rate of change, etc. Finally, students will be able to identify the appropriate function to model a situation as well as explore how transformations of functions relate to their parent function.

*(See CCSS CA Algebra 2 Framework for more details).*

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<tr>
<td><strong>Key Terms for Word Bank:</strong></td>
<td>• Use real life examples to illustrate when to apply the various functions</td>
<td></td>
</tr>
<tr>
<td>• Radical</td>
<td>• Use multiple representations (graphs, tables, equations) to help students make connections to other functions</td>
<td></td>
</tr>
<tr>
<td>• Ratio</td>
<td>• Use software or calculators to illustrate radical and rational functions to help students make connections between them as well as other functions</td>
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<tr>
<td>• Rational</td>
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<td>• Fractional</td>
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<tr>
<td>• Integer</td>
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<tr>
<td>• End behavior</td>
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<tr>
<td>• Rate of change</td>
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</tbody>
</table>

**Teacher Notes:**
### Unit 5: Modeling and Function Connections

**Big Idea**
How a system changes is directly related to the rules in which it functions. Similar families of systems behave in similar ways helping to identify certain qualities and key features making them useful as models in various situations.

### Essential Questions
- What are functions and what are their key features?
- How can functions be represented in multiple ways?
- How can functions be useful in modeling given situations?

### Performance Task
- **The Savings Plan** C3 2014 p.8-9
- **Maximum Revenue** C3 2013 p.8-9
- **The Rollarcoast Ride** C3 2012 p.6-7

### Problem of the Month
- **Friends You Can Count On** and **Teacher’s Notes**

### Unit Topics/Concepts
- Recognize key features of graphs and tables including: intercepts; intervals where the function is increasing/decreasing, positive/negative; relative maximum/minimum, symmetries; end behavior; and periodicity.
- Understand that functions represent quantitative change.
- Know the definition of a function.
- Identify the similarities and commonalities in “families” of functions. Graph and differentiate between a square root, cube root, step-function, absolute value, polynomial, exponential, and logarithmic function.
- Write and recognize a function and their properties in multiple yet equivalent forms.
- Identify patterns different representations, domain, and range of functions including:
  - Linear
  - Quadratic
  - Cubic
  - Square Root
  - Exponential
  - Even/Odd
- Create equations and inequalities in one and two variables and use them to solve problems including absolute value, quadratic,

### Content Standards
- **N.RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. E.g., we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^4$ must equal 5.
- **N.RN.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **A-SSE.1** Interpret expressions that represent a quantity in terms of its context.
  a. Interpret parts of an expression, such as terms, factors, and coefficients.
  b. Interpret complicated expressions by viewing one or more of their parts as a single entity. E.g., interpret $P(1 + r)^n$ as the product of $P$ and a factor not depending on $P$.
- **A-REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g. using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
- **F-IF.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **F.IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
  a. Graph square root, cube root, and piecewise-

### Resources
- **Essential Resources:**
  - **Algebra II Mathematics Framework**
  - **Unit of Study**
  - **Real World Application:** Textbook: Real World Applications:
    - **Real World Application:** Ch. 1 Project
    - **Real World Application:** Ch. 3 Project
    - **Real World Application:** Ch. 10 Project
- **MARS:**
  - **Functions and Everyday Situations**
  - **Equations of Circles 1**
  - **Equations of Circles 2**
  - **Ferris Wheel**
  - **Manipulating Radicals**
- **Additional Resources:**
  - **Geogebra**
  - **Dan Meyer 3-act videos** (list and interactive link to Dan Meyer’s videos by standard)

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<table>
<thead>
<tr>
<th>rational, and exponential functions</th>
<th>defined functions.</th>
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</thead>
<tbody>
<tr>
<td>Represent constraints by equations or inequalities, and systems and interpret solutions</td>
<td></td>
</tr>
<tr>
<td>Represent and solve graphically and explain the point of intersection are the solutions to the equations</td>
<td></td>
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<tr>
<td>Revisit systems of equations with advanced functions.</td>
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</table>

F.BF.1 Write a function that describes a relationship between two quantities.*

b. Combine standard functions types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to decaying exponential, and relate these functions to the model.

F.BF.3 Identifying the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
Students can make good use of graphing software to investigate the effects of replacing a function \( f(x) \) by \( f(x) + k \), \( kf(x) \), \( f(kx) \), and \( f(x + k) \) for different types of functions (MP.5). For example, starting with the simple quadratic function \( (x) = x^2 \), students see the relationship between these transformed functions and the vertex-form of a general quadratic, \( (x) = (x - h)^2 + k \). They understand the notion of a family of functions, and characterize such function families based on their properties. These ideas will be explored further with trigonometric functions (F-TF.5).

In F-BF.4a, students learn that some functions have the property that an input can be recovered from a given output, i.e., the equation \( (x) = c \) can be solved for \( x \), given that \( c \) lies in the range of \( f \). They understand that this is an attempt to “undo” the function, or to “go backwards.” Tables and graphs should be used to support student understanding here. This standard dovetails nicely with standard F-LE.4 described below and should be taught in progression with it. Students will work more formally with inverse functions in advanced mathematics. (See CCSS CA Algebra 2 Framework for more details).

| Key Terms for Word Bank: Functions, linear, quadratic, exponential, cubic, logarithmic, rational, growth, rate of change, x and y intercepts, domain, range, multiple representations, match, corresponding | Activity examples:  
• Create multiple representations of functions Activity (4 person match-up). Teacher will create several function examples that can be represented in four ways. All students receive a card with one of the following function representations  
  • Table of Values  
  • Graph/Plot  
  • Pattern Growth image (ex. . . . ... .... )  
  • Verbal Explanation of the aforementioned image  
Students then seek to find others that match their function. As a group, students present how they determined their group | This unit is a cumulative review of previously taught concepts dealing with functions. |
| --- | --- | --- |
| Use of sentence frames (appropriate for language level) to facilitate academic language and conversations  
Use of visual organizers (thinking maps) to assist processing mathematical ideas  
Explicitly teaching key academic vocabulary  
Use of mathematical tools (graphing calculator) to facilitate conceptual understanding  
Flexible grouping to support language acquisition and target instruction  
Use of collaboration to promote socio-cultural learning | | |

**Teacher Notes:**
# Unit 6: Trigonometric Functions

**Big Idea**
Situations that repeat over a given period can be predicted and modeled.

**Essential Questions**
- How do you evaluate trigonometric functions for given values, periods, and intervals?
- How trigonometric functions relate to the unit circle?
- How do we model “Real world” scenarios to trigonometric functions?

**Performance Task**
- **Ramps** C2 2005 p.17
- **Remote Measures** C3 2013 p.4-5

**Problem of the Month**
- Turn Up the Volume Level D and Teacher’s Notes

## Unit Topics/Concepts

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<th>Content Standards</th>
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<tbody>
<tr>
<td><strong>F-TF.1</strong> Extend the domain of trigonometric functions using the unit circle. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</td>
</tr>
<tr>
<td><strong>F-TF.2</strong> Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</td>
</tr>
<tr>
<td><strong>F-TF.2.1</strong> Graph all 6 basic trigonometric functions. (CA)</td>
</tr>
<tr>
<td><strong>F-TF.3</strong> Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.</td>
</tr>
<tr>
<td><strong>F-TF.8</strong> Prove and apply trigonometric identities. Prove the Pythagorean identity ( \sin^2(\theta) + \cos^2(\theta) = 1 ) and use it to calculate trigonometric ratios.</td>
</tr>
<tr>
<td><strong>FBF.1b</strong> Combine standard functions types using arithmetic operations.</td>
</tr>
<tr>
<td><strong>F-IF.4</strong> Interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features, such as intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior, given a verbal description of the relationship.</td>
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<tr>
<td><strong>F-IF.5</strong> Relate the domain of a function to its graph and to the quantitative relationship it describes. <em>E.g., if the function ( h(n) ) gives the number of hours it takes to assemble ( n ) engines in a factory, then the positive integers would be an appropriate domain for the function.</em></td>
</tr>
<tr>
<td><strong>F-IF.6</strong> Calculate, interpret, and/or estimate the average rate of change of a function (presented symbolically or as a table) over a specified interval or from a graph.</td>
</tr>
<tr>
<td><strong>F-IF.7e</strong> Graph trigonometric functions showing period, midline, an amplitude.</td>
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<tr>
<td><strong>F-IF.8</strong> Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</td>
</tr>
</tbody>
</table>

**Resources**
- **Essential Resources:**
  - Algebra II Mathematics Framework
  - Resource Packet
  - Real World Application: Ch. 13 Project
  - Real World Application: Ch. 14 Project
- **Additional Resources:**
  - Geogebra algebra/geometry modeling software
  - Dan Meyer 3-act videos (list and interactive link to Dan Meyer’s videos by standard)
**Unit 6: Trigonometric Functions**

*(Instructional Support & Strategies)*

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<tr>
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| In this unit students will learn about the properties of trigonometric functions and how they relate to the unit circle. Students will also be able to interpret and identify various concepts such as period, amplitude and midline. In geometry, students began trigonometry through the study of right triangles. In this unit they will be able to extend the three basic functions to the unit circle. They will also be able to explore trigonometric identities and connect them to what they already learned about various geometric concepts such as the Pythagorean Theorem. Students will also model how trigonometric functions relate to the “real world” with various concepts involving waves, amplitude, trends, etc. (See CCSS CA Algebra 2 Framework for more details).*

<table>
<thead>
<tr>
<th>Academic Language Support</th>
<th>Instructional Tool/Strategy Examples</th>
<th>Preparing the Learner</th>
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</table>
| Key Terms for Word Bank:  | • Have students construct unit circles to discover their properties including radians • Use software that models tides and periodicity, real world examples such as ferris wheels videos showing motion with graphing simultaneously (several websites offer this visual) • Connect the trig identities to the Pythagorean Theorem, cutting out right triangles and fitting them to the unit circle, working hands on with the triangles | Student in geometry begin with an understanding of trigonometric ratios through the study of right triangles. They will then be able to relate the concept of trigonometric functions not only by exploring ratios of triangles, but they will then relate trigonometric functions to the unit circle. Students also explored the Pythagorean Theorem in geometry. They will later explore the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \).*

| Teacher Notes: |
| Big Idea | Unit 7: Probability and Statistics  
(3 weeks 5/16-6/3) |
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<tbody>
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<td>Situations can be evaluated and used in order to make decisions.</td>
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### Essential Questions
- How can data collection help in making decisions?
- How can situations be modeled and approximated using probability and statistics?
- How can parameters be adjusted to help in making the best fit for a given situation?

### Performance Task
- **Bird's Egg** C2 2004 p.28-29
- **Taxi Times** C2 2003 p.3-4
- **Media Surfing** C1 2012 p.4-5
- **Scatter Diagrams** C1 2005 p.14-15
- **Population** C1 2004 p.19-20

### Problem of the Month
- Data $en$Se and Teacher Notes

### Unit Topics/Concepts
- Use the mean and standard deviation of data
- Fit normal distribution to data set
- Estimate population percentages
- Use calculators, spreadsheets, tables to estimate areas under the normal curve
- Make inferences about population parameters
- Decide if a specified model is consistent with results
- Recognize purpose and difference of sample surveys, experiments and observational studies
- Explain randomization
- Use data from sample survey
- Estimate a population mean
- Develop a margin of error
- Compare two treatments using data
- Use simulations to decide if differences between parameters are significant
- Evaluate reports based on data
  - Modeling
  - Using Best-Fit curves/lines
  - Data Collection
  - Sampling
  - Experiments, Simulations, Randomness

### Content Standards
- **S-ID.4** Summarize, represent, and interpret data on a single count or measurement variable. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
- **S-IC.1** Making Inferences and Justifying Conclusions. Understand and evaluate random processes underlying statistical experiments. Understand statistics as a process for making inferences to be made about population parameters based on a random sample from that population.
- **S-IC.2** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?
- **S-IC.3** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Recognize the purposes and differences among sample surveys, experiments, and 880 observational studies; explain how randomization relates to each.
- **S-IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
- **S-IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
- **S-IC.6** Evaluate reports based on data. Write a function that describes a relationship between two quantities.

### Essential Resources:
- Algebra II Mathematics Framework
- CPM. Org Probability/Statisitics Supplement: Student Version
- Teacher Version
- Textbook: 12.1-12.7
- Real World Application: Ch. 12

### Additional Resources
- Geogebra algebra/geometry modeling software
- MARS –
  - Using Box Plots
  - Frequency Curves
  - Devising a Method For Correlation
  - Interpreting Statistics
  - Modeling: Having Kittens
- Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard)
Unit 7: Probability and Statistics  
(Instructional Support & Strategies)

Framework Description/Rationale
The purpose of this unit is for students to use what they have learned about various topics in algebra II to model situations, make predictions, and identify trends using statistics and probability. Students will identify functions to model a situation and adjust parameters to improve the model. They compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. Students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data including surveys, experiments, and simulations. They will also learn the role that randomness and careful design play in the conclusions that can be drawn.

(See CCSS CA Algebra 2 Framework for more details)

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| Key Terms for Word Bank:  | • Collect data based on certain criteria or something relevant to students/class  
                           • Graph and analyzing data using mean, standard deviation  
                           • Find out which data sets apply to the statistics tools, if not why? (normal distribution, etc)  
                           • Compare and contrast the different graphical displays (box plot--previously referred to as a box-whisker, stem-leaf, bar, pie, etc)  
                           • Explore a variety of statistical experiments to arrive at statistical techniques (graphs, distribution) | This unit is a culmination of all the skills that students have acquired in previous grades, units, etc. so that they can apply what they have learned and use reasoning and make decisions based on statistics and probability. This unit relies heavily on student understanding of models and “real world” situations so that they can make judgments based on the appropriate mathematical models.  
Students can use statistics concepts that they have learned from previous grades and units (i.e. data collection, scatter plots, mean, median, mode, etc.) to explore further judgments based on complex topics such as sampling, randomness, best-fit, etc. |

Teacher Notes: