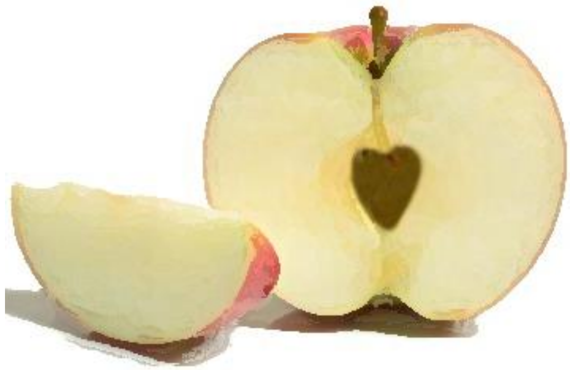


SAUSD

Mathematics Resource Packet

Math 7 Unit 1:

Ratios and Proportional Reasoning

(Content Includes: Material adapted from Engage NY, Georgia Department of Education, Silicon Valley Math Initiative, as well as original content)

Student Name ________________________________ **Period** ______

Teacher ___________________________

Ratios and Proportional Reasoning

Big Idea of the Unit:

If two quantities vary proportionally, that relationship can be represented in multiple ways.

Essential Questions:

(*Questions I will be able to answer by the end of the unit*):

*What is the constant of proportionality?

*How can two quantities be identified as proportional or non-proportional?

*How can the constant of proportionality (unit rate) be determined given a table? Graph? Equation? Diagram? Verbal description?

*What does a specific point on a graph (x,y) represent?

Problem of the Month:

First Rate

Ratios & Proportional Reasoning

Standards Connections

- ☐ I can write a *ratio* to describe a given situation.
- ☐ I can compare quantities using *ratios*.
- ☐ I can find the *unit rate* and use it to compare quantities.
- ☐ I can find the *unit rates* of multiple items and use them to determine which is the best deal.
- ☐ I can test for equivalent ratios using tables and graphs.
- ☐ I can graph ratios for a given situation.
- ☐ I can decide when two quantities are proportional and non-proportional.
- ☐ I can identify the *constant of proportionality* in tables, graphs, equations, diagrams and verbal descriptions.
- ☐ I can represent proportional relationships the equation y=kx.
- ☐ I can explain what a point (r,1) on a graph of *proportional* relationships mean in terms of the situation.
- ☐ I can solve problems involving *proportional* relationships.

Key Vocabulary:

- ☐ Ratio
- ☐ Rate
- ☐ Unit Rate
- ☐ Ratio Table
- ☐ Equivalent Ratio
- ☐ Quantities
- ☐ Constant
- ☐ Constant of Proportionality
- ☐ Proportional Relationships
- ☐ Proportional

Lesson 1: Nana's Chocolate Milk

Act One: Watch the video

Act Two: Answer the questions below.

1. What did he do wrong?

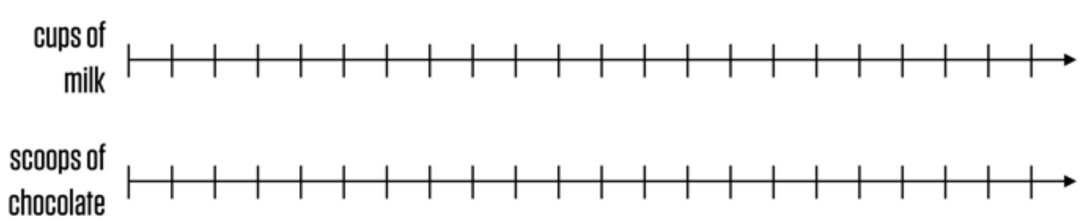
2. How can he fix it? How can he make it the right mix of chocolate and milk? Explain the strategy or strategies you used (even if you don't come up with a workable solution.)

3. Did you find more than 1 solution? Explain.

4. The glass won't hold two cups of liquid. How does this affect your solution?

Act Three: Watch Video – "How I fixed it"

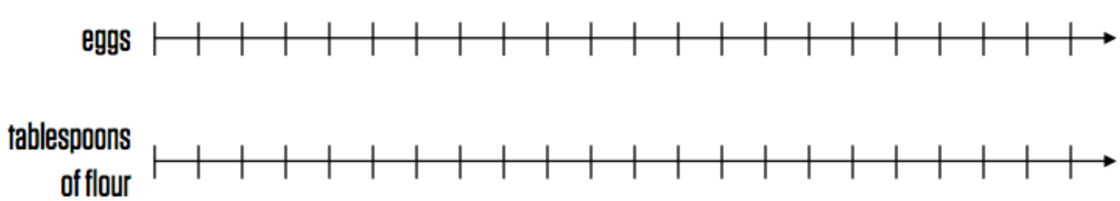
5. How was his solution similar or different from yours?



The Sequel: Nana's Eggs

1. What did he do wrong?

2. How can he fix it? Explain the strategy you used.



3. Watch Video – "How I fixed it". Explain how he fixed the problem.

4. What is a ratio?

Lesson 2: Ratios as a Measure of Rate

Activity 1: How fast is our class?

Record the results from the paper passing exercise in the table below.

Trial	Number of Papers Passed	Time (in seconds)	Ratio of Number of Papers Passed to Time	Rate	Unit Rate
1					
2					
3					
4					

Activity 2: Our Class by Gender

	Number of boys	Number of girls	Ratio of boys to girls	Unit Rate
Class 1				
Class 2				
Class 3				
Whole 7^{th} grade				

Analysis

1. Using the information in the table above, predict how many boys and how many girls are in the whole 7th grade at our school. Show your work.

Discussion

1. How many different ways can a ratio be written?

2. What is a **Unit Rate?**

Activity 3: The Amazing Tillman

Watch the video clip of Tillman the English bulldog, the Guinness World Record holder for Fastest Dog on a Skateboard.

1. At the conclusion of the video, your classmate takes out his or her calculator and says, "Wow that was amazing! That means the dog went about 5 meters in 1 second!" Is your classmate correct, and how do you know?

2. After seeing this video, another dog owner trained his dog, Lightning, to try to break Tillman's skateboarding record. Lightning's fastest recorded time was on a 75-meter stretch where it took him 15.5 seconds. Based on this data, did Lightning break Tillman's record for fastest dog on a skateboard? Explain how you know.

Lesson 3: Interpreting Unit Rates

Activity 1: In each problem, find both possible rates. Use division to find the unit rates.

a) 6 bags of flour weigh 30 pounds

Rate 1:	Unit rate:	Interpretation:
Rate 2:	Unit rate:	Interpretation:

b) 9 tennis balls come in 3 cans

Rate 1:	Unit rate:	Interpretation:
Rate 2:	Unit rate:	Interpretation:

c) 5 gallons of gas cost $6.50

Rate 1:	Unit rate:	Interpretation:
Rate 2:	Unit rate:	Interpretation:

d) In 25 minutes, Jenny can run 10 laps

Rate 1:	Unit rate:	Interpretation:
Rate 2:	Unit rate:	Interpretation:

Activity 2:

a) At Frank's Fruit Stand, 3 apples cost 90 cents.
You want to buy 7 apples. How much will they cost?

1. What are the two possible rates for this problem?

Apples	Cost

2. Show each rate as a unit rate.

3. What does each rate tell you?

4. Which rate will help you solve this problem?

5. If it costs ________ cents to buy 1 apple, how much will 2 apples cost? 4 apples? Complete the table.

6. What pattern do you see in the table?

7. Since you know the unit rate, write a number sentence for the cost of 7 apples.

b) At the bank of England, Mrs. Brown wanted to exchange 20 U.S. dollars for British pounds. The exchange rate is 3 U.S. dollars for 2 British pounds. How many pounds did Mrs. Brown receive?

Dollars	Pounds

1. What are the two possible rates for this problem?
2. Show each rate as a unit rate.
3. What does each rate tell you?
4. Which rate will help you solve this problem?
5. How many British pounds can be exchanged for 1 U.S. dollar? For 2 U.S. dollars? Complete the table.
6. What number pattern do you see?
7. Since you know the unit rate, write a number sentence for the number of pounds you can exchange for 20 U.S. dollars.

Activity 3:

Anne is painting her house blue. To make the color she wants, she must add 3 cans of white paint to every 2 cans of blue paint. How many cans of white paint will she need to mix with 6 cans of blue paint?

Rate Needed:	Unit rate:	Interpretation:

Show your work & explain your thinking:

Solution:

Practice: Unit Rates

1. Ryan is making a fruit drink. The directions say to mix 5 cups of water with 2 scoops of powdered fruit mix. How much water should he use with 9 scoops of fruit powder?

Rate Needed:	Unit rate:	Interpretation:

Show your work & explain your thinking:

Solution:

2. Yvette is running around a track. It takes her 10 minutes to run 6 laps. If she keeps running at the same speed, how long will it take her to run 5 laps?

Rate Needed:	Unit rate:	Interpretation:

Show your work & explain your thinking:

Solution:

3. Mark's model train can go 12 laps around its track in 4 minutes. If it runs at the same speed, how many laps can the train go in 9 minutes?

Rate Needed:	Unit rate:	Interpretation:

Show your work & explain your thinking:

Solution:

4. Find each rate and unit rate.
    a. 420 miles in 7 hours
    b. 360 customers In 30 days
    c. 40 meters in 16 seconds
    d. $7.96 for 5 pounds

5. Write three ratios that are equivalent to the one given:
    18 right-handed students for every 4 left-handed students

6. Mr. Rowley has 16 homework papers and 14 exit tickets to return. Ms. Rivera has 64 homework papers and 60 exit tickets to return. For each teacher, write a ratio to represent the number of homework papers to number of exit tickets they have to return. Are the ratios equivalent? Explain.

7. Jonathan's parents told him that for every 5 hours of homework or reading he completes, he will be able to play 3 hours of video games. His friend Lucas's parents told their son that he can play 30 minutes for every hour of homework or reading time he completes. If both boys spend the same amount of time on homework and reading this week, which boy gets more time playing video games and how do you know?

8. At Euclid Middle School, of the 30 girls who tried out for the soccer team, 12 were selected and of the 40 boys who tried out, 16 were selected. Does a boy or a girl have a better chance of making the team?

9. Devon is trying to find the unit price on a 6-pack of energy drinks on sale for $2.99. His sister says that at that price, each energy drink would cost just over $2.00. Is she correct and how do you know? If she is not, how would Devon's sister find the correct price?

10. Each year Lizzie's school purchases student agenda books, which are sold in the school store. This year, the school purchased 350 books at a cost of $1,137.50. If the school would like to make a profit of $1,500 to help pay for field trips and school activities, what is the least amount they can charge for each agenda book? Explain how you found your answer.

Lesson 4: Proportional Relationships

Activity 1: Pay by the Ounce Frozen Yogurt

A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed their dish and this is what they found.

Weight (ounces)	12.5	10	5	8
Cost ($)	5	4	2	3.20

1. What do you notice about these weights and the cost of the yogurt?
   a) Does everyone pay the same cost per ounce? How do you know?

   b) Isabelle's brother takes an extra-long time to create his dish. When he puts it on the scale, it weighs 15 ounces. If everyone pays the same rate in this store, how much will his dish cost? How did you calculate this cost?

   c) Is the cost ***proportional*** to the weight?

   d) What happens if you don't serve yourself any yogurt or toppings, how much do you pay? Does the relationship still hold true for zero ounces?

   e) For any weight (x), how do we find the cost (y)?

Activity 2: A Cooking Cheat Sheet!

In the back of a recipe book, a diagram provides easy conversions to use while cooking.

a) What does the diagram tell us?

b) Is the number of ounces proportional to the number of cups? How do you know?

c) How many ounces are there in 4 cups? 5 cups? 8 cups? How do you know?

d) For any number of cups (x), how do we find the number of ounces (y)?

Practice

1. During Jose's physical education class today, students visited activity stations. Next to each station was a chart depicting how many calories (on average) would be burned by completing the activity.

Calories burned while Jumping Rope

   a. Is the number of Calories burned proportional to time? How do you know?

   b. If Jose jumped rope for 6.5 minutes, how many calories would he expect to burn?

   c. For any number of minutes (x), how do we find the number of calories burned (y)?

Activity 3: Summer Job

Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

To check his assumption, he decided to make a table. He entered his total money earned at the end of week 1 and his total money earned at the end of Week 4.

Week	0	1	2	3	4	5	6	7	8
Total Earnings		$28			$112				

a. Work with a partner to answer Alex's question.

b. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?

c. For any number of weeks (x), how do we find the total earnings (y)?

Practice

1. A cran-apple juice blend is mixed in a ratio of cranberry to apple of 3 to 5.

    a. Complete the table to show different amounts that are proportional.

Amount of Cranberry			
Amount of Apple			

    b. Why are these quantities proportional?

2. John is filling a bathtub that is 18 inches deep. He notices that it takes two minutes to fill the tub with three inches of water. He estimates it will take ten more minutes for the water to reach the top of the tub if it continues at the same rate. Is he correct? Explain.

Discussion

1. How do we know if two quantities are proportional to each other?

2. How can we recognize a proportional relationship when looking at a table or a set of ratios?

Lesson 5: Identifying Proportional and Non-Proportional Relationships in Tables

Activity 1:

1. You have been hired by your neighbors to babysit their children on Friday night. You are paid $8 per hour. Complete the table relating your pay to the number of hours you worked.

x Hours Worked	y Pay
1	
2	
3	
4	
4 ½	
5	
6	
6.5	

a. Explain how you completed the table.

b. How did you determine the pay for 4 ½ hours?

c. How could you use the information to determine the pay for a week in which you worked 20 hours?

d. Based on the table above, is pay proportional to hours worked? How do you know?

e. Describe the relationship between the amount of money earned and the number of hours worked in this example.

In exercises 1-4, determine if y is proportional to x. Justify your answer.

1. The table below represents the inches of snowfall in 5 counties to the hours of a recent winter storm.

x Time (hrs)	y Snowfall (In)
2	10
6	12
8	16
2.5	5
7	14

2. The table below shows the relationship between the costs of renting a movie to the number of days on rent.

x Number of Days	y Cost
6	2
9	3
24	8
3	1

3. The table below shows the relationship between the amount of candy (pounds) bought and the total cost.

x Pounds	y Cost
5	10
4	8
6	12
8	16
10	20

4. Randy drove from New Jersey to Florida. He recorded the distance traveled and the number of gallons used every time he stopped for gas. To complete the table, assume *Miles Driven* is proportional to *Gallons Consumed.*

Gallons Consumed	2	4		8	10	12
Miles Driven	54		189	216		

Practice

1. In each table determine if y is proportional to x. Explain why or why not.

a.

x	y
3	12
5	20
2	8
8	32

b.

x	y
3	15
4	17
5	19
6	21

c.

x	y
6	4
9	6
12	8
3	2

a. Proportional or Non-proportional	**b.** Proportional or Non-proportional	**c.** Proportional or Non-proportional
Explain:	**Explain:**	**Explain:**

2. Kayla made observations about the selling price of a new brand of coffee that sold in three different sized bags. She recorded those observations in the following table:

Ounces of Coffee	6	8	16
Price in Dollars	$2.10	$2.80	$5.60

   a. Is the price proportional to the amount of coffee? ________
   Explain why or why not?

   b. Use the relationship to predict the cost of a 20 oz. bag of coffee?

3. You and your friends go to the movies. The cost of admission is $9.50 per person.

   a. Create a table showing the relationship between number of people going to the movies and the total cost of admission.

   b. Explain why the cost of admission is proportional to the amount of people.

4. For every 5 pages Gil can read, his daughter can read 3 pages. Let ***g*** equal the number of pages Gil reads and let ***d*** equal the number of pages his daughter reads.

   a. Create a table showing the relationship between the number of pages Gil reads and the number of pages his daughter reads.

   b. Is the number of pages Gil's daughter reads proportional to the number of pages he reads? ______Explain why or why not.

5. The table shows the relationship between the number of parents in a household and the number of children in the same household. Is the number of children proportional to the number of parents in the household? Explain why or why not.

Number of Parents	Number of Children
0	0
1	3
1	5
2	4
2	1

6. The table below shows the relationship between the number of cars sold and money earned for a car salesperson. Is the money earned proportional to the number of cars sold? Explain why or why not.

Number of Cars Sold	Money Earned
1	250
2	600
3	950
4	1076
5	1555

7. Make your own example of a relationship between two quantities that are **NOT** proportional. Describe the situation and create a table to model it. Explain why one quantity is not proportional to the other.

Activity 2: Which Team Will Win the Race?

You have decided to run in a long distance race. There are two teams that you can join. Team A runs at a constant rate of 2.5 miles per hour. Team B runs 4 miles the first hour and then 2 miles per hour after that.

Task: Create a table for each team showing the distances that would be run for times of 1, 2, 3, 4, ,5 and 6 hours.

Team A	
Time (hrs)	Distance (miles)

Team B	
Time (hrs)	Distance (miles)

a. For which team is distance proportional to time? Explain your reasoning.

b. Explain how you know distance for the other team is not proportional to time.

c. If the race were 2.5 miles long, which team would win? Explain.

d. If the race were 3.5 miles long, which team would win? Explain.

e. If the race were 4.5 miles long, which team would win? Explain.

f. For what length race would it be better to be on Team B than Team A? Explain.

g. Using this relationship, if the members on the team ran for 10 hours, how far would each member run on each team?

h. Will there always be a winning team, no matter what the length of the course? Why or why not?

i. If the race is 12 miles long, which team should you choose to be on if you wish to win? Why would you choose this team?

j. How much sooner would you finish on that team compared to the other team?

Discussion:

How can you use a table to determine whether the relationship between two quantities is proportional?

How does knowing that two quantities are proportional help answer questions about the quantities? For example, if we know 1 cup = 8 oz, what does that allow us to do?

Practice

1. Joseph earns $15 for every lawn he mows. Is the amount of money he earns proportional to the number of lawns he mows? Make a table to help you identify the type of relationship.

Number of Lawns Mowed				
Earnings ($)				

2. At the end of the summer, Caitlin had saved $120 from her summer job. This was her initial deposit into a new savings account at the bank. As the school year starts, Caitlin is going to deposit another $5 each week from her allowance. Is her account balance proportional to the number of weeks of deposits? Use the table below. Explain your reasoning.

Time (in weeks)				
Account Balance ($)				

3. Lucas and Brianna read three books each last month. The table shows the number of pages in each book and the length of time it took to read the entire book. What observations can you make about the reading rates of the two students. Explain.

Pages Lucas Read	208	156	234
Time (hours)	8	6	9

Pages Brianna Read	168	120	348
Time (hours)	6	4	12

Lesson 6: Identifying Proportional Relationships in a Graph

Activity 1: Weighing Nickels

a. Weigh nickels and complete the table

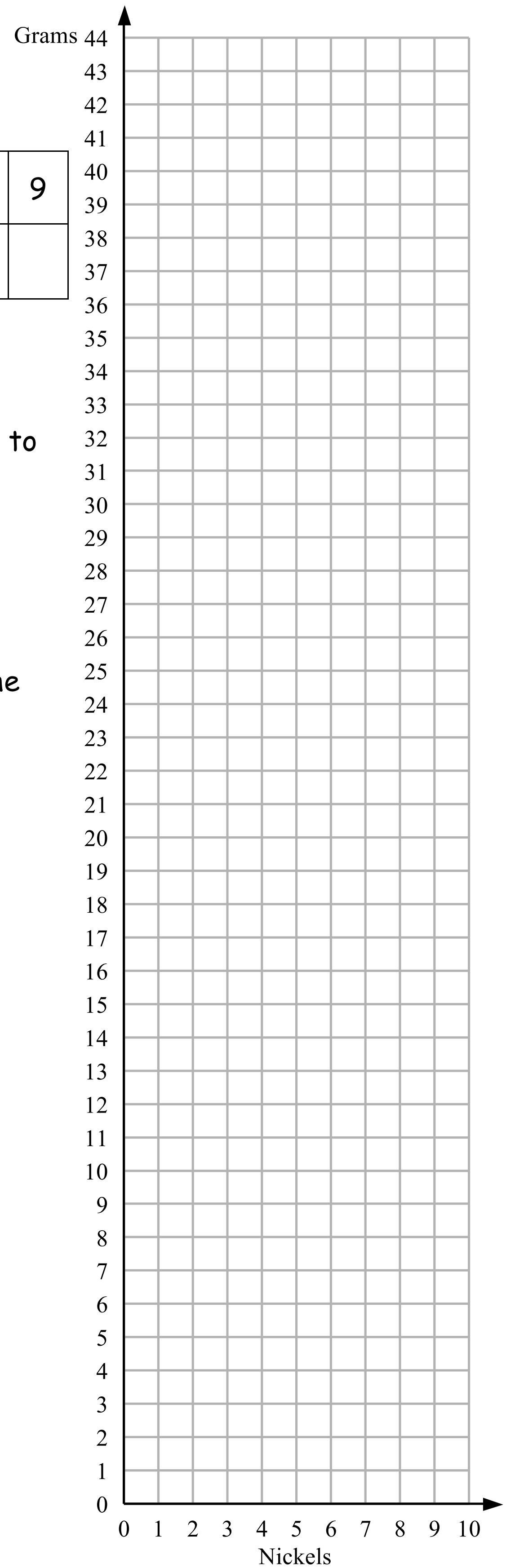
Nickels	0	1	2	3	4	5	6	7	8	9
Grams										

b. Graph Results

c. Is the number of the nickels proportional to their weight?

d. What observations can you make about the arrangement of points?

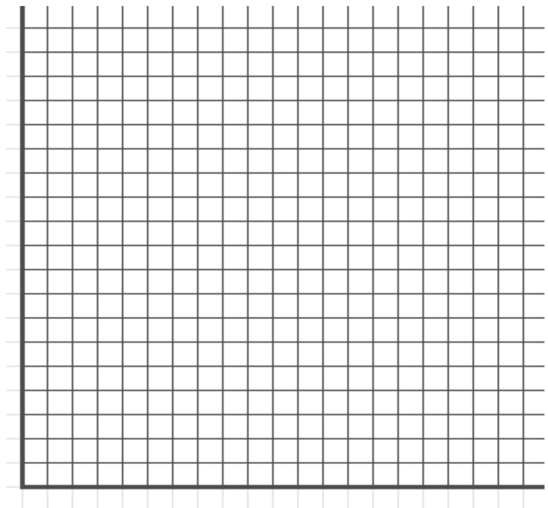
e. Do we extend the line in both directions? Explain why or why not.



Activity 2: From a Table to a Graph

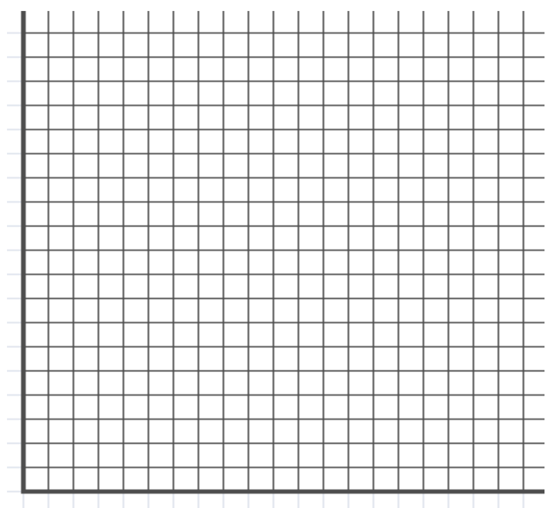
a. Isaiah sold candy bars to help raise money for his scouting troop. The table shows the amount of candy he sold to the money he received. Is the amount of candy bars sold proportional to the money Isaiah received? How do you know?

x Candy Bars Sold	y Money Received ($)
2	3
4	5
8	9
12	12



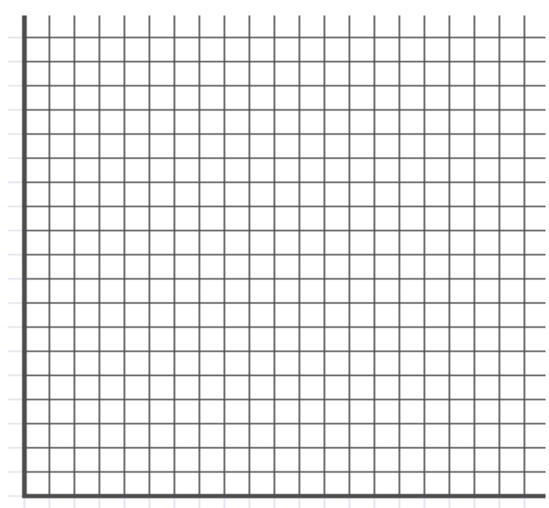
b. Create another ratio table that contains two sets of quantities that <u>are</u> proportional to each other using the first ratio on the table.

x Candy Bars Sold	y Money Received ($)
2	3



c. Graph the coordinates in this table.

x	y
0	6
3	9
6	12
9	15
12	18



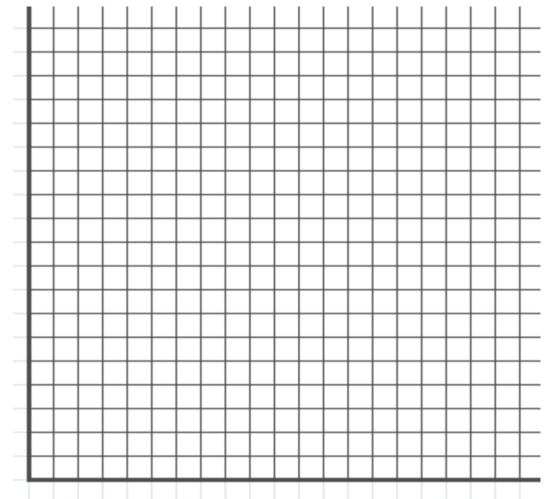
How are graphs **a** & **c** similar or different?

How are graphs **b** & **c** similar or different?

Activity 3: Graphing Proportional relationships

1. Go back to Lesson 4 - Activity 1 (Frozen Yogurt):
   a. Graph the data in the table. Be sure to label the x & y-axis.

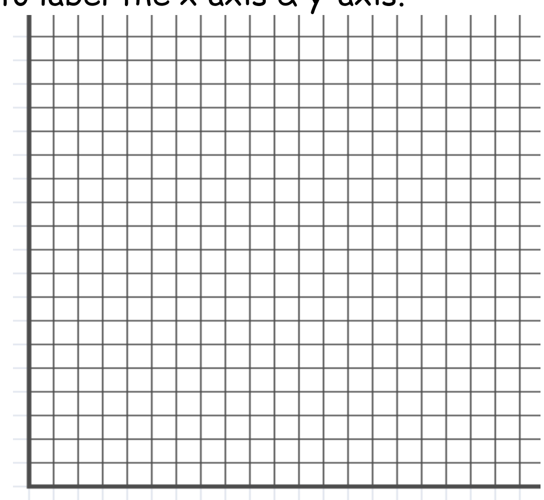
Weight (ounces)	Cost ($)
	0
	5
10	4
5	2
8	3.2
20	



   b. What do you notice about the graph?

2. Go back to Lesson 5 – Activity 1 (Babysitting):
   a. Graph the data in the table. Be sure to label the x axis & y-axis.

Hours Worked (x)	Pay (y)
0	
1	
2	
3	
4	
4 ½	
5	
6.5	



   b. What do you notice about the graph?

Discussion

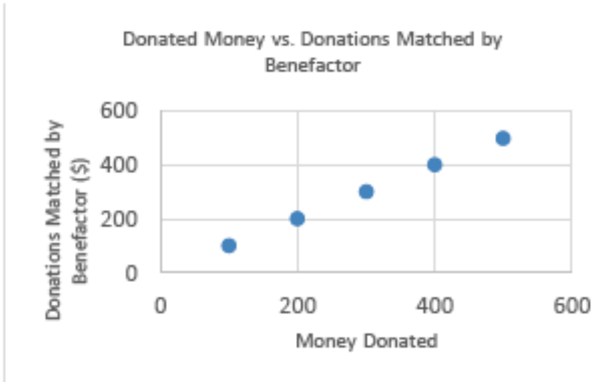
1. How are proportional quantities represented in a graph?

2. What can you infer about graphs of two quantities that are proportional to each other?

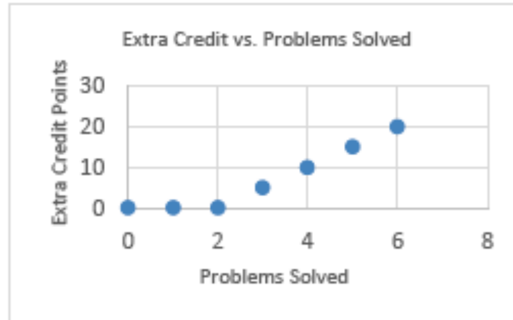
Practice

1. Determine whether or not the following graphs represent two quantities that are proportional to each other. Give reasoning.

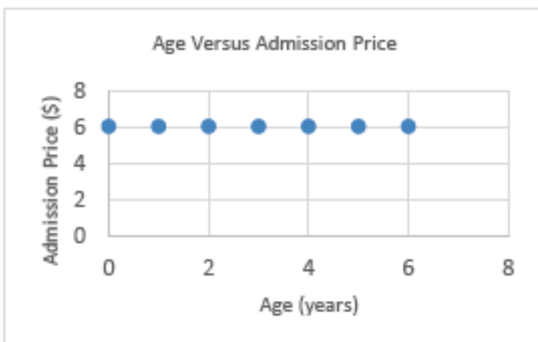
a.



c.

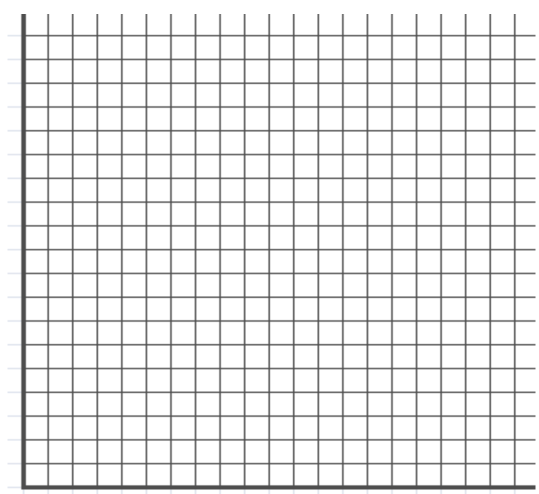


b.



2. Create a table and a graph for the ratios 2:22, 3 to 15 and 1/11. Does the graph show that the two quantities are proportional to each other? Explain why or why not.

x	y



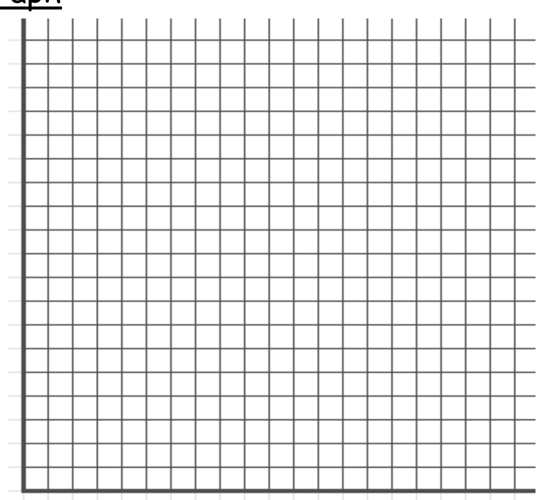
Lesson 7: Identifying Proportional and Non-Proportional Relationships in Tables and Graphs

Activity 1: Today you will be working in groups to make a table, graph and identify whether or not the two quantities are proportional to each other.

You are Group ________

Team Responsibilities:
Recorder: Folds the paper in quarters
Reader: Opens the envelope and reads the situation
Team: Create the poster, demonstrating the situation

Poster Layout - **Do the sample problem below**

Problem	Table
Graph 	Proportional or not? Explain

Gallery Walk

1. As you walk around and reflect on the other posters, answer the following questions:

    a. Which graphs in the art gallery walk represented proportional relationships and which did not? List the group number.

<u>Proportional Relationship</u>	<u>Non-proportional Relationship</u>

    b. What are the characteristics of the graphs that represent proportional relationships?

    c. For the graphs representing proportional relationships, what does (0,0) mean in the context of the given situation?

    d. Were there any differences found in groups that had the same ratios?

    e. Did you notice any common mistakes? How might they be fixed?

f. Was there a group that stood out by representing their problems and findings exceptionally clearly?

2. Self-Critique: Evaluate your own poster

a. What does it mean for a display to be both visually appealing and informative?

b. How much time did your group spend on the content of your poster

c. Suppose we invited people from another school, state or country to walk through our gallery. Would they be able to learn about ratio and proportion from our posters? Explain.

d. What would you do differently next time we do a collaborative activity?

Lesson 8: Unit Rate and Constant of Proportionality

Activity 1: National Forest Deer Population in Danger?

Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 216 deer in a 24 square mile area of the forest. Do conservationists need to be worried?

1. Why does it matter if the deer population is not constant in a certain area of the national forest?

2. What is the population density of deer per square mile?

Table:

   a. The Unit Rate of deer per 1 square mile is ______________________.

   b. The unit rate is also called the ***Constant of Proportionality (k)***. In this problem ***k=*** _____

   c. The meaning of the ***Constant of Proportionality*** in this problem is

3. Use the unit rate of deer per square mile to determine how many deer are there for every 207 square miles.

4. Use the unit rate to determine the number of square miles in which you would find 486 deer?

Activity 2: You Need WHAT???

Brandon came home from school and informed his mother that he had volunteered to make cookies for his entire grade level. He needed 3 cookies for each of the 96 students in 7th grade. Unfortunately, he needed the cookies for an event at school on the very next day! Brandon and his mother determined that they could fit 36 cookies on two cookie sheets.

1. Is the number of cookies proportional to the number of sheets used in baking? Create a table that shows data for the number of sheets needed for the total number of cookies needed.

Table:

   a. The Unit Rate of cookies per sheet: ________________________

   b. Constant of Proportionality: ***k=*** _____

   c. Meaning of Constant of Proportionality in this problem:

2. It took 2 hours to bake 8 sheets of cookies. If Brandon and his mother begin baking at 4:00 pm, when will they finish baking the cookies?

Activity 3: French Class Cooking

Suzette and Margo want to prepare crepes for all of the students in their French class. A recipe makes 20 crepes with a certain amount of flour, milk, and 2 eggs. The girls know that they already have plenty of flour and milk but need to determine the number of eggs needed to make 50 crepes because they are not sure they have enough eggs for the recipe.

1. Considering the amount of eggs necessary to make the crepes, what is the constant of proportionality?

2. What does the constant or proportionality mean in the context of this problem?

3. How many eggs will be needed for 50 crepes?

Practice:

For each of the following problems, identify and define the constant of proportionality to answer the follow-up question.

1. Bananas are $0.59/pound.
    a. What is the constant of proportionality?
    b. How much does 25 pounds of bananas cost?
2. The dry cleaning fee for 3 pairs of pants is $18.
    a. What is the constant of proportionality?
    b. How much will the dry cleaner charge for 11 pairs of pants?
3. For every $5 that Micah saves, his parents give him $10.
    a. What is the constant of proportionality?
    b. If Micah saves $150, how much money will his parents give him?

4. Each school year, the 7th graders who study Life Science participate in a special field trip to the city zoo. In 2010, the school paid $1260 for 84 students to enter the zoo. In 2011, the school paid $1050 for 70 students to enter the zoo. In 2012, the school paid $1395 for 93 students to enter the zoo.

    a. Is the price the school pays each year in entrance fees proportional to the number of students entering the zoo?

    b. Explain why or why not.

    c. Identify the constant of proportionality and explain what it means in the context of this situation.

    d. What would the school pay if 120 students entered the zoo?

    e. How many students would enter the zoo if the school paid $1,425?

Lesson 9: Representing Proportional Relationships with Equations

Review: Points to remember:

- Proportional relationships have a constant ratio, or unit rate.
- The constant ratio, or unit rate, can also be called the constant of proportionality (k).

Activity 1:

1. Go back to Lesson 5 – Activity 1 (Babysitting): What did you do to the number of hours worked (x) to find the amount of pay she will receive (y)?

2. Complete the table

Hours Worked (x)	Pay (y)	Equation
0	0	
1	8	1 • 8 = 8
2		2 • 8 =
3		
4		
4 ½		
5		
6		
6.5		
x	**y**	

Discussion Notes

How could we use what we know about the constant of proportionality to write an equation?

Activity 2: Do We Have Enough Gas to Make it to the Gas Station?

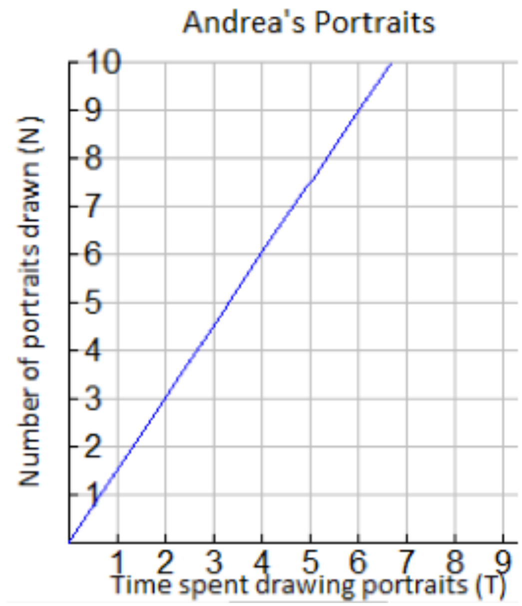
Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Mother's Gas Record	
Gallons	Miles Driven
8	224
10	280
4	112

1. Find the constant of proportionality and explain what it represents in this situation.	
2. Write an equation that will relate the miles driven to the number of gallons of gas.	3. Knowing that there is a half-gallon left in the gas tank when the light comes on, will she make it to the nearest gas station? Explain why or why not.
4. Using the equation found in part b, determine how far your mother can travel on 18 gallons of gas. Solve the problem in two ways.	5. Using the equation found in part b, determine how many gallons of gas would be needed to travel to Bend, Oregon, which is 750 miles north of Santa Ana?

Activity 2: Andrea's Portraits

Andrea is a street artist in New Orleans. She draws caricatures (cartoon-like portraits of tourists). People have their portrait drawn and then come back later to pick it up from her. The graph below shows the relationship between the number of portraits she draws and the amount of time in hours needed to draw the portraits.



1. Write several ordered pairs from the graph and explain what each coordinate pair means in the context of this graph.

2. Write several equations that would relate the number of portraits drawn to the time spent drawing the portraits.

3. Determine the constant of proportionality and explain what it means in this situation.

Practice

1. There are 3 cans that store 9 tennis balls. Consider the number of balls per can.

a. Find the constant of proportionality for this situation.	b. Write an equation to represent the relationship.	c. How many cans are needed for 16 tennis balls?

2. In 25 minutes Li can run 10 laps around the track. Consider the number of laps she can run per minute.

a. Find the constant of proportionality for this situation.	b. Write an equation to represent the relationship.	c. How long will it take her to run 5 laps?

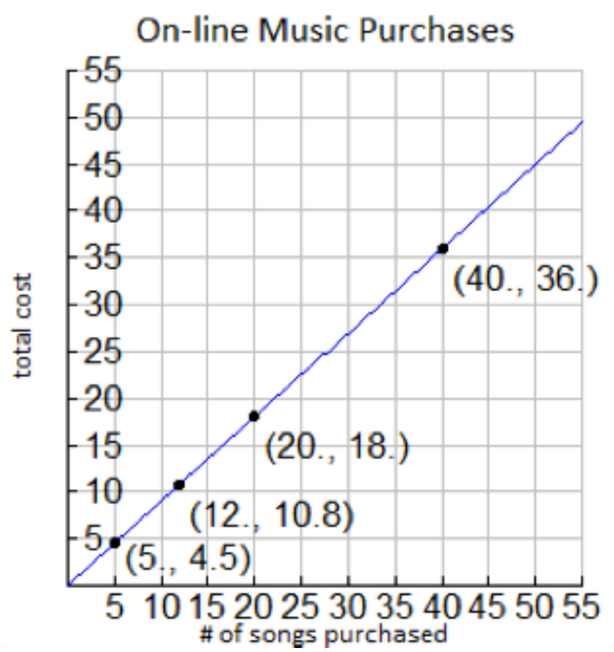
3. Jennifer is shopping with her mother. They pay $2 per pound for tomatoes at the vegetable stand.

a. Find the constant of proportionality for this situation.	b. Write an equation to represent the relationship.	c. How much will she pay for 2.5 pounds of tomatoes?

4. It costs $5 to send 6 packages through a certain shipping company. Consider the number of packages per dollar.

a. Find the constant of proportionality for this situation.	b. Write an equation to represent the relationship.	c. How much would it cost to send 11 packages?

5. On average, Susan downloads 60 songs per month. An online music vendor sells package prices for songs that can be downloaded onto personal digital devices. The graph below shows the package prices for the most popular promotions. Susan wants to know if she should buy her music from this company or pay a flat fee of $58.00 for the month offered by another company. Which is the better buy?



a. Find the constant of proportionality for this situation.

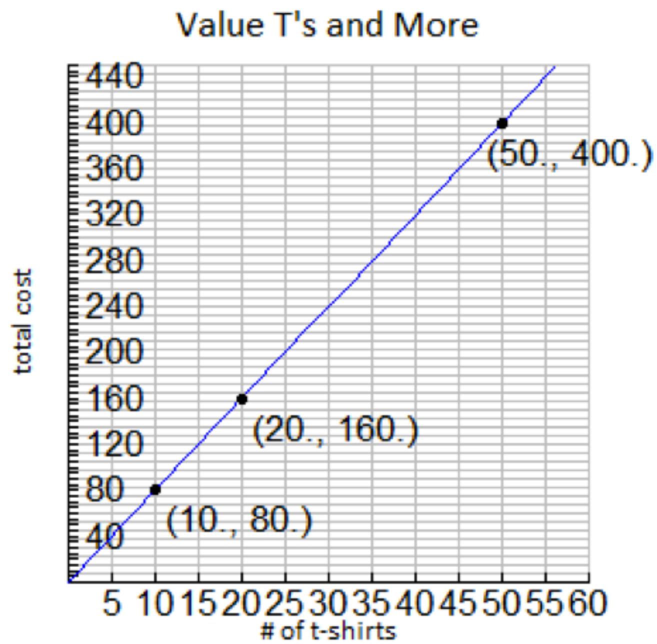
b. Write an equation to represent the relationship.

c. Use your equation to find the answer to Susan's question above. Justify your answer with mathematical evidence and a written explanation.

6. Allison's middle school team has designed t-shirts containing their team name and color. Allison and her friend Nicole have volunteered to call local stores to get an estimate on the total cost of purchasing t-shirts. Print-o-Rama charges a set-up fee as well as a fixed amount for each shirt ordered. The total cost is shown below for the given number of shirts. Value T's and More charges $8 per shirt. Which company should they use?

Print-o-Rama

# shirts	Total cost
10	95
25	
50	375
75	
100	



a. Does either pricing model represent a proportional relationship between quantity of t-shirts and total cost? Explain.

b. Write an equation relating cost and shirts for Value T's and More.

c. What is the constant of proportionality for Value T's and More? What does it represent?

d. How much is Print-o-Rama's set-up fee?

e. Write a proposal to your teacher indicating which company the team should use. Be sure to support your choice. Determine the number of shirts that you need for your team.

Lesson Summary

If a proportional relationship is described by the set of ordered pairs that satisfies the equation $y = kx$, where k is a positive constant, then k is called the constant of proportionality. The constant of proportionality expresses the multiplicative relationship between each x-value and its corresponding y-value.

Activity 3: Jackson's Birdhouses

Jackson and his grandfather constructed a model for a birdhouse. Many of their neighbors offered to buy the birdhouses. Jackson decided that building birdhouses could help him earn money for his summer camp, but he is not sure how long it will take him to fill all of the requests for birdhouses. If Jackson can build 7 birdhouses in 5 hours, write an equation that will allow Jackson to calculate the time it will take him to build any given number of birdhouses.

a. Write an equation that you could use to find out how long it will take him to build any number of birdhouses.	b. How many birdhouses can Jackson build in 40 hours?
c. How long will it take Jackson to build 35 birdhouses? Use the equation from part a to solve the problem.	d. How long will it take to build 71 birdhouses? Use the equation from part **a** to solve the problem.

Activity 4: Al's Produce Stand

Al's Produce Stand sells 7 ears of corn for $1.50. Barbara's Produce stand sells 13 ears of corn for $2.85. Write two equations, one for each produce stand that models the relationship between the number of ears of corn sold and the cost. Then use each equation to help complete the tables below.

Al's Produce Stand

Ears	7	14	21	
Cost	$1.50			$50.00

Barbara's Produce Stand

Ears	13	14	21	
Cost	$2.85			$50.00

Lesson 10: Focus on the Dependant and Independent Variables

1. Watch the video and take notes

2. Suppose that the cost of renting a snowmobile is $37.50 for 5 hours.
    a. If the c = cost and h = hours, which variable is the dependent variable? Explain why?

    b. What would be the cost of renting 2 snow mobiles for 5 hours each?

3. In your car, the number of miles driven is proportional to the number of gallons of gas used.

Gallons	Miles driven
4	112
6	168
	224
10	

a. Which variable is the independent variable? Explain why.

b. Write the equation that will relate the number of miles driven to the gallons of gas.

c. What is the constant of proportionality?

d. How many miles could you go if you filled your 22-gallon tank?

e. If your family takes a trip of 600 miles, how many gallons of gas would be needed to make the trip?

f. If you drive 224 miles during one week of commuting to school and work, how many gallons of gas would you use?

Practice

1. A person who weighs 100 pounds on Earth weighs 16.6 lb on the moon.

    a. Which variable is the independent variable? Explain why.

    b. What is an equation that relates weight on Earth to weight on the moon?

    c. How much would a 185 pound astronaut weigh on the moon? Use an equation to explain how you know.

    d. How much would a man that weighed 50 pounds on the moon weigh back on Earth?

2. Use this table to answer the following questions.

Gallons	Miles driven
0	0
2	62
4	124
10	310

a. Which variable is the dependent variable? Explain why.

b. Is miles driven proportionally related to gallons of gas consumed? If so what is the equation that relates miles driven to gallons?

c. In any ratio relating gallons and miles driven, will one of the values always be larger, if so, which one?

d. If the number of gallons is known, can you find the miles driven? Explain how this value would be calculated.

e. If the number of miles driven is known, can you find the number of gallons consumed?

f. How many miles could be driven with 18 gallons of gas?

g. How many gallons are used when the car has been driven 18 miles?

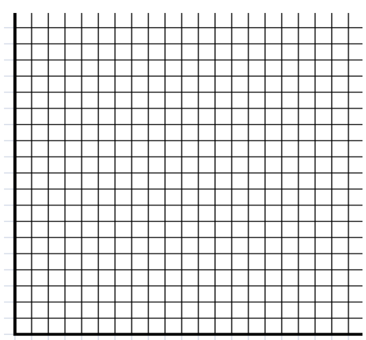
h. How many miles have been driven when 1/2 of a gallon is used?

i. How many gallons have been used when the car has been driven 1/2 mile?

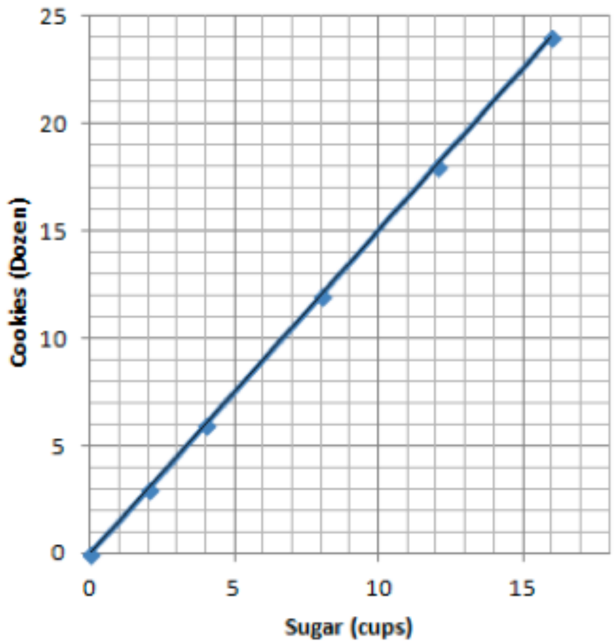
Lesson 11: Representing Proportional Relationships with Equations & Graphs

Activity 1: Baking Cookies – View image, what do you know about baking cookies?

Grandma's Special Chocolate Chip Cookie recipe, which yields 4 dozen cookies, calls for 3 cups of flour. Using this information, complete the chart.

Table – Create a chart comparing the amount of flour used to the amount of cookies.	**Table** – Is the number of cookies proportional to the amount of flour used? Explain.
Graph – Model the relationship on a graph. 	**Unit Rate** – What is the unit rate and what is the meaning in the context of the problem?
Does the **graph** show the two quantities being **proportional** to each other? Explain	**Equation** – Write an equation that can be used to represent the relationship

Activity 2: Below is a graph modeling the amount of sugar required to make Grandma's Chocolate Chip Cookies.



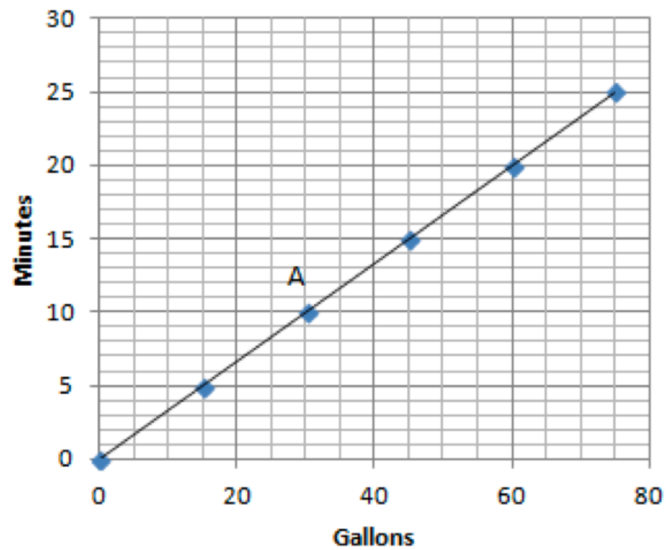
a. In a table, record the x and y values for each point on the graph. What do these ordered pairs (values) represent?

b. Grandma has 1 remaining cup of sugar. How many dozen cookies will she be able to make? Plot the point on the graph above.

c. How many dozen cookies can grandma make if she has no sugar? Can you graph this on the grid provided above? What do we call this point?

Practice:

1. The graph below shows the amount of time a person can shower with a certain amount of water.



a. Can you determine by looking at the graph whether the length of the shower is proportional to the number of gallons of water? Explain how you know.

b. How long can a person shower with 15 gallons of water? With 60 gallons of water?

c. What are the coordinates of point A? Describe point A in the context of the problem.

d. Can you use the graph to identify the unit rate?

e. Plot and label the unit rate on the graph **(1,r)**. Is the point on the line of this relationship?

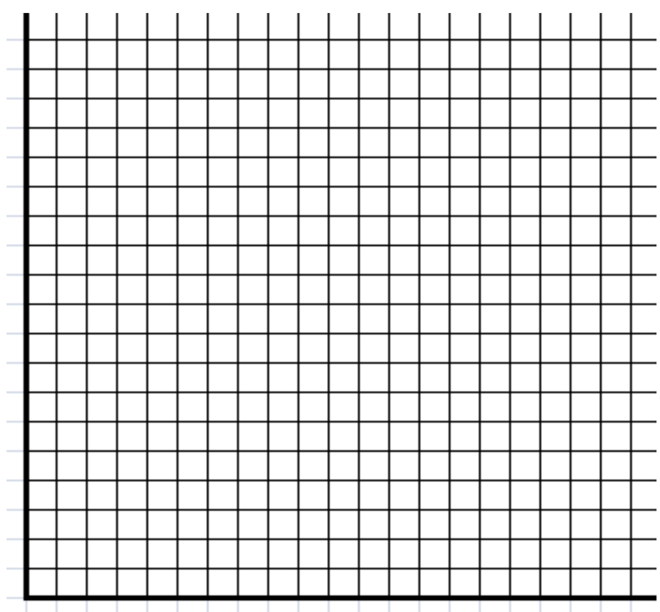
2. Your friend uses the equation $C = 50P$ to find the total cost of P people entering the local Amusement Park.

   a. Create a table and record the cost of entering the amusement park for several different-sized groups of people.

   b. Is the cost of admission proportional to the amount of people entering the Amusement Park? Explain why or why not.

   c. What is the unit rate and what does it represent in the context of the situation?

   d. Create a graph to represent this relationship. Remember to label the x-axis and y-axis.



e. What point MUST be on the graph of the line if the two quantities represented are proportional to each other? Explain why and describe this point in the context of the problem.

f. Would the point (5, 250) be on the graph? What does this point represent in the context of the situation?

3. Jackie is making a snack mix for a party. She is using M&M's and peanuts. The table below shows how many packages of M&M's she needs to how many cans of peanuts she needs to make the mix.

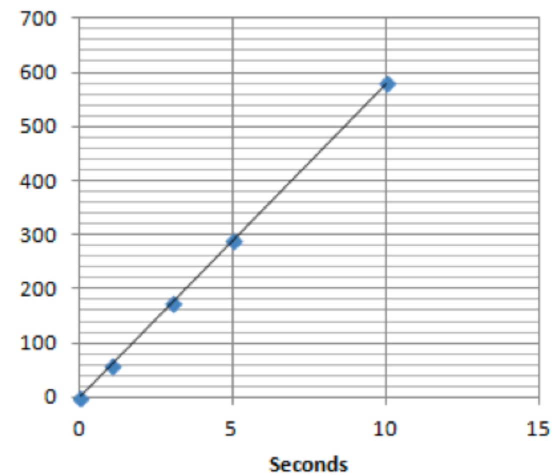
   a. What points MUST be on the graph for the number of cans of peanuts to be proportional to the packages of M&M's? Explain why.

Packages of M&M's	Cans of Peanuts
0	0
1	2
2	4
3	6
4	8

   b. Write an equation to represent this relationship.

   c. Describe the ordered pair (12, 24) in the context of the problem.

4. The graph to the right shows the distance (in ft.) run by a Jaguar.



a. What does the point (5, 290) represent in the context of the situation?

b. What does the point (3, 174) represent in the context of the situation?

c. Is the distance run by the Jaguar proportional to the time? Explain why or why not.

d. Write an equation to represent the distance run by the Jaguar. Explain or model your reasoning.

e. How far can a jaguar run in 2.5 seconds?

f. How long will it take the jaguar to run 986 feet?

5. Championship T-shirts sell for $22 each.

   a. What point(s) MUST be on the graph for the quantities to be proportional to each other?

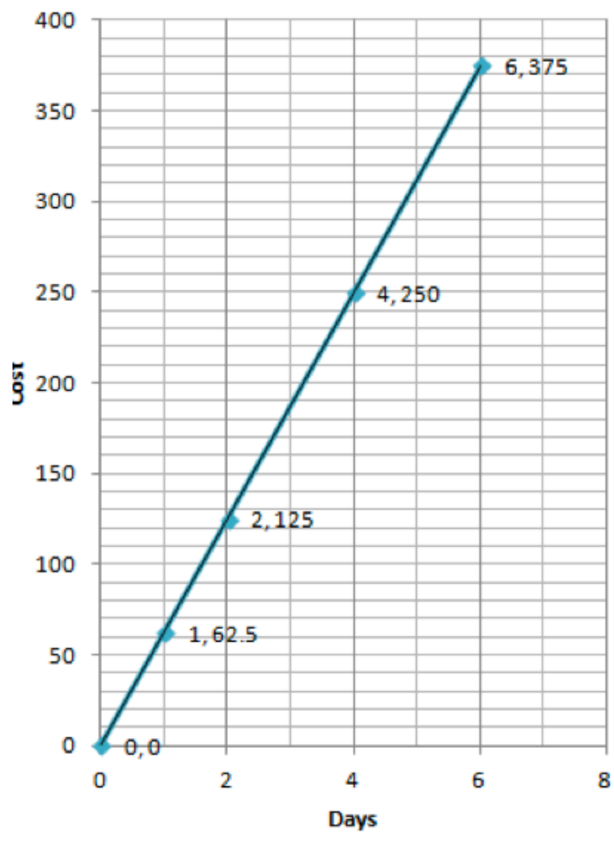
   b. What does the ordered pair (5, 110) represent in the context of this problem?

   c. How many T-shirts were sold if you spent a total of $88?

6. The following graph represents the total cost of renting a car. The cost of renting a car is a fixed amount each day regardless of how many miles the car is driven. It does not matter how many miles you drive; you just pay an amount per day.

   a. What does the ordered pair (4, 250) represent?

   b. What would be the cost to rent the car for a week? Explain or model your reasoning.



7. The following table shows the amount of candy and price paid.

Amount of Candy (pounds)	2	3	5
Cost (Dollars)	5	7.5	12.5

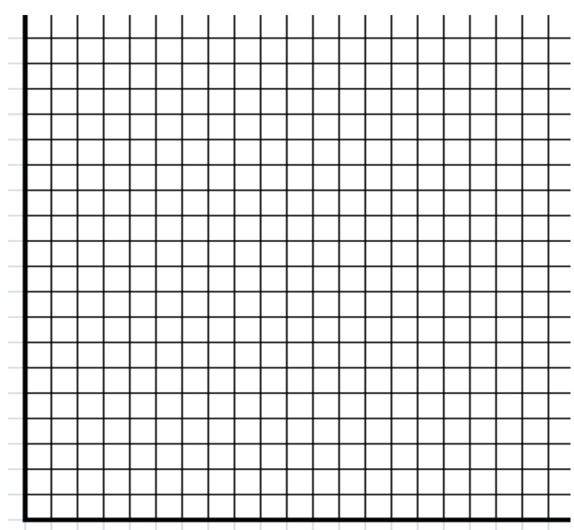
a. Is the cost of candy proportional to the amount of candy?

b. Write an equation to illustrate the relationship between the amount of candy and the cost.

c. Using the equation, predict how much it will cost for 12 pounds of candy?

d. What is the maximum amount of candy you can buy with $60?

e. Graph the relationship.



Lesson 12: Art Class

Activity 1:

1. Watch demonstration. Yellow and blue will be mixed to make _________. To make green, the teacher is using _____ parts yellow water and ____parts blue water. (Mixture A)

2. Predict how the shade of green will change when we mix <u>one</u> part yellow water with <u>seven</u> parts blue water. (Mixture B)

Write a **ratio** to represent each of the water mixtures. How could we write these ratios if we doubled the mixtures to make a bigger batch? Tripled?

Mixture A

$\frac{yellow}{blue} =$

Mixture B

$\frac{yellow}{blue} =$

An equation showing two equivalent ratios is called a _____________________.

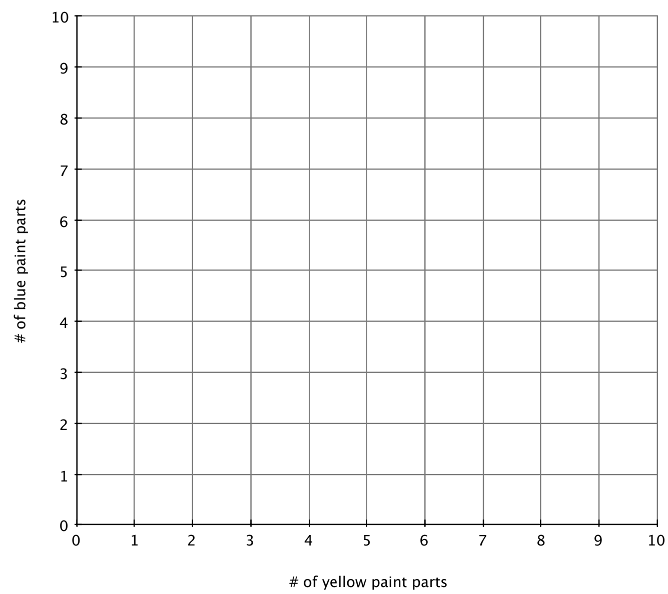
Activity 2:

Students in Ms. Baca's art class are mixing yellow and blue paint to make green. She told them that two mixtures of green will be the same shade of green if the blue and yellow paint are the same ratio. The table below shows the different mixtures students made.

	Mixture A	Mixture B	Mixture C	Mixture D	Mixture E
Yellow	1 part	2 parts	3 parts	4 parts	6 parts
Blue	2 parts	3 parts	6 parts	6 parts	9 parts

1. Determine how many different *shades* of the green paint the students made. Explain how you know.

2. Plot a point for each mixture on the coordinate plane below.



x = yellow paint parts

y = blue paint parts

3. Draw a line connecting each point to (0,0). What do you notice about the lines? What do you notice about the slope of the lines? What does a steeper slope mean? Explain.

4. Create a new shade of green and write a ratio.

5. Using the same coordinate grid, plot a point for a new mixture. Sketch a line connecting the point to (0,0). Describe that mixture of green and compare it to the other mixtures on the graph.

6. We were able to see the different shades of green represented on the graph. Find another method to represent the relationship between the yellow and blue in each of the mixtures.

7. An additional mixture of paint has the ratio of $\frac{yellow}{blue} = \frac{6}{1}$. Describe this mixture and compare it to the others.

Lesson 13: Orange Fizz Experiment

A famous cola company is trying to decide how to change their drink formulas to produce the best tasting soda drinks on the market. The company has three different types of formulas to test with the public. The formula consists of two ingredients: orange juice concentrate and carbonated water.

You are a scientist working for this company, and you will get paid a large commission if you can find the right formula that will sell the best. Your job is to find out which of the formulas people like best.

Activity 1:
Using the company's new formulas, you must follow the recipe to the strict guidelines:

Formula A: 1 tablespoon of orange concentrate to 2 tablespoons of carbonated water

Formula B: 2 tablespoons of orange concentrate to 5 tablespoons of carbonated water

Formula C: 2 tablespoons of orange concentrate to 3 tablespoons of carbonated water

1. Which formula will make a drink that has the strongest orange taste? Show your work and explain your choice.

2. Which formula has the highest amount of carbonated water in the mixture? Estimations may be used. Show your work and justify your answer.

Activity 2:
For researchers to test their product, they will need to produce enough of each of the three drink formulas to take to various locations around the area for taste testing. Researchers would like for at least 100 people to sample each formula. Each sample will contain 1 cup of liquid.

Fill in the table to determine the least amount of concentrate and carbonated water that you would have to use to give 1-cup servings to 100 people.

Formula A:		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
1	**2**	**3 cups (3 servings)**
2	4	6 cups (6 servings)
3	6	9 cups (9 servings)
...	...	,,,
17	34	51 cups (51 servings)
...	...	...
33	66	_____ cups (_____ servings)
34		_____ cups (_____ servings)
	70	_____ cups (_____ servings)

For **Formula A**, How much orange concentrate and carbonated water is needed to serve at least 100 people?

Formula B:		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	**5**	**7 cups (7 servings)**
4	10	14 cups (14 servings)
8		28 cups (28 servings)
...	...	,,,
	35	______cups (49 servings)
...	...	...
26		______ cups (_____ servings)
		98 cups (98 servings)
		______cups (servings)

For **Formula B**, How much orange concentrate and carbonated water is needed to serve at least 100 people?

Formula C:		
Orange Concentrate (cups)	Carbonated Water (cups)	Total Amount (cups/servings)
2	**3**	**5 cups (5 servings)**
4	6	10 cups (10 servings)
...	...	,,,
20		_____ cups (_____ servings)
...	...	...
36		_____ cups (_____ servings)
	57	_____ cups (_____ servings)
		_____ cups (_____ servings)

For **Formula C**, How much orange concentrate and carbonated water is needed to serve at least 100 people?

2. Your lab technicians will be bringing you all of the supplies that you will need in order to make the formulas at the sites. How much of each ingredient will you need?

	Orange Concentrate	Carbonated Water
Formula A		
Formula B		
Formula C		
Total (cups)		

3. Your lab technician calls to tell you that he only has gallon jugs. (Hint: one gallon=16 cups)
    a. How many gallons of orange concentrate do you need to make Formula A? Justify your answer.
    b. How many gallons of carbonated water do you need to make Formula B? Justify your answer.

4. Write a letter to the company manager explaining which formula the company should use based on the data you collected. Also, what other information do you think the company should collect before they make a final decision?

Lesson 14: Pulse Rate

1. Find your pulse on your left wrist. Count how many pulse beats you feel in 15 seconds. Record your results.

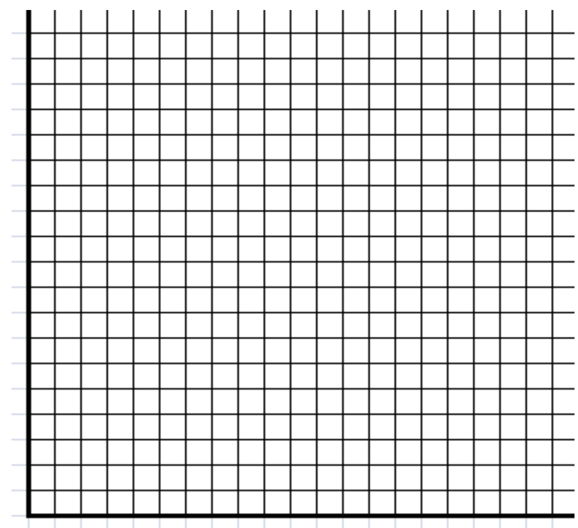
	Heart Beats in 15 seconds
Trail 1	
Trial 2	
Trial 3	
Average	

2. Write your results as a ratio of beats per 15 seconds. ______________

3. Now change the ratio into a **unit rate:** _____________ **beats/second.**

4. Pulse rate is usually given in beats per minute (BPM).
Calculate your unit pulse rate in **beats/minute:** ____________________

5. Write an equation to show how you can calculate the number of beats for any number of minutes.



6. Graph your data.

7. Approximately how many times will your heart beat in:

a. 1 hour

b. 1 week

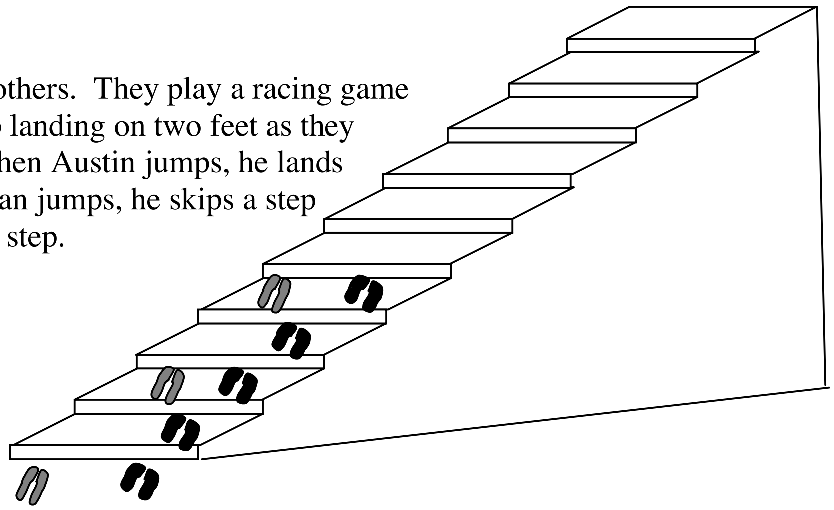
c. 1 day

d. 1 year

Problem of the Month
First Rate

Level A

Dylan and Austin are brothers. They play a racing game up the stairs. They jump landing on two feet as they race up the staircase. When Austin jumps, he lands on each step. When Dylan jumps, he skips a step and lands on every other step.



1. Who has to take more jumps to get to the top of the stairs?

2. When Dylan jumps up the staircase, how many jumps does he make?

3. When Austin jumps up the staircase, how many jumps does he make?

4. If Austin and Dylan each took 5 jumps, who would be farthest up the stairs?

5. At the end of the race who took less jumps?

6. Who do you think won the race? Explain your answer.

Level B

Tom and Diane start to race. Tom took 4 seconds to run 6 yards. Diane ran 5 yards in 3 seconds.

If they continued to run at the same speeds, who would get to 30 yards first? Show how you figured out.

Who runs faster? How can you compare their speeds?

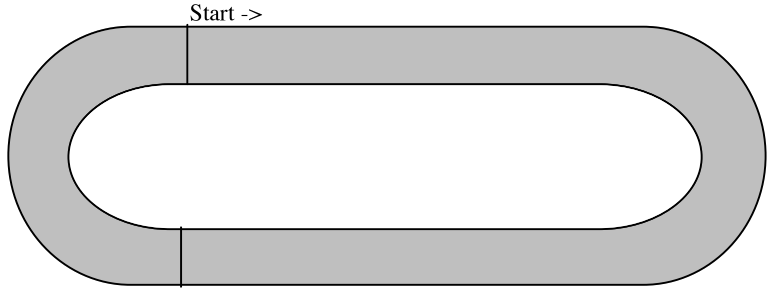
Level C

The Environmental Club at school attends an annual community clean-up event. They have recycling games. A team is assigned an area of land that is scattered with litter. The goal is for a pair of participants to clean up the area in the fastest time possible.

Tammy, working alone, could clean one-half the area in one hour. Her partner Melissa, working alone, could clean one-third of the area in one hour. During the contest when they work together, how long will it take them to clean the area? Explain how you found your solution.

Level D

You are an Olympic runner. You have just qualified to be in the finals of the 1,500-meter race. The track is 400 meters in an oval shape. The race is three and three-fourth laps around the track.



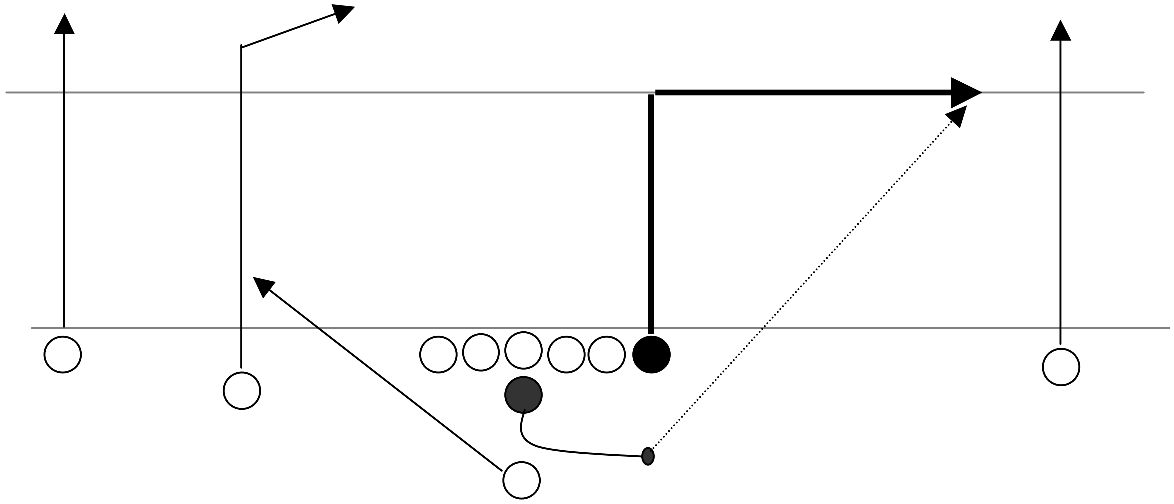
The favorite to win the race is a Kenyan, who holds the current best time, which is 3 minutes 29.4 seconds. The Kenyan runs a very steady race. Each of the Kenyan's lap times (400 meters) are within a second of each other.

You run a completely different type of race. You have a very strong kick, which means you usually lag behind for the first three laps to save energy and then when the leader has 300 meters to go you pour it on to win at the tape. You like to save energy in the first three laps, but you don't want to be more than 50 meters behind when you start your kick to the finish line.

Determine your strategy to win this race. What is the average speed you need to run the first part of the race? What is the average speed you need to run during your kick to win the race? How might your race change if the Kenyan runs two seconds faster?

Level E

It is third down, ten yards to go for a first down. The quarterback calls his favorite play, a roll out to the right and a square out pass to his tight end. See the diagram of the play below:



On the snap from center, the tight end runs straight ahead for ten yards, makes a sharp right turn and runs towards the side lines. The quarterback rolls to his right and stops directly behind where the tight end began, but six yards behind the line of scrimmage. The quarterback does not make the pass until after the tight makes his break towards the sidelines. The tight end is running towards the sideline at a speed of 8 yards/sec. The quarterback tracks the receiver deciding when to throw the pass and the flight path of the ball. If the tight end makes the catch 12 yards after the break, how far does the quarterback throw the pass (in straight line) and at what rate is the distance between the receiver and quarterback changing?

Suppose the quarterback threw the pass sooner, and the receiver is running at the same speed. The distance the ball traveled was 17.3 yards. How many yards after the break was the ball caught and at what rate is the distance between the receiver and quarterback changing?

Given the constant speed of the receiver, consider several locations where the square out pass could be completed. Explain the relationship between the spot of the completion, the distance of the pass and at what rate is the distance between the receiver and quarterback changing?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
1. Make sense of problems and persevere in solving them. • Interpret and make meaning of the problem to find a starting point. Analyze what is given in order to explain to themselves the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor their progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask themselves, "Does this make sense?" Can understand various approaches to solutions.	How would you describe the problem in your own words? How would you describe what you are trying to find? What do you notice about...? What information is given in the problem? Describe the relationship between the quantities. Describe what you have already tried. What might you change? Talk me through the steps you've used to this point. What steps in the process are you most confident about? What are some other strategies you might try? What are some other problems that are similar to this one? How might you use one of your previous problems to help you begin? How else might you organize...represent... show...?
2. Reason abstractly and quantitatively. • Make sense of quantities and their relationships. • Decontextualize (represent a situation symbolically and manipulate the symbols) and contextualize (make meaning of the symbols in a problem) quantitative relationships. • Understand the meaning of quantities and are flexible in the use of operations and their properties. • Create a logical representation of the problem. • Attends to the meaning of quantities, not just how to compute them.	What do the numbers used in the problem represent? What is the relationship of the quantities? How is _______ related to ________? What is the relationship between ______and ______? What does_______mean to you? (e.g. symbol, quantity, diagram) What properties might we use to find a solution? How did you decide in this task that you needed to use...? Could we have used another operation or property to solve this task? Why or why not?
3. Construct viable arguments and critique the reasoning of others. • Analyze problems and use stated mathematical assumptions, definitions, and established results in constructing arguments. • Justify conclusions with mathematical ideas. • Listen to the arguments of others and ask useful questions to determine if an argument makes sense. • Ask clarifying questions or suggest ideas to improve/revise the argument. • Compare two arguments and determine correct or flawed logic.	What mathematical evidence would support your solution? How can we be sure that...? / How could you prove that...? Will it still work if...? What were you considering when...? How did you decide to try that strategy? How did you test whether your approach worked? How did you decide what the problem was asking you to find? (What was unknown?) Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not? What is the same and what is different about...? How could you demonstrate a counter-example?
4. Model with mathematics. • Understand this is a way to reason quantitatively and abstractly (able to decontextualize and contextualize). • Apply the mathematics they know to solve everyday problems. • Are able to simplify a complex problem and identify important quantities to look at relationships. • Represent mathematics to describe a situation either with an equation or a diagram and interpret the results of a mathematical situation. • Reflect on whether the results make sense, possibly improving/revising the model. • Ask themselves, "How can I represent this mathematically?"	What number model could you construct to represent the problem? What are some ways to represent the quantities? What is an equation or expression that matches the diagram, number line.., chart..., table..? Where did you see one of the quantities in the task in your equation or expression? How would it help to create a diagram, graph, table...? What are some ways to visually represent...? What formula might apply in this situation?

Summary of Standards for Mathematical Practice	Questions to Develop Mathematical Thinking
5. Use appropriate tools strategically. • Use available tools recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	What mathematical tools could we use to visualize and represent the situation? What information do you have? What do you know that is not stated in the problem? What approach are you considering trying first? What estimate did you make for the solution? In this situation would it be helpful to use...a graph..., number line..., ruler..., diagram..., calculator..., manipulative? Why was it helpful to use...? What can using a ______ show us that _____may not? In what situations might it be more informative or helpful to use...?
6. Attend to precision. • Communicate precisely with others and try to use clear mathematical language when discussing their reasoning. • Understand the meanings of symbols used in mathematics and can label quantities appropriately. • Express numerical answers with a degree of precision appropriate for the problem context. • Calculate efficiently and accurately.	What mathematical terms apply in this situation? How did you know your solution was reasonable? Explain how you might show that your solution answers the problem. What would be a more efficient strategy? How are you showing the meaning of the quantities? What symbols or mathematical notations are important in this problem? What mathematical language...,definitions..., properties can you use to explain...? How could you test your solution to see if it answers the problem?
7. Look for and make use of structure. • Apply general mathematical rules to specific situations. • Look for the overall structure and patterns in mathematics. • See complicated things as single objects or as being composed of several objects.	What observations do you make about...? What do you notice when...? What parts of the problem might you eliminate..., simplify...? What patterns do you find in...? How do you know if something is a pattern? What ideas that we have learned before were useful in solving this problem? What are some other problems that are similar to this one? How does this relate to...? In what ways does this problem connect to other mathematical concepts?
8. Look for and express regularity in repeated reasoning. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results	Explain how this strategy work in other situations? Is this always true, sometimes true or never true? How would we prove that...? What do you notice about...? What is happening in this situation? What would happen if...? Is there a mathematical rule for...? What predictions or generalizations can this pattern support? What mathematical consistencies do you notice ?