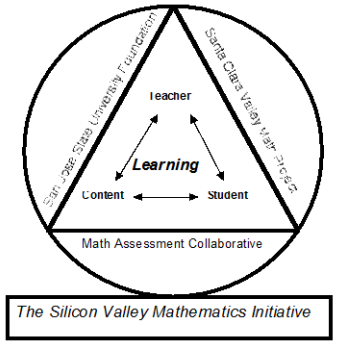
The Common Core State Standards

The transition to the CCSSM and the new Assessment Consortia



David Foster
Silicon Valley Mathematics Initiative
www.svmimac.org

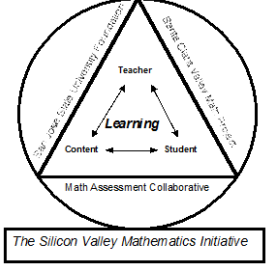
Optimism

"Optimism is an essential ingredient for innovation. How else can the individual welcome change over security, adventure over staying in safe places? A significant innovation has effects that reach much further than can be imagined at the time, and creates its own uses. It will not be held back by those who lack the imagination to exploit its use, but will be swept along by the creative members of our society for the good of all. Innovation cannot be mandated any more than a baseball coach can demand that the next batter hit a home run. He can, however, assemble a good team, encourage his players, and play the odds."

Robert N. Noyce

Silicon Valley Mathematics Initiative

87 Members - School Districts, Charter School Networks, and Schools



Albany USD
Alvord SD (Riverside County)
Antioch Unified SD
Aspire Charter School Network
Assumption School (San Leandro)
Bayshore SD
Belmont-Redwood Shores SD
Berryessa SD
Bolinas – Lagunitas SD
Brisbane SD
Buckeye SD
Cambrian SD
Campbell Union ESD
Castro Valley USD
Charter School of Morgan Hill
Chicago Public School
Creative Arts Charter (SF)
CSU San Bernardino
Cotati – Rohnert Park
Cupertino SD
Dade County Schools (GA)
Del Mar USD (San Diego Co)
Discovery Charter School
Dioceses of Santa Clara
Dublin USD
East Side UHSD
Edmonds Community College
Emery SD
Etiwanda SD (San Bernardino Co)
Gilroy (Brownell MS)
Fairfield-Suisun USD
Fremont USD
Forsyth County School (GA)
Hamilton County (Tn)
Hayward USD
Jefferson ESD
Jefferson HSD
Las Lomitas SD
La Honda-Pescadero Sd
Livermore USD
Los Altos SD
Los Gatos SD
Menlo Park SD
Monterey Peninsula USD
Moreland SD
Mountain SD
National Council of La Raza
New York City PS
New Visions for Public Schools
Oakland Unified SD
Pacifica SD
Pajaro Valley USD
Palo Alto USD
Pittsburgh USD
Portola Valley SD
Ravenswood City SD
Riverside COE
Redwood City Schools
Sacramento City USD
Salinas City Schools
San Carlos CLC
San Diego COE
San Diego Unified School District
San Francisco USD
SMFC (Park School)
San Jose Unified SD
San Leandro USD
San Ramon Valley USD
Santa Clara USD
Santa Cruz City Schools
Saint Michael's School (Poway)
Saint Patrick's School (San Jose)
Saratoga
Scotts Valley USD
SCCOE County Court Schools
Sequoia HSD
SMCOE County Court Schools
South Cook Service District
South San Francisco USD
Sumter County (GA)
The Nueva School
Union SD
University of Illinois, Chicago
Valley Christen (Dublin)
Valdosta City (GA)
Walnut Creek SD
Woodside SD

Supporting Teaching and Learning of Mathematics Since 1996

The state of mathematics education in America

After a decade of high-stakes accountability promising to equalize performances among students of all demographic classifications, what is happening to the “***Gap”***?

The Achievement Gap

NAEP 2009	US 4th Grade	US 8th Grade
All Students	240	283
Black	222	260
White	248	292
Hispanic	227	266
Asian	255	300
Low Income	228	266
Mid-High Income	250	293
English Learner	218	243
English Fluent	242	284

Source: U.S. Department of Education

National Achievement of Education Progress (NAEP 2009 Math) The Nation's Report Card

Approximately 10 scale points is equivalent to a grade level of learning

NY TIMES April 29, 2009

Persistent Racial Gap Seen in Students Test Scores

By SAM DILLON

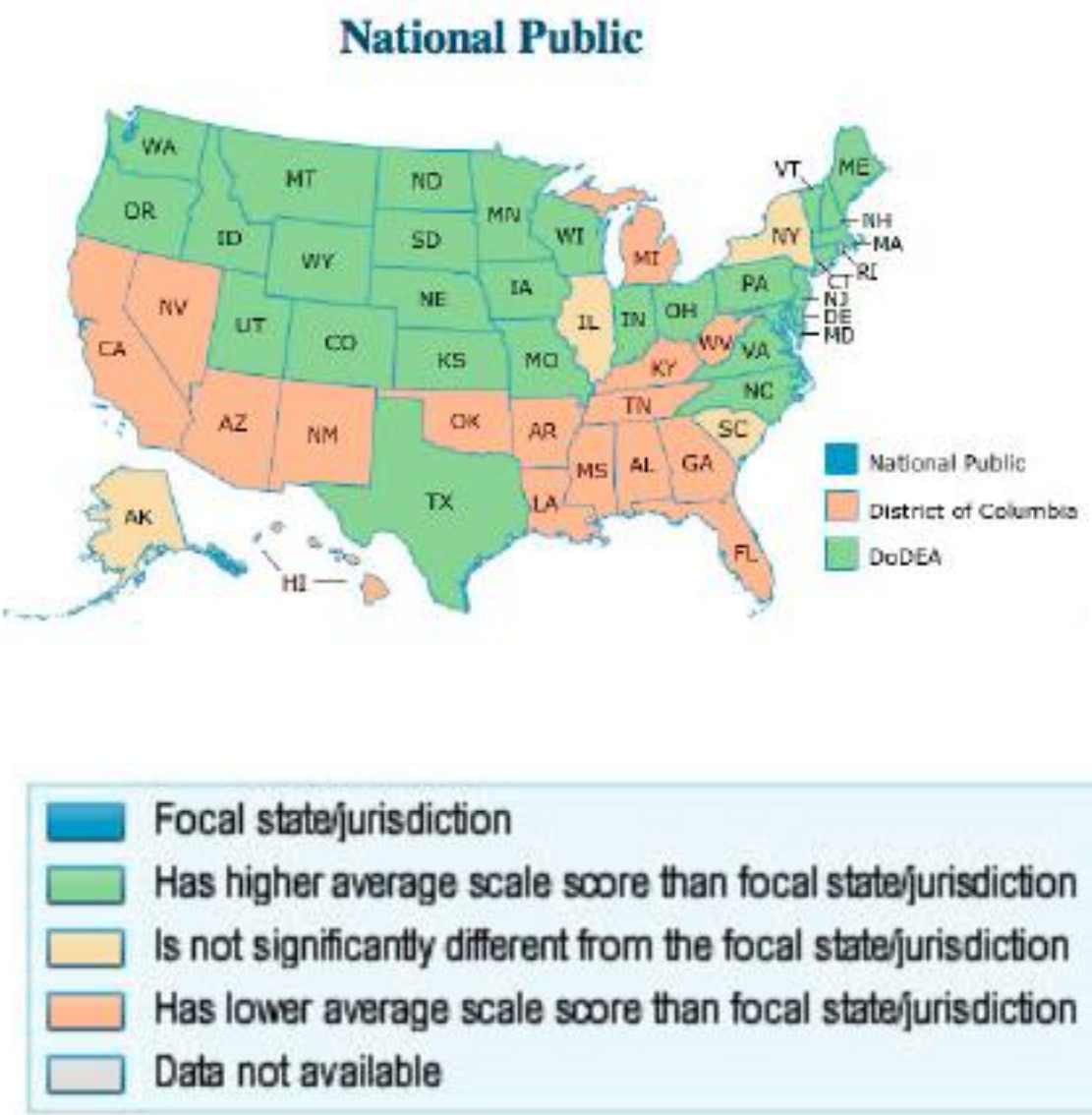
The achievement gap between white and minority students has not narrowed in recent years, despite the focus of the No Child Left Behind law on improving black and Hispanic scores, according to results of a federal test considered to be the nation's best measure of long-term trends in math and reading proficiency.

NAEP 2009 8th grade

National Ave. 282

Massachusetts (1st) 299

California (49th) 270



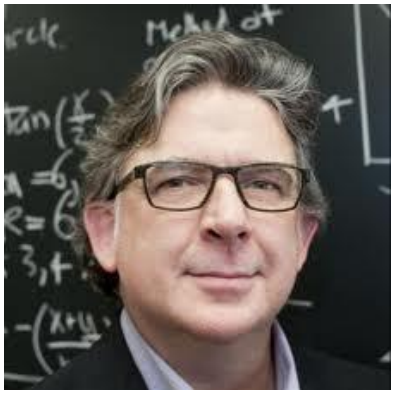
Where you live and your background correlates to how you score on tests.

Common Core Standards:

A New Direction linking Instruction and Assessment

Three Central Authors

Common Core State Standards in Mathematics



Bill McCallum

Phil Daro

Jason Zimba

Charges given to the authors:

- All students College and Career Ready by 11th grade
- Internationally Benchmarked
- Make the standards “Fewer, Clear and Higher”

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Domains K–8

Counting & Cardinality						Ratios & Proportional Relationships		
Operations and Algebraic Thinking						The Number System		
Number and Operations in Base Ten						Expressions and Equations		
			Fractions					Functions
Measurement and Data								
Geometry						Geometry		
						Statistics and Probability		
K	1	2	3	4	5	6	7	8

State Senator Hancock

"Several implementation issues arise by California adopting a different set of grade 8 math standards from other participating states. For example, instructional materials for use in California would need to be different from those used by other states - the unique additional standards may increase the costs of those materials for our local school districts. In addition, assessment consortia will be developing assessment aligned to the common core standards and not the variation adopted in California. This may result in issues with our Algebra standards and curriculum not being aligned with our assessment and accountability system."

California SB 1200

Establishes an 11-member standards review commission to review and recommend modifications to the 8th grade mathematics standards, to be appointed as follows:

Requires the recommendations of the standards review commission and modifications approved by the SBE to ensure all of the following:

a) The rigor of the state common core standards is maintained so that all high school graduates are prepared for college and careers, as specified in the common core standards.

b) All of the common core standards developed by the national Common Core Standards Initiative consortium are adopted.

c) Modifications, including additions and revisions, amount to no more than 15% of the state common core standards adopted by the SBE.

CALIFORNIA DEPARTMENT OF EDUCATION
NEWS RELEASE

TOM TORLAKSON
State Superintendent
of Public Instruction

California Adopts Modified Math Standards to Restore Local Decision Making

Required by Legislation, Move Allows Progress Toward Common Core

The move rescinds action by the prior Board in 2010, which adopted standards that contained a unique Grade 8 Algebra I course inconsistent with the published *Common Core State Standards for Mathematics*.

Torlakson recommended the unique Grade 8 Algebra I course be replaced with Algebra I and Mathematics I courses based upon the *Common Core State Standards for Mathematics*.

Date: Wed, 16 Jan 2013

Two Mathematics Pathways

Two Regular Sequences:

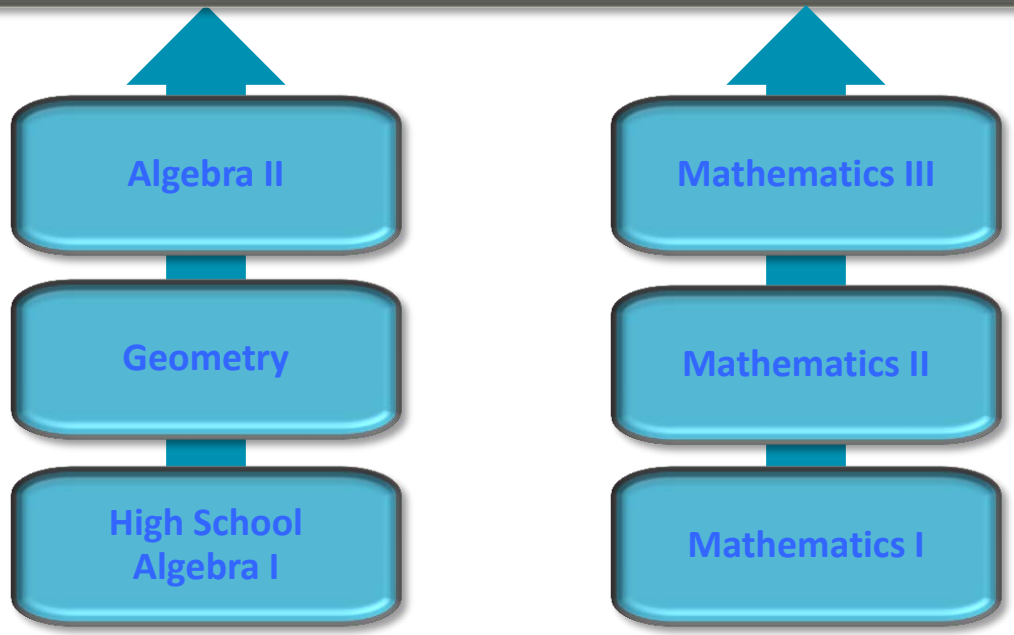
Traditional Pathway

- 2 Algebra courses,1 Geometry course, with Probability and Statistics interwoven

Integrated Pathway

- 3 courses that attend to Algebra, Geometry, and Probability and Statistics each year

Courses in higher level mathematics: Precalculus, Calculus*, Advanced Statistics, Discrete Mathematics, Advanced Quantitative Reasoning, or courses designed for career technical programs of study.



Traditional Pathway
Typical in U.S.

Integrated Pathway
Typical outside of U.S.

Common Core State Standards

- Define the knowledge and skills students need for college and career
- Developed voluntarily and cooperatively by states; more than 40 states have adopted
- Provide clear, consistent standards in English language arts/literacy and mathematics

Source: www.corestandards.org

Smarter Balanced Assessment Consortium

States have joined Assessment Consortia funded by RttT

Partnership for Assessment of Readiness of College and Careers (PARCC) is being managed by Achieve, Inc., a Washington-based non-profit. There are 23 states and DC in PARCC.

The SMARTER-Balanced Assessment Consortium is being managed by San Francisco-based WestEd and its senior program director, Stanley Rabinowitz. SMARTER-Balanced enlisted 31 states.

At this point, both consortia are targeting the first test administration by 2014-15. Both say they will integrate summative or end-of-the-year tests with interim and formative assessments that can guide instruction during the year. Both are promising to include performance-based tasks, such as conducting a science experiment and writing short answers to questions, that are intended to show deeper levels of learning and thinking than multiple choice questions supposedly can measure. Both indicate that technology will play a major role.

Goals of Assessment

"We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics."

Everybody Counts

CST – Released Items Algebra 1

The total cost (c) in dollars of renting a sailboat for n days is given by the equation

$$c = 120 + 60n.$$

If the total cost was $360, for how many days was the sailboat rented?

A 2

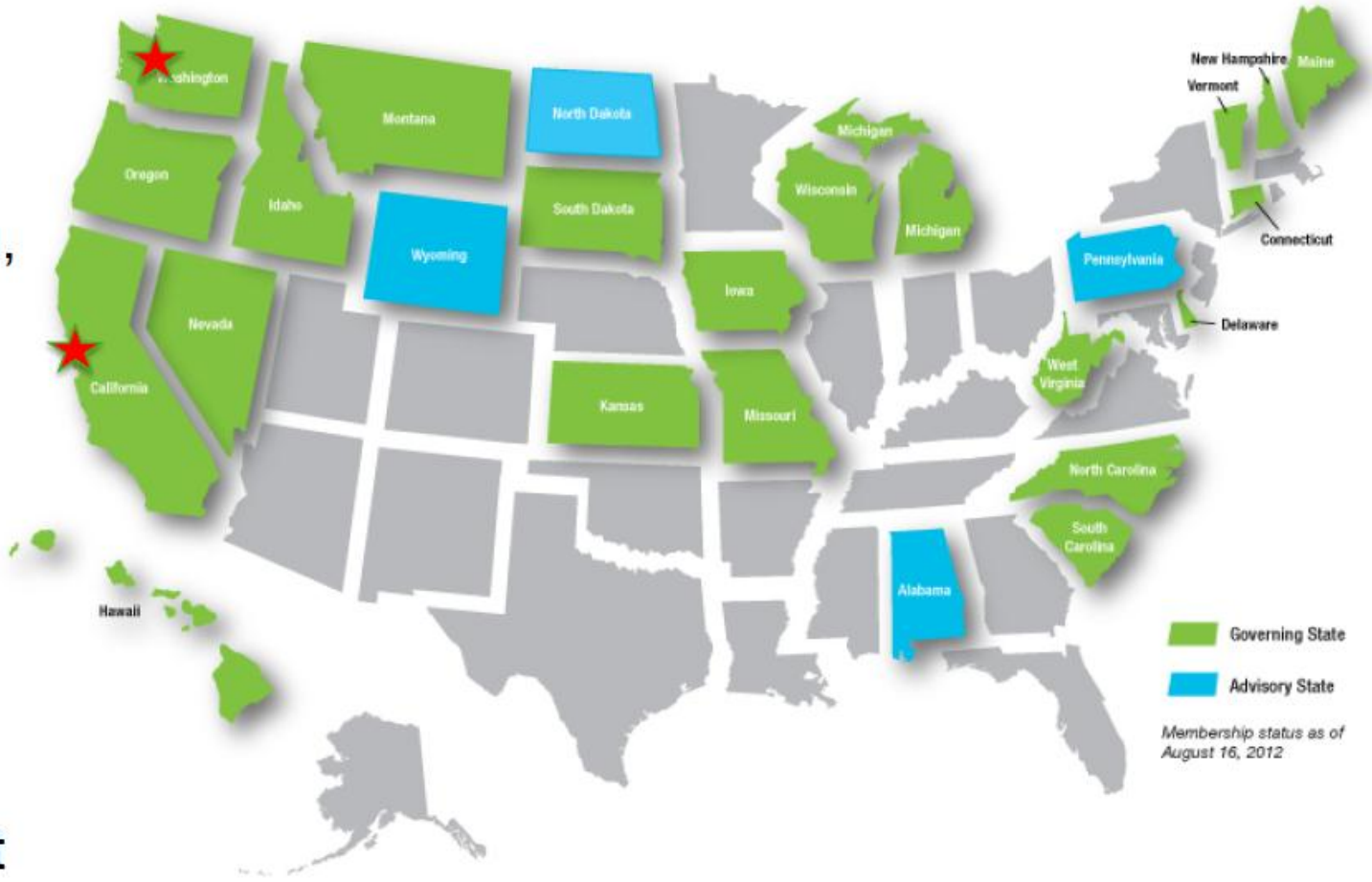
B 4

C 6

D 8

A National Consortium of States

- 25 states representing 40% of K-12 students
- 21 governing, 4 advisory states
- Washington state is fiscal agent
- WestEd provides project management services



Smarter Balanced Assessment Consortium

Current vs. CCSS

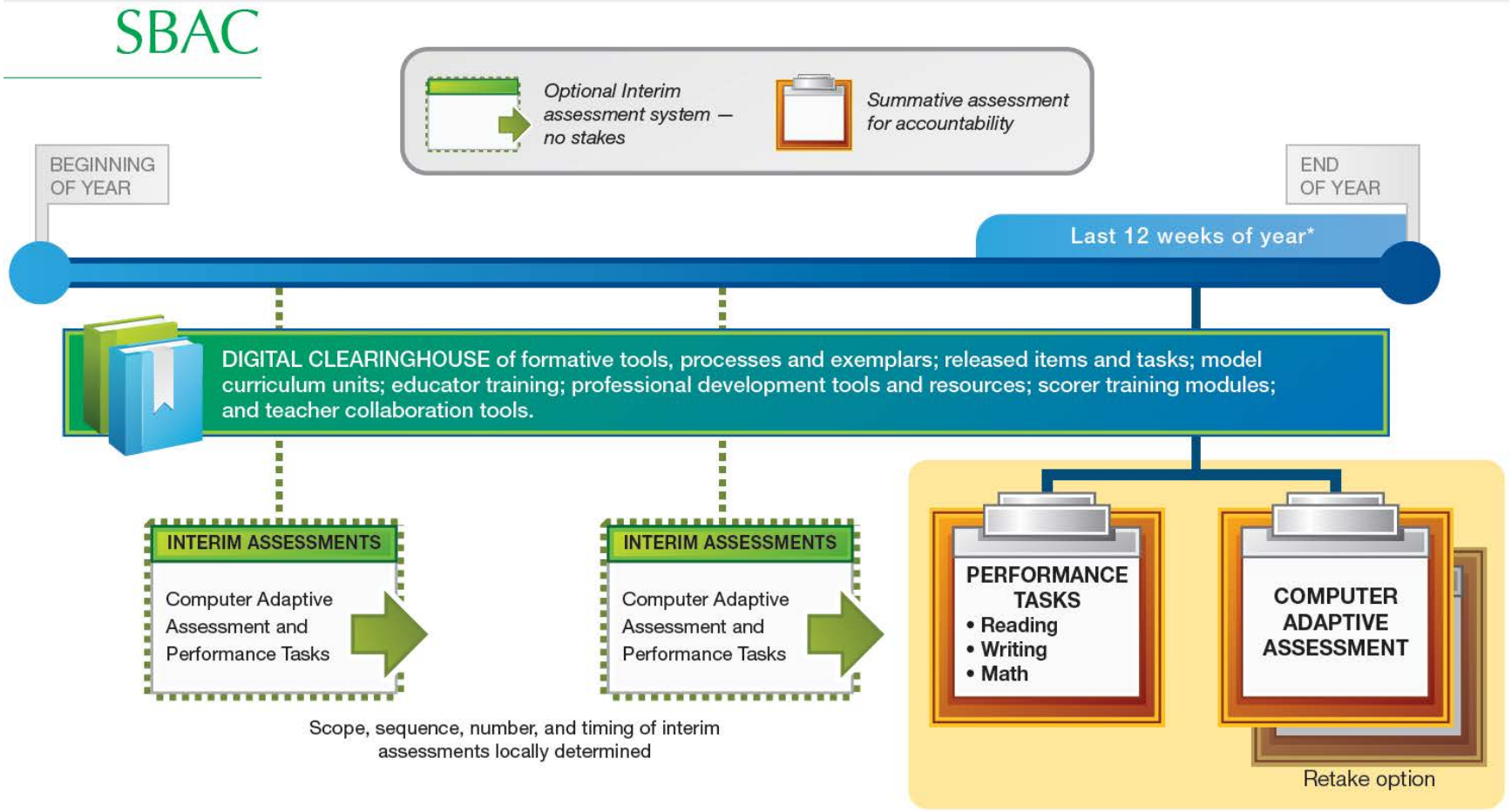
Current STAR Assessments

- Grades 2-11, writing at 4th and 7th
- Only paper & pencil option
- Taken around 85% of the instructional days
- Only multiple choice
- Part of the state and federal accountability system

Proposed CCSS Assessments

- Grades 3-8 and 11, Grades 9 and 10 available for states that choose to use them
- Delivered via computer (Paper and pencil option available for 3 years) and are computer adaptive
- Taken during the final 12 weeks of school
- Performance tasks and comprehensive end-of -year computer adaptive assessment which will some selected response items
- Accountability system has not been established yet

The System
(Possible Scenario)



Claim-Evidence-Warrant

A Model for Analyzing Arguments

(adapted from the work of Stephen Toulmin)

SMARTER
Balanced Assessment Consortium

PARCC States

Content Specifications
for the Summative assessment of the
Common Core State Standards for Mathematics

DRAFT TO ACCOMPANY GOVERNING STATE VOTE ON ASSESSMENT CLAIMS

March 20, 2012

Developed with input from content experts and Smarter Balanced Assessment Consortium Staff, Work Group Members, and Technical Advisory Committee

Acknowledgements

Alan Schoenfeld, University of California at Berkeley and **Hugh Burkhardt**, Shell Centre, University of Nottingham served as principal authors of this paper. Sections of the document were also authored by **Jamal Abedi**, University of California at Davis; **Karin Hess**, National Center for the Improvement of Educational Assessment; **Martha Thurlow**, National Center on Educational Outcomes, University of Minnesota

Significant contributions and organization of this second draft were provided by **Shelbi Cole**, Connecticut State Department of Education, and **Jason Zimba**, Student Achievement Partners. The project was facilitated by **Linda Darling-Hammond** at Stanford University.

Others who offered advice and feedback on the document include:
Rita Crust, Lead Designer, Mathematics Assessment Resource Service
Past President, Association of State Supervisors of Mathematics
Brad Findell, Former Mathematics Initiatives Administrator, Ohio Department of Education
David Foster, Director, Silicon Valley Mathematics Initiative
Henry Pollak, Adjunct Professor, Columbia University, Teachers College,
Former Head of Mathematics and Statistics, Bell Laboratories
W. James Popham, Emeritus Professor, University of California, Los Angeles
Cathy Seeley, Senior Fellow, Charles A. Dana Center, The University of Texas at Austin
Malcolm Swan, Professor of Mathematics Education, Centre for Research in Mathematic Education,
University of Nottingham

SMARTER – Balance

Content Specifications - CCSSM

Four Major Claims for the SMARTER Balanced Assessment Consortium's assessments of the *Common Core State Standards for Mathematics*

Claim #1 **- Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.**

Claim #2 **- Students can frame and solve a range of complex problems in pure and applied mathematics.**

Claim #3 **- Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.**

Claim #4 **- Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.**

Assessment: What We Know

- Assessments will begin in 2014-15.
- California is a governing state in the SMARTER Balanced Assessment Consortium.
- Assessments will include:
  - Computer Adaptive Assessments (interim & summative)
  - Performance Assessments (interim & summative)
    - Selected Response
    - Technology Enhanced
    - Constructed Response
    - Performance Task
    - Extended Performance Event

Traditional Selected Response CST Example

A company has 6 big trucks. Each truck has 18 wheels. How many wheels is this in all?

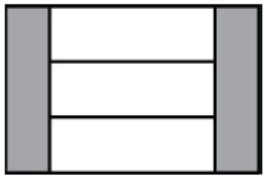
A 24
B 96
C 108
D 116

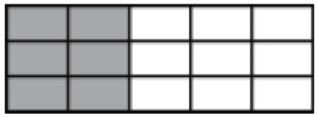
2009 California Standards Test Released Test Question pg. 14, #34

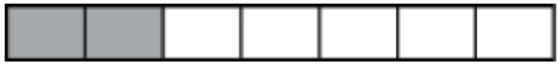
Non-Traditional Selected Response

For numbers 1a-1d, state whether or not each figure has $\frac{2}{5}$ of its whole shaded.

1a.  (Y) Yes (N) No

1b.  (Y) Yes (N) No

1c.  (Y) Yes (N) No

1d.  (Y) Yes (N) No

Non-Traditional Selected Response Rubric

Scoring Rubric

Responses to this item will receive 0-2 points, based upon the following:

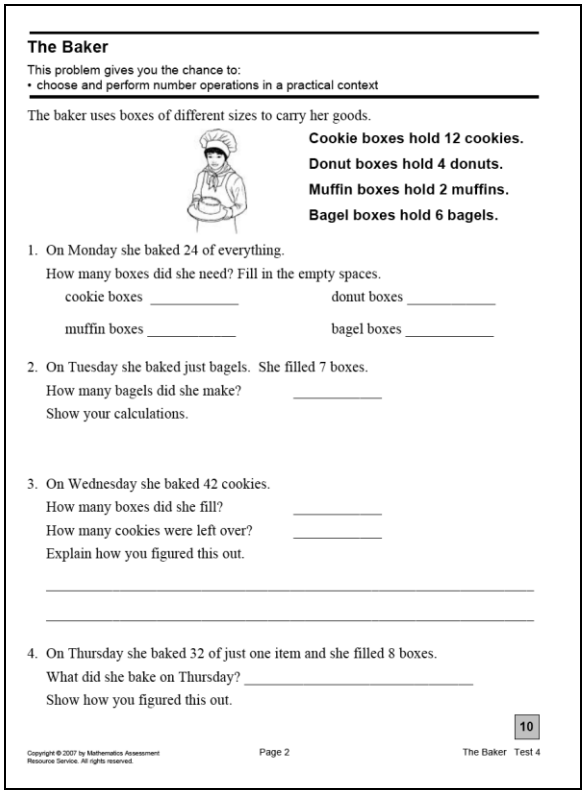
2 points: YNYN The student has a solid understanding of 2/5 as well as an equivalent form of 2/5.

1 point: YNNN, YYNN, YYYN The student has only a basic understanding of 2/5. Either the student doesn't recognize an equivalent fraction for 2/5 or doesn't understand that all 5 parts must be equal-sized in figure 1b.

0 points: YYYY, YNNY, NNNN, NNYY, NYYN, NYNN, NYYY, NYNN, NNNN, NYNY, NNYN, NNNY The student demonstrates inconsistent understanding of 2/5 or answers "Y" to figure 1d, clearly showing a misunderstanding of what 2/5 means. Figure 1d is considered a "disqualifier" and an answer of "Y" to this part of the item would cancel out any other correct responses as "guesses" on the part of the student.

Performance Assessments

To Inform Instruction And Measure Higher Level Thinking



The Baker

This problem gives you the chance to:
• choose and perform number operations in a practical context

The baker uses boxes of different sizes to carry her goods.

Cookie boxes hold 12 cookies.
Donut boxes hold 4 donuts.
Muffin boxes hold 2 muffins.
Bagel boxes hold 6 bagels.

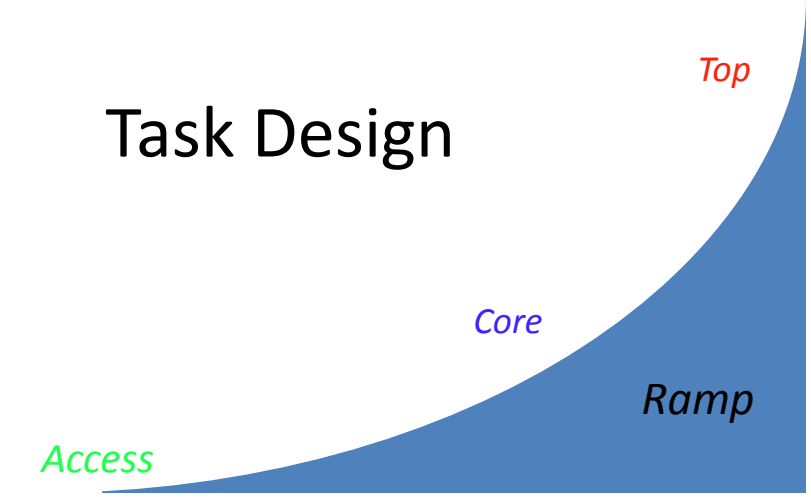
1. On Monday she baked 24 of everything.
   How many boxes did she need? Fill in the empty spaces.
   cookie boxes _____ donut boxes _____
   muffin boxes _____ bagel boxes _____

2. On Tuesday she baked just bagels. She filled 7 boxes.
   How many bagels did she make? _____
   Show your calculations.

3. On Wednesday she baked 42 cookies.
   How many boxes did she fill? _____
   How many cookies were left over? _____
   Explain how you figured this out.

4. On Thursday she baked 32 of just one item and she filled 8 boxes.
   What did she bake on Thursday? _____
   Show how you figured this out.

10



Entry level (access into task)

Core Mathematics - (meeting standards)

Top of Ramp (conceptually deeper, beyond)

- The Mathematics Assessment Resource Service (MARS) is an NSF funded collaboration between U.C. Berkeley and the Shell Centre in Nottingham England.
- The Assessments target grades 2- Geometry and are aligned with the State and NCTM National Math Standards.

SHELL CENTRE

BALANCED ASSESSMENT

MARS

Apprentice Task

CR 4: Baseball Jerseys

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks 2 local companies to give him a price.

1. 'Print It' will charge $21.50 each for the jerseys.

   Using *n* for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

   _____

2. 'Top Print' has a Set-Up cost of $70 and then charges $18 for each jersey.

   Using *n* to stand for the number of jerseys ordered and *c* for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

   _____

3. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to order for the price from 'Top Print' to be less than from 'Print It'.
   Explain how you figured it out.

   _____

   _____

   _____

   _____

   _____

4. Bill decides to order 30 jerseys from 'Top Print'.
   How much more would the jerseys have cost if he had bought them from 'Print It'?
   Show all your calculations.

   _____

Baseball Jerseys

This problem gives you the chance to:

- work with equations that represent real life situations

Bill is going to order new jerseys for his baseball team.

The jerseys will have the team logo printed on the front.

Bill asks two local companies to give him a price.

1. 'Print It' will charge \$21.50 each for the jerseys.

   Using n for the number of jerseys ordered, and c for the total cost in dollars, write an equation to show the total cost of jerseys from 'Print It'.

2. 'Top Print' has a one-time setting up cost of \$70 and then charges \$18 for each jersey.

   Using n to stand for the number of jerseys ordered, and c for the total cost in dollars, write an equation to show the total cost of jerseys from 'Top Print'.

3. Bill decides to order 30 jerseys from 'Top Print'.
How much more would the jerseys cost if he buys them from 'Print It'?
Show all your calculations. _____

4. Use the two equations from questions 1 and 2 to figure out how many jerseys Bill would need to buy for the price from 'Top Print' to be less than from 'Print It'.
Explain how you figured it out. _____

_____

_____

_____

_____

Performance Exams

40,000 – 70,000 students per year since 1999

Students in grades 2 through 10th/11th grade are administered performance exams (5 apprentice tasks per exam).

Task 1: Candies — Rubric

The core elements of performance required by this task are:
- work with fractions and ratios

Based on these, credit for specific aspects of performance should be assigned as follows

		points	section points
1.	Gives correct answer: 2/3 or 6/9	1	1
2.	Gives correct answer: 3	1	
	Shows work such as: 1 + 3 = 4 12 ÷ 4 =		
	Accept diagrams.	1	2
3	Gives correct answer: 18	2	
	Shows work such as: 2 + 3 = 5 30 ÷ 5 = 6 6 x 3 =		
	Accept diagrams.	1	3
4.	Gives correct answer: 6	1	
	Gives a correct explanation such as: Anthony mixes a ratio of one cup of cream to two cups of chocolate. The ratio stays the same for different amounts. So I wrote the numbers in a chart like this 1 to 2 = a total of 3 2 to 4 = a total of 6 3 to 6 = a total of 9	1	
	Accept diagrams.		2
	Total Points		8

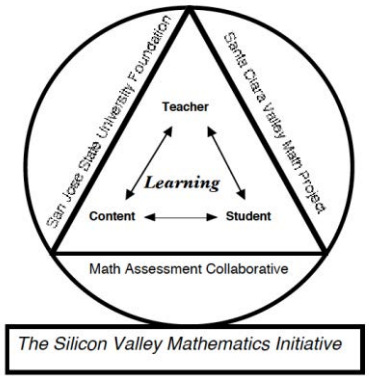
District scoring leaders are trained in using task specific rubrics

Student tests are hand scored by classroom teachers trained and calibrated using standard protocols.

Random sample of student papers are audited and rescored by SJSU math & CS students. (Two reader correlation >0.95)

EDUCATIONAL DATA SYSTEMS

Student results are collected, analyzed, and reported by an independent data contractor.



MAC vs. CST 2012

Silicon Valley Mathematics Initiative

Mathematics Assessment Collaborative
Performance Assessment Exam 2012

MAC vs CST 2012

2nd Grade	MAC Level 1	MAC Level 2	MAC Level 3	MAC Level 4
Far Below Basic	1.0%	0.3%	0.1%	0.0%
Below Basic	1.9%	2.4%	1.2%	0.0%
Basic	1.3%	4.8%	5.5%	0.3%
Proficient	0.4%	3.5%	17.7%	3.4%
Advanced	0.3%	0.9%	23.4%	31.4%

2nd Grade	MAC Below	MAC At/Above	Total
CST Below	11.7%	7.1%	18.8%
CST At/Above	5.1%	75.9%	81.0%
Total	16.8%	83.0%	100%

Elementary Grades

3rd Grade	MAC Below	MAC At/Above	Total
CST Below	15.9%	5.2%	21.1%
CST At/Above	13.7%	65.4%	79.1%
Total	29.6%	70.6%	100%

4th Grade	MAC Below	MAC At/Above	Total
CST Below	16.9%	2.8%	19.7%
CST At/Above	20.3%	60.0%	80.3%
Total	37.2%	62.8%	100%

5th Grade	MAC Below	MAC At/Above	Total
CST Below	20.6%	3.8%	24.4%
CST At/Above	18.7%	56.9%	75.6%
Total	39.3%	60.7%	100%

Middle School

6th Grade	MAC Below	MAC At/Above	Total
CST Below	37.2%	1.4%	38.6%
CST At/Above	25.1%	36.5%	61.6%
Total	62.3%	37.9%	100%

7th Grade	MAC Below	MAC At/Above	Total
CST Below	33.3%	2.1%	35.4%
CST At/Above	27.4%	37.1%	64.5%
Total	60.7%	39.2%	100%

Course 1	MAC Below	MAC At/Above	Total
CST Below	34.5%	3.6%	38.1%
CST At/Above	30.3%	31.5%	61.8%
Total	64.8%	35.1%	100%

8th Graders Taking HS Geometry

Course 2	MAC Below	MAC At/Above	Total
CST Below	3.1%	0.8%	3.9%
CST At/Above	51.3%	44.8%	96.1%
Total	54.4%	45.6%	100%

Expert Tasks

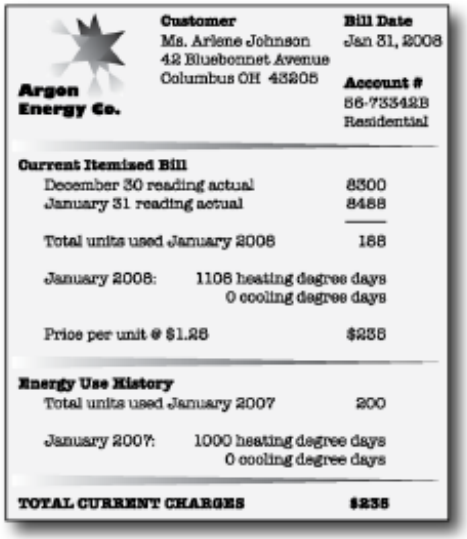
The main point in mathematics teaching is to develop the tactics of problem solving.

George Polya

Gas Bills, Heating Degree Days, and Energy Efficiency

Here is a typical story about an Ohio family concerned with saving money and energy by better insulating their house.

Kevin and Shana Johnson's mother was surprised by some very high gas heating bills during the winter months of 2007. To improve the energy efficiency of her house, Ms. Johnson found a contractor who installed new insulation and sealed some of her windows. He charged her $600 for this work and told her he was pretty sure that her gas bills would go down by "at least 10 percent each year." Since she had spent nearly $1,500 to keep her house warm the previous winter, she expected her investment would conserve enough energy to save at least $150 each winter (10% of $1,500) on her gas bills.



Argon Energy Co.

Customer
Ms. Arlene Johnson
42 Bluebonnet Avenue
Columbus OH 43205

Bill Date
Jan 31, 2008

Account #
86-73342B
Residential

Current Itemized Bill

December 30 reading actual	8300
January 31 reading actual	8488
Total units used January 2008	188
January 2008:	1106 heating degree days 0 cooling degree days
Price per unit @ $1.28	$235

Energy Use History

Total units used January 2007	200
January 2007:	1000 heating degree days 0 cooling degree days

TOTAL CURRENT CHARGES **$235**

Ms. Johnson's gas bill in January 2007 was $240. When she got the bill for January 2008, she was stunned that the new bill was $235. If the new insulation was going to save only $5 each month, it was going to take a very long time to earn back the $600 she had spent. So she called the insulation contractor to see if he had an explanation for what might have gone wrong. The contractor pointed out that the month of January had been very cold this year *and* that the rates had gone up from last year. He said her bill was probably at least 10% less than it would have been without the new insulation and window sealing.

Ms. Johnson compared her January bill from 2008 to her January bill from 2007. She found out that she had used 200 units of heat in January of 2007 and was charged $1.20 per unit (total = $240). In 2008, she had used 188 units of heat but was charged $1.25 per unit (total = $235) because gas prices were higher in 2008. She found out the average temperature in Ohio in January 2007 had been 32.9 degrees, and in January of 2008, the average temperature was more than 4 degrees colder, 28.7 degrees. Ms. Johnson realized she was doing well to have used less energy (188 units versus 200 units), especially in a month when it had been colder than the previous year.

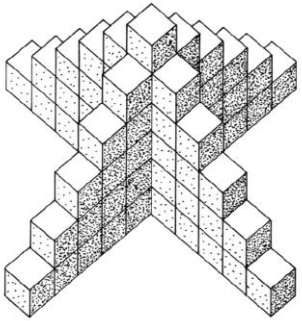
Since she used gas for heating only, Ms. Johnson wanted a better estimate of the savings due to the additional insulation and window sealing. She asked Kevin and Shana to look into whether the "heating degree days" listed on the bill might provide some insight.

Ohio Performance Assessment

Ohio Department of Education • Stanford University • ESC of Central Ohio

Pilot Project

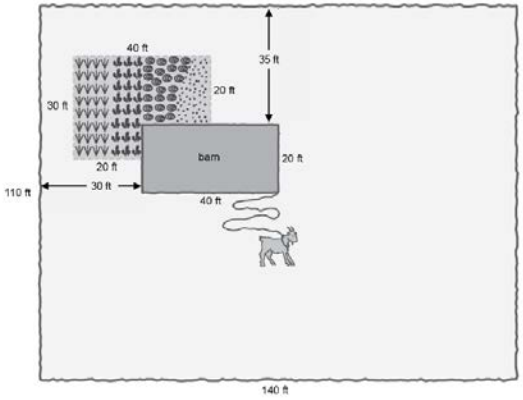
SALLY'S SUGAR STACK



Grazing Area

A farmer tethers her goat to the corner of a 40-by-20-foot barn in a fenced lot that is 140-by-110 feet. She also has an herb garden next to part of the barn. The goat is tethered on a 50-foot rope to the corner of the barn farthest from the herb garden.

After leaving the goat out on the rope for one day, the farmer discovers that a large area of her herb garden has been nibbled to the ground! Where can she tether the goat so that her herb garden is not within reach of the goat, but without decreasing the grazing area of the goat? The original grazing area includes all the grass area the goat could reach including that section of the herb garden that the goat ate.



Open for Business

Malena is a student who wants to raise $5,000 to tour South America next summer. To raise the money, she decides to open her own business on eBay.

The owner of an electronics shop offers to sell Malena some of his products at the wholesale price. She needs to decide which items to sell and how to price those items in order to maximize her profit.

She does some market research and finds the information provided in the table below about some of the items she is considering selling. Her research results include the cost to buy these items from the wholesale supplier, the retail price at which different items were sold at different times, and the number of items sold at these different prices during the month.

SCALE

Stanford Center for Assessment, Learning, & Equity

SRN LEADS
STANFORD UNIVERSITY

Mathematics Claim #1
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

The measurement instrument will be novice [short] tasks. The items assessing this claim will account for 40% of the total score administered by computer.

Mathematics Claim #2
Students can frame and solve a range of complex problems in pure and applied mathematics.

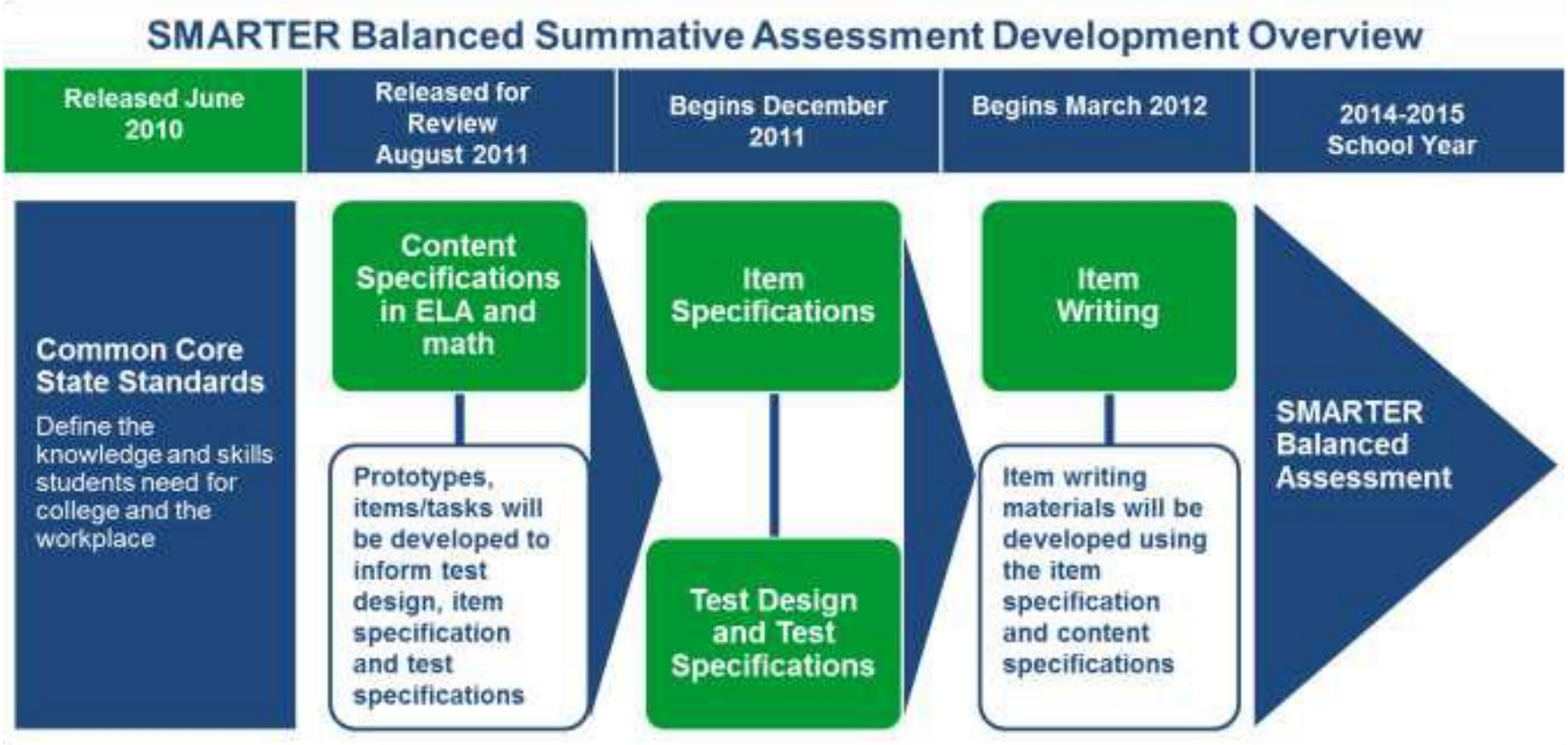
The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.

Mathematics Claim #3
Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.

Mathematics Claim #4
Students can analyze complex, real-world scenarios and can use mathematical models to interpret and solve problems.

The measurement instrument will be by apprentice [scaffold] tasks or expert [unscaffold] tasks. The items assessing this claim will account for 20% of the total score and will be human scored.


SMARTER Balanced Summative Assessment Development Overview
Released June 2010
Released for Review August 2011
Begins December 2011
Begins March 2012
2014-2015 School Year
Common Core State Standards
Define the knowledge and skills students need for college and the workplace
Content Specifications in ELA and math
Prototypes, items/tasks will be developed to inform test design, item specification and test specifications
Item Specifications
Test Design and Test Specifications
Item Writing
Item writing materials will be developed using the item specification and content specifications
SMARTER Balanced Assessment

Problem Sources

Part I: Short items

1: MARS
2: MARS
3: SBAC
4: MARS
5: PISA
6: MARS
7: PISA
8: MARS
9: MARS
10: MARS
11: SBAC
12: SBAC
13: SBAC

Part II: Selected Response Tasks

CR 1: SBAC
CR 2: MARS
CR 3: MARS
CR 4: MARS
CR 5: MARS
CR 6: MARS
CR 7: MARS
CR 8: MARS
CR 9: MARS
CR 10: MARS

Part III: Extended Performance Task

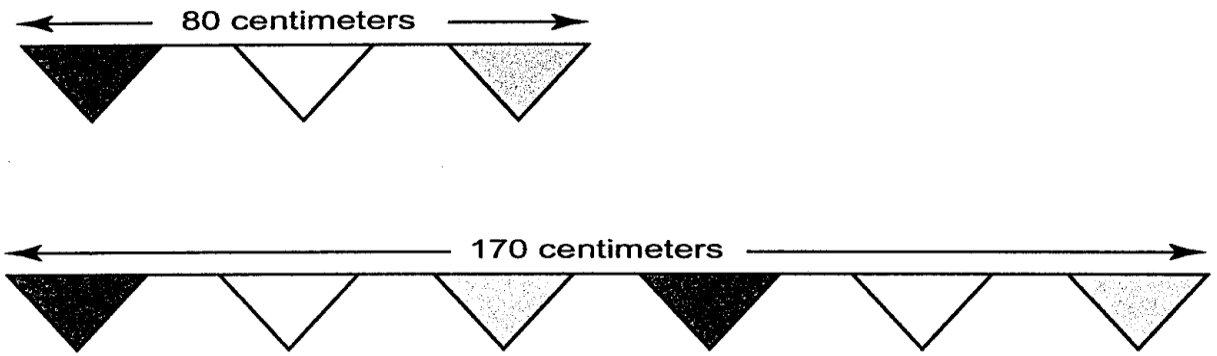
Ohio Department of Education and the Stanford University School Redesign Network

Teaching for Meaning

Party Flags

This problem gives you the chance to:

- find sizes by interpreting a diagram
- express a function by a formula



Erica is putting up lines of colored flags for a party.

The flags are all the same size and are spaced equally along the line.

1. Calculate the length of the sides of each flag, and the space between flags.

Show all your work clearly.

2. How long will a line of *n* flags be?

Write down a formula to show how long a line of *n* flags would be.

Algebra students had been working on system of linear equations for weeks.

$$6x + 5y = 170$$

$$3x + 2y = 80$$

$$6x + 5y = 170$$

$$-6x + -4y = -160$$

$$y = 10$$

The Findings from Party Flags

- The task may be approached as a system of simultaneous equations, almost no algebra students used such an approach.
- 49% of algebra students had no success.
- 44% accurately found the two lengths (most commonly by an estimation strategy only using one constraint).
- 21% correctly used both constraints (the length of three flags is 80 cm. and the length of 6 flags is 170 cm.).
- 7% of the students were able to develop a valid generalization for n flags.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

Mathematical Practice

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

CCSS Mathematical Practices

OVERARCHING HABITS OF MIND

1. Make sense of problems and persevere in solving them
6. Attend to precision

REASONING AND EXPLAINING

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

4. Model with mathematics
5. Use appropriate tools strategically

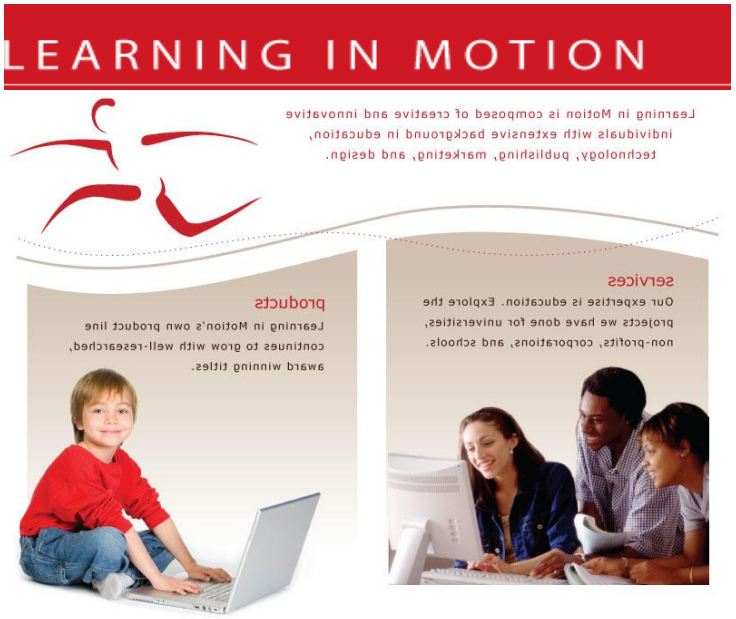
SEEING STRUCTURE AND GENERALIZING

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

New K-12 Math Curriculum Inspired by The Common Core State Standards

BILL & MELINDA GATES *foundation*

PEARSON FOUNDATION



The Gates Foundation and the Pearson Foundation are funding a large scale project to create a system of courses to support the ELA and Mathematics CCSS. These will be a modular, electronic curriculum spanning all grade levels. A Santa Cruz based company, Learning In Motion, is working to write the lessons.

Think in Terms of Units

Phil Daro has suggested that it is not the lesson or activity, but rather the **unit** that is the "optimal grain-size for the learning of mathematics". Hence that was the starting point for our Scope and Sequence.

Developers of High School:
Patrick Callahan, Dick Stanley,
David Foster, Brad Findell,
Phil Daro, and Marge Cappo

Two Mathematics Pathways

Two Regular Sequences:

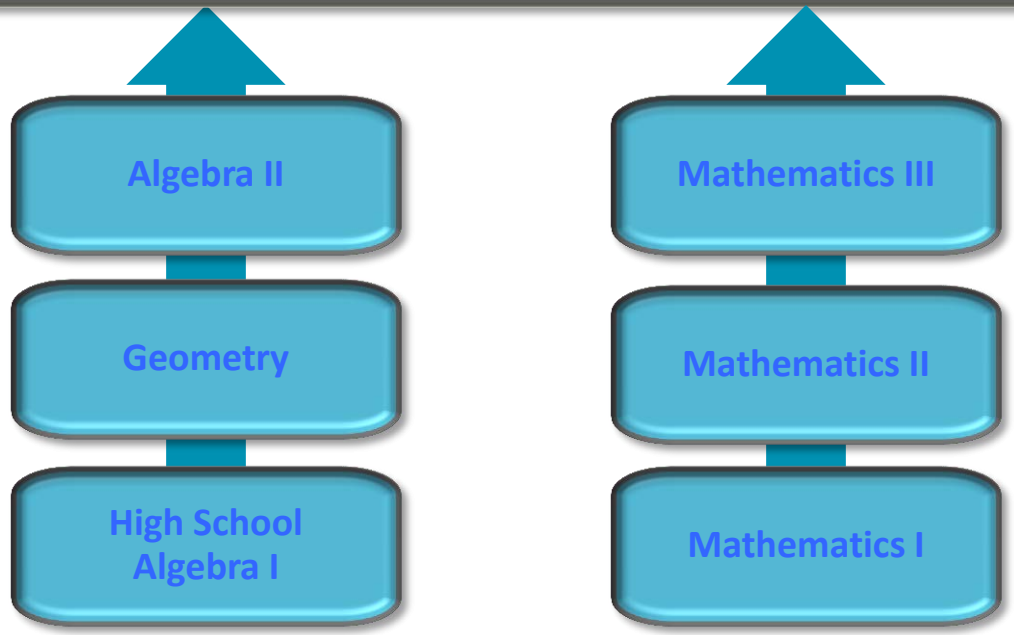
Traditional Pathway

- 2 Algebra courses,1 Geometry course, with Probability and Statistics interwoven

Integrated Pathway

- 3 courses that attend to Algebra, Geometry, and Probability and Statistics each year

Courses in higher level mathematics: Precalculus, Calculus*, Advanced Statistics, Discrete Mathematics, Advanced Quantitative Reasoning, or courses designed for career technical programs of study.



Traditional Pathway
Typical in U.S.

Integrated Pathway
Typical outside of U.S.

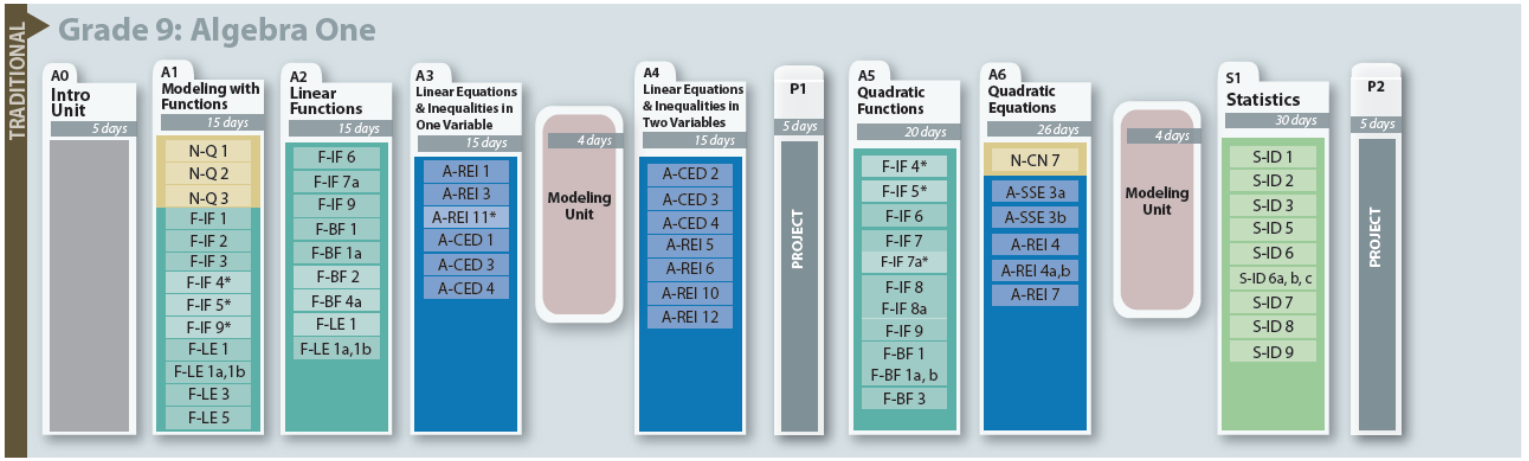
CCSS High School Units

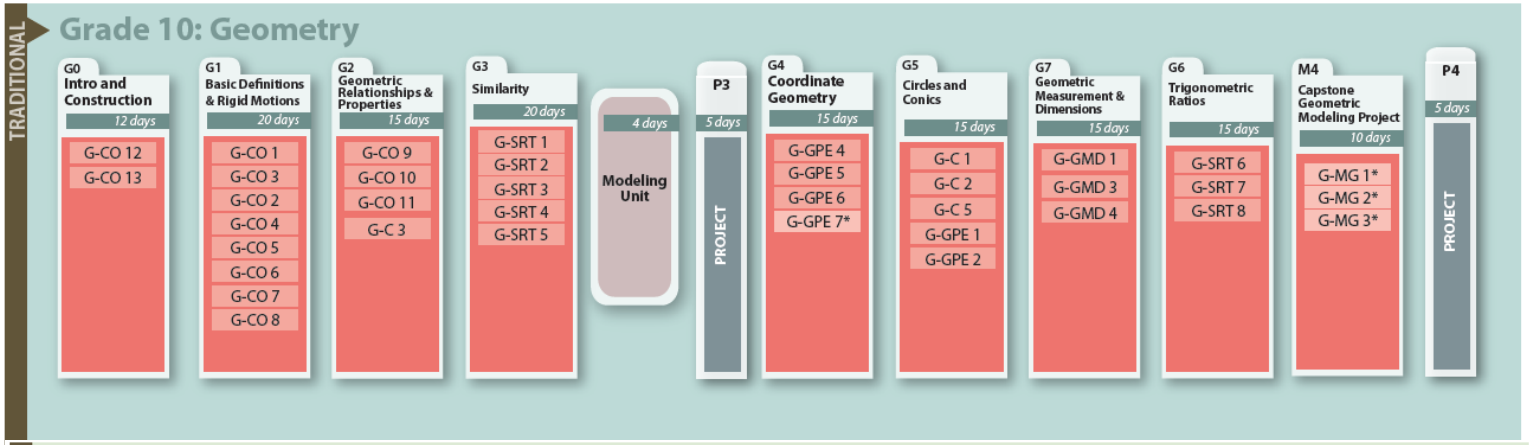
High School Algebra Units:
A0 Introductory Unit
A1 Modeling with Functions
A2 Linear Functions
A3 Linear Equations and Ineq in One Var
A4 Linear Equations and Ineq in Two Var
A5 Quadratic Functions
A6 Quadratic Equations
A7 Exponential Functions
A8 Trigonometric Functions
A9 Functions
A10 Rational and Polynomial Expressions

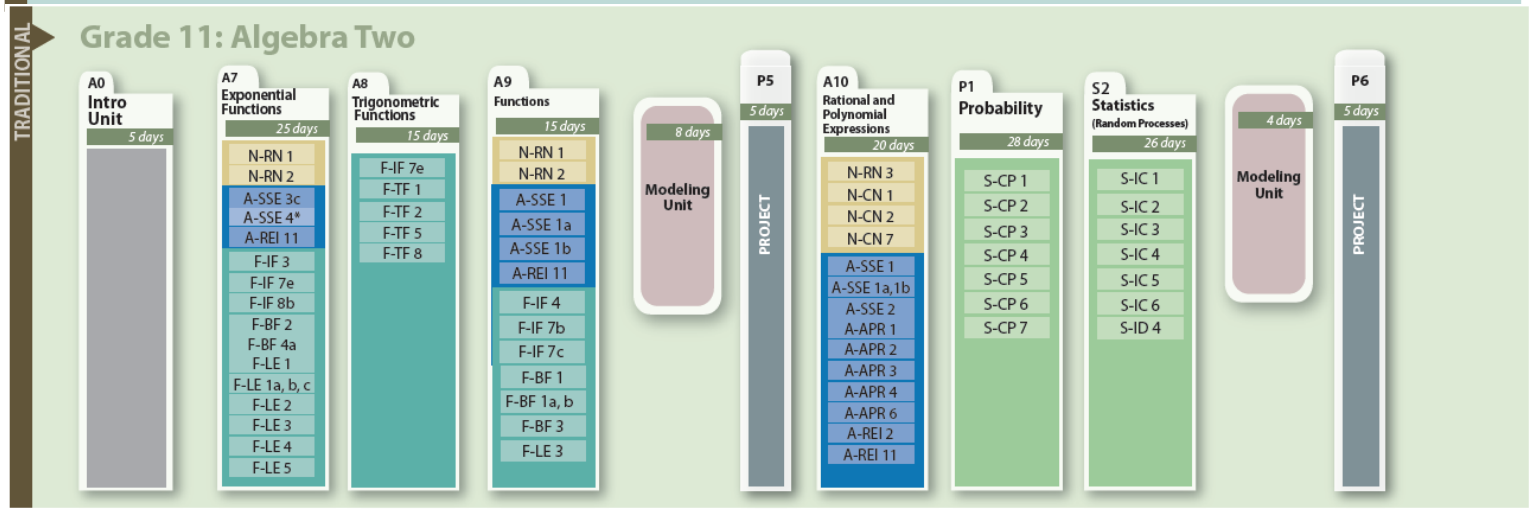
High School Geometry Units:
G0 Introduction and Construction
G1 Basic Definitions and Rigid Motions
G2 Geometric Relationships and Properties
G3 Similarity
G4 Coordinate Geometry
G5 Circle and Conics
G6 Trigonometric Ratios
G7 Geometric Measurement and Dimension
M4 Capstone Geometric Modeling Project

High School Prob & Stat Units:
P1 Probability
S1 Statistics
S2 Statistics (Random Process)

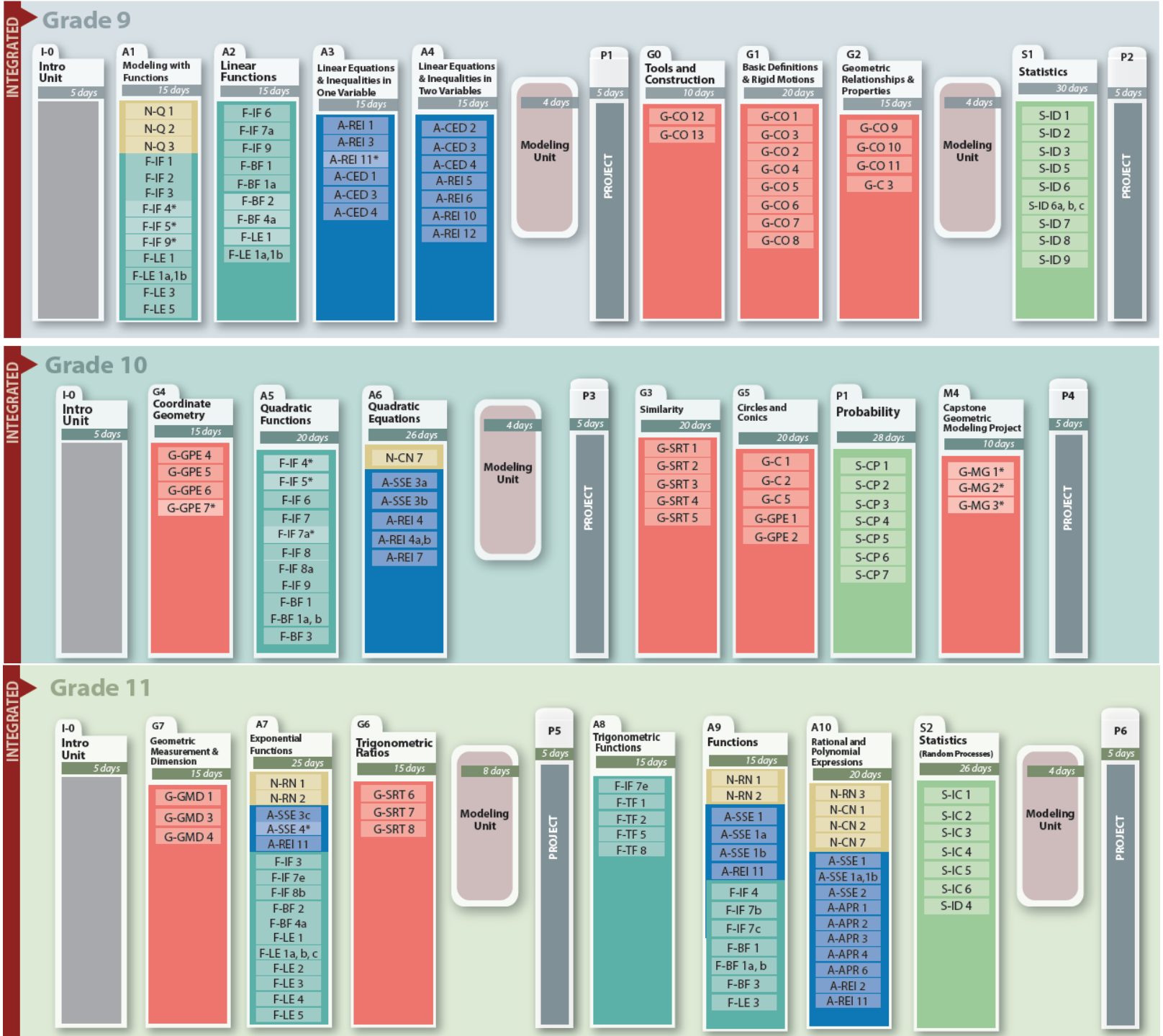
TRADITIONAL US







INTERNATIONAL



A Mathematical Big Idea

Linear Equations in Two Variables

$$6x + 5y = 170$$

$$3x + 2y = 80$$

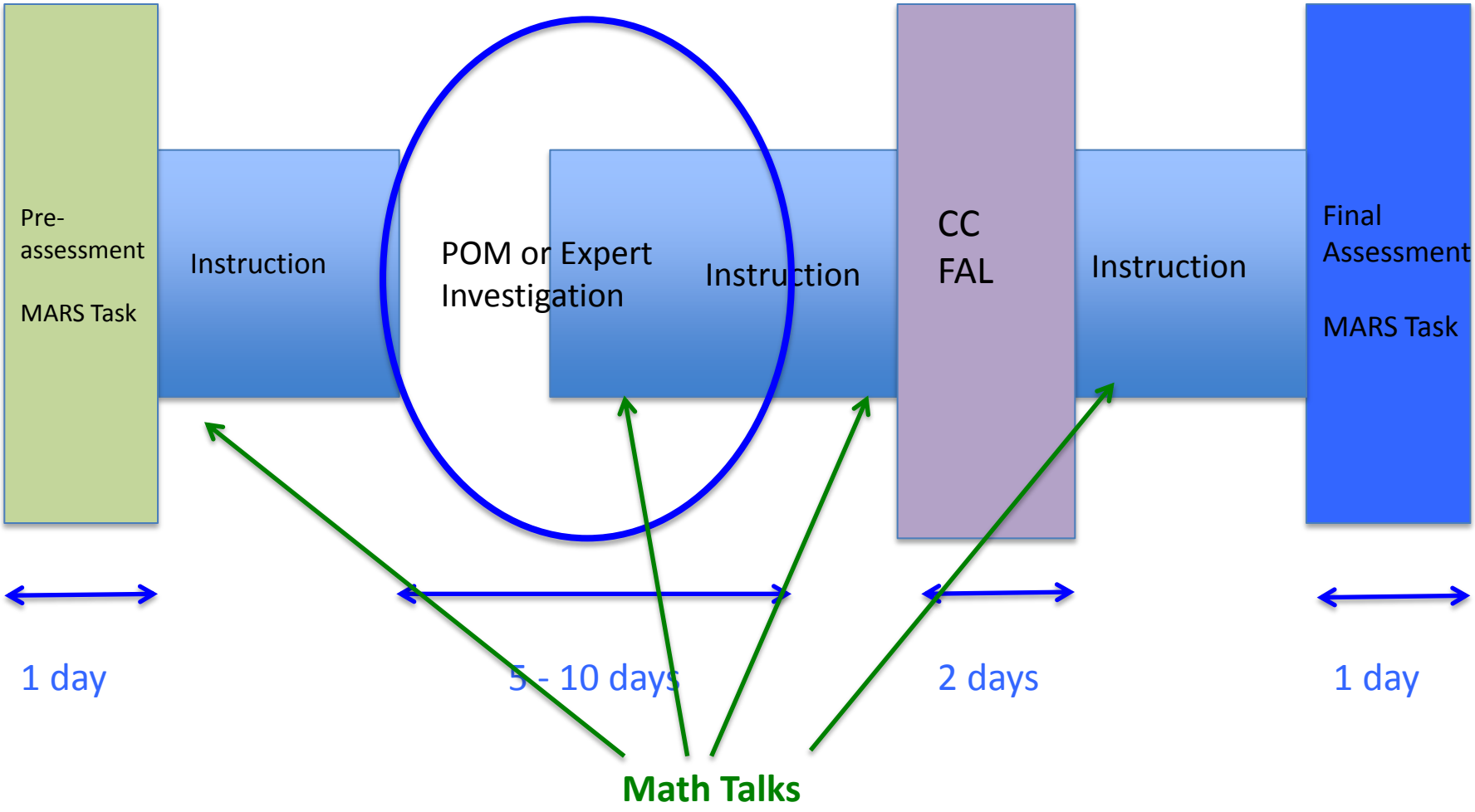
Linear Equations & Inequalities in Two Variables

15 days

- A-CED 2
- A-CED 3
- A-CED 4
- A-REI 5
- A-REI 6
- A-REI 10
- A-REI 12

Unit of Study

Around a Big Idea



MATH TALKS

Math Talks

- A daily ritual with the entire class for the purpose of developing conceptual understanding of and efficiency with numbers, operations and other mathematics such as geometry and algebra. (no more than 10 minutes per day)

Math Talks are used to:

- Support active student engagement through signaling
- Review and practice procedures and concepts
- Introduce a concept before diving into the lesson of the day
- Support students in deepening their understanding of the Properties of Arithmetic and our Place Value System
- Explore mathematical connections and relationships
- Encourage students to construct viable arguments and critique the reasoning of others
- Support students in using precise mathematical language in sharing their different strategies and approaches

Math Talk – Relational Thinking

 +  = 7 **and**  x  = 12

____ + ____ = 7 **and** ____ x ____ = 12

Math Talk – Relational Thinking

 +  = 10 **and**  ÷  = 4

____ + ____ = 10 **and** ____ ÷ ____ = 4

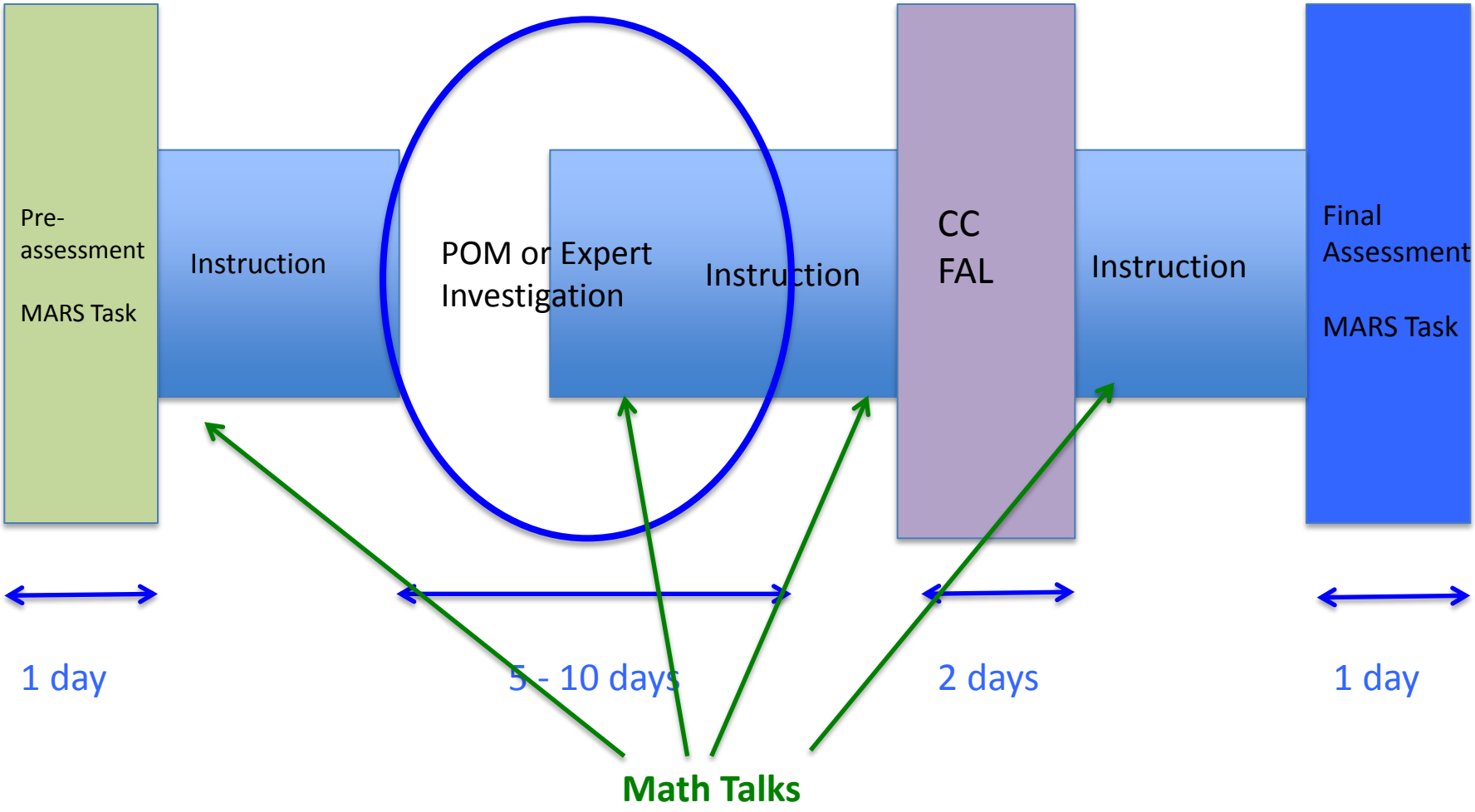
Math Talk – Relational Thinking

 ÷  = 3 **and**  -  = 6

____ ÷ ____ = 3 **and** ____ - ____ = 6

Unit of Study

Around a Big Idea



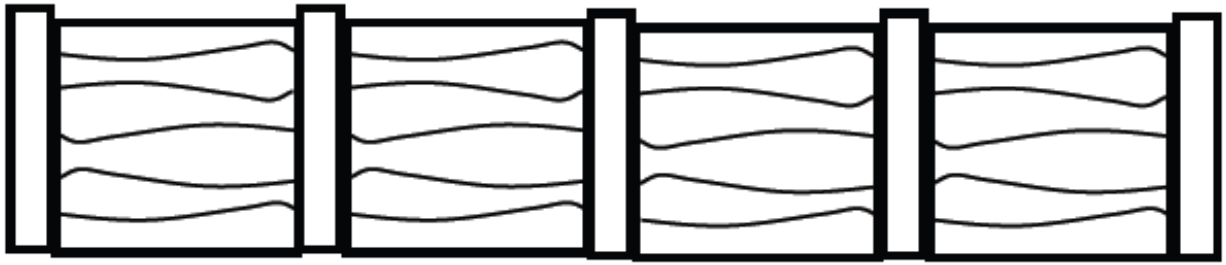
Fencing

This problem gives you the chance to:

- interpret given information
- choose and use an appropriate method to solve a problem

Jon buys fencing for his yard.

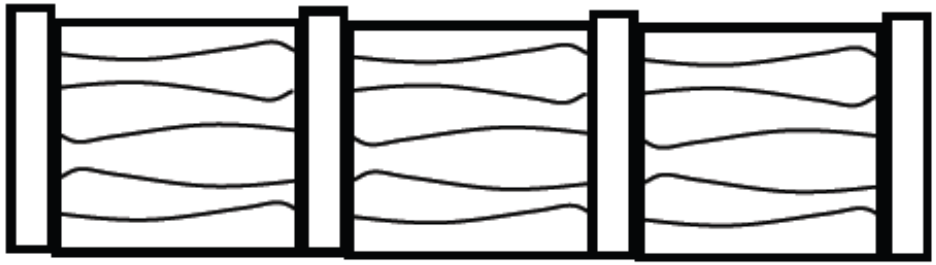
He pays $122 for 5 fence posts and 4 fence panels.



He pays $570 for 21 fence posts and 20 fence panels.

How much does he pay for 4 fence posts and 3 fence panels? $_____

Show how you figured it out.

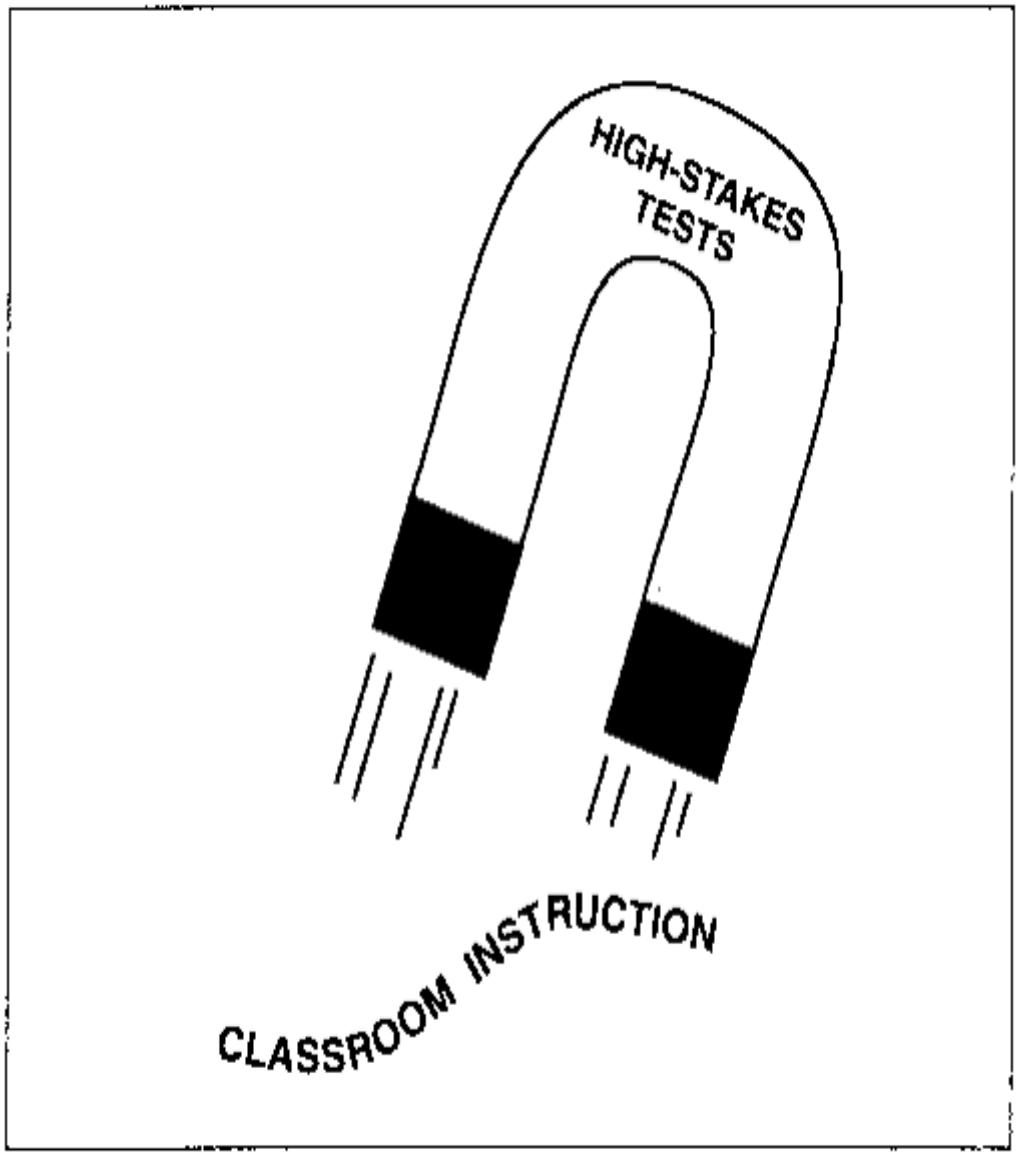


What Ever Happened to Problem Solving In Math Class?

Silicon Valley Math Initiative

Problems of the Month

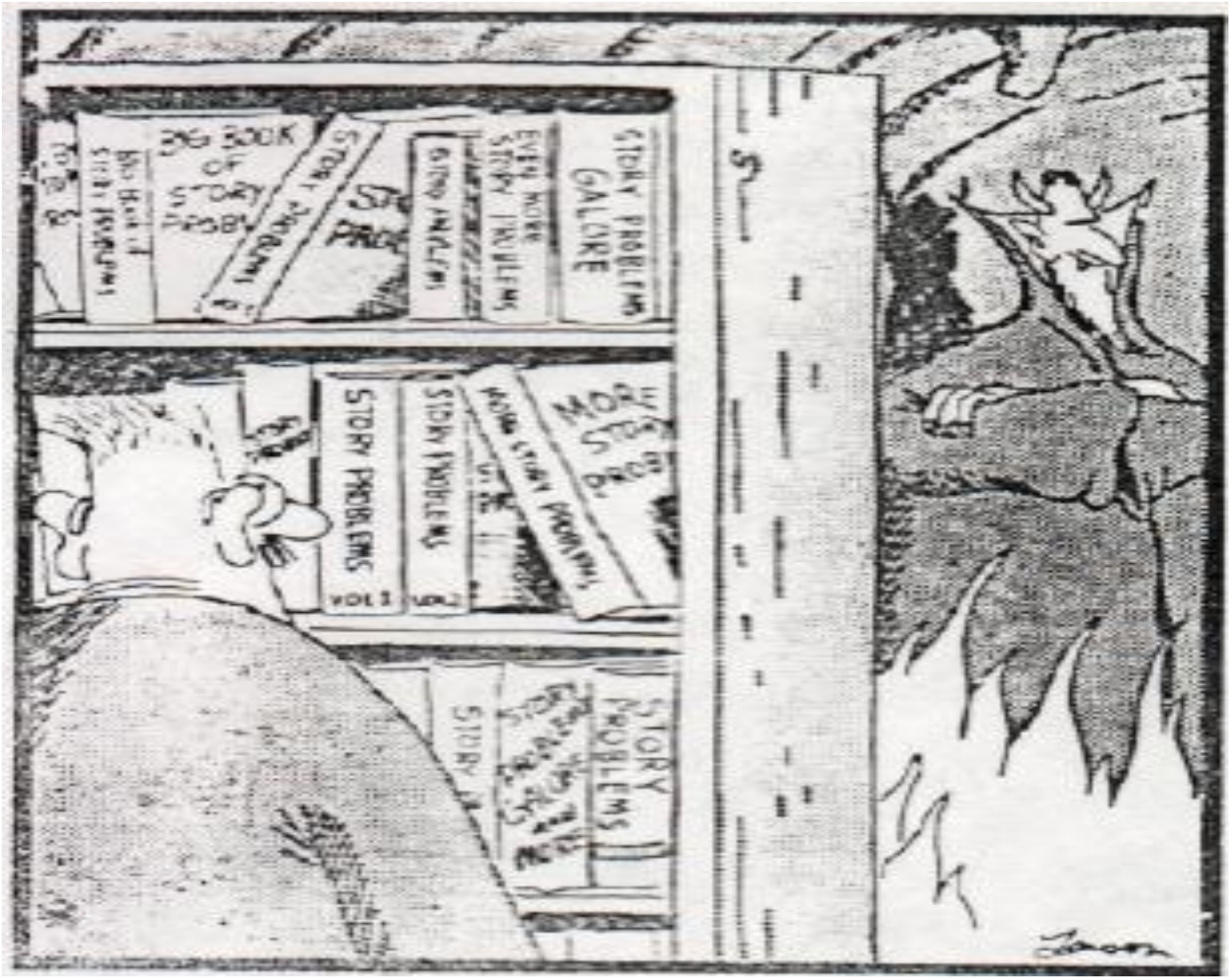
Promoting Problem Solving in Mathematics



"High-stakes testing has encouraged a drill-and-practice form of instruction among teachers who are perfectly capable of developing deep understanding on the part of their students."

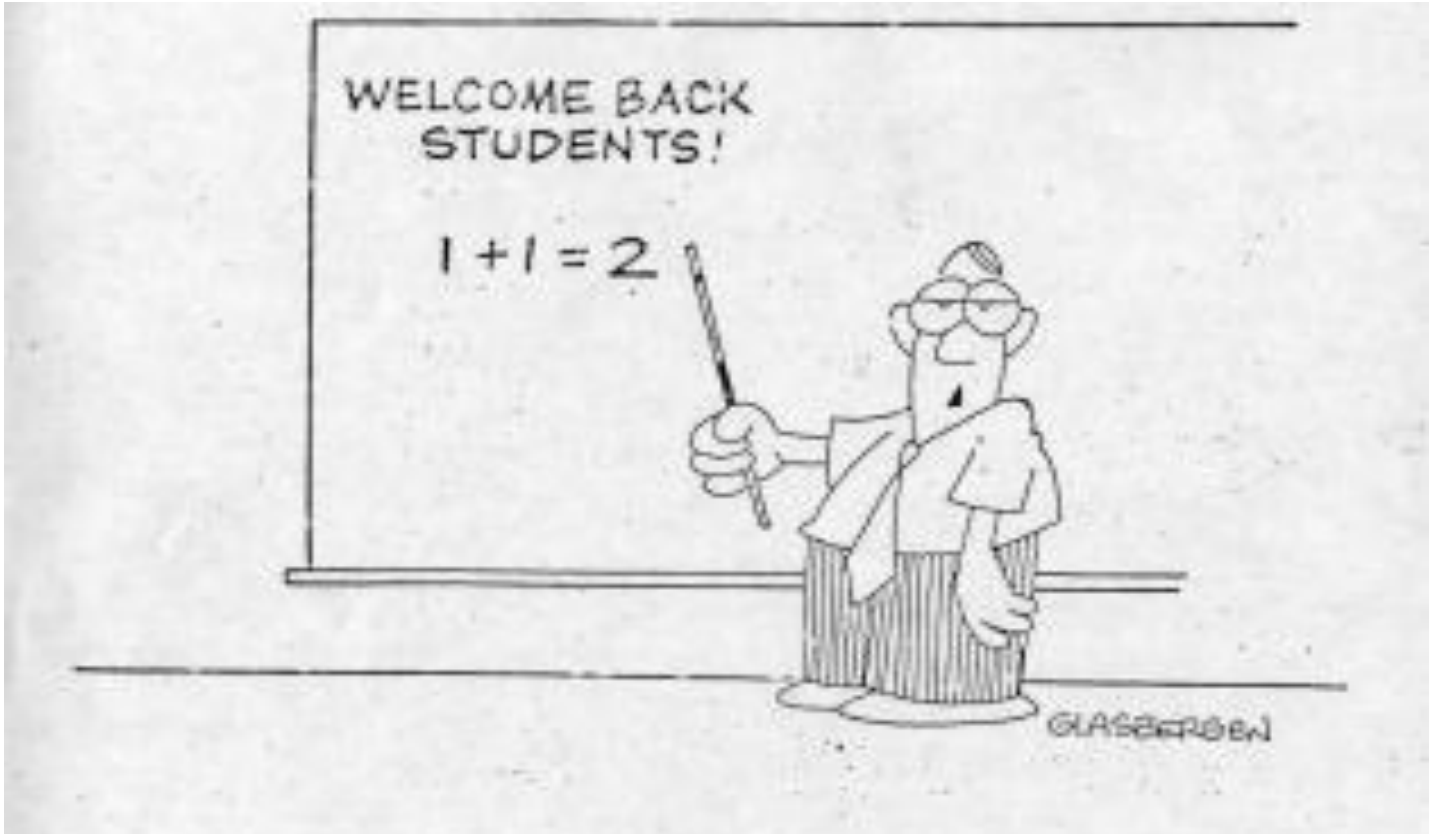
Learning from Leadership Project,
Univ. of Minnesota, Toronto

Problem solving is more than the word problems at then end of the problem set or chapter.



Hell's Library

Problem Solving requires a depth of mathematical knowledge.



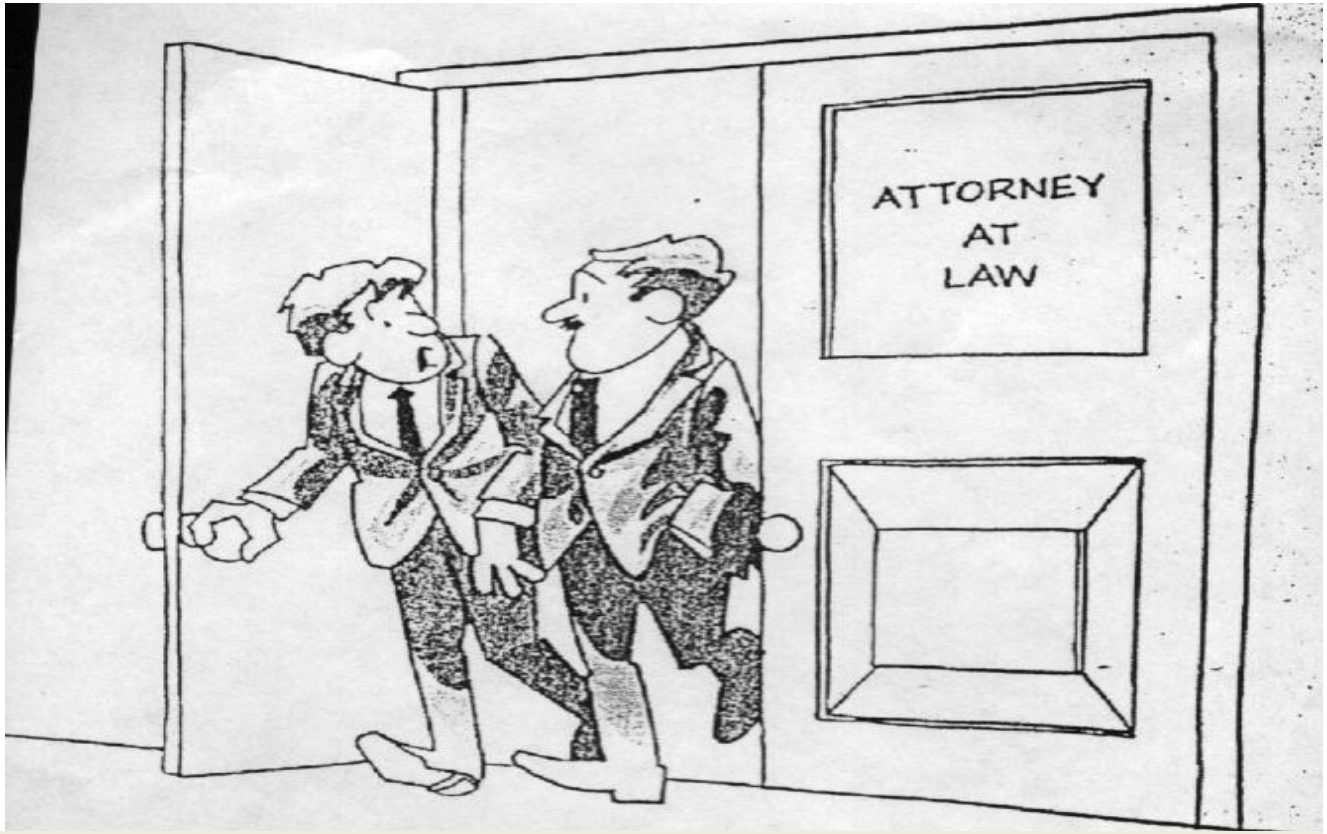
"Before we begin advanced calculus, let's review any information we may have forgotten over the summer."

"Mr. Osborne, may I be excused?
My brain is full."

Problem solving requires perseverance

Problem Solving is about

non-routine problems not merely procedural knowledge.



“He was never any good at arithmetic, he left a third to you, a third to me, a third to Shirley and a third to Max”

Everyone can become a good problem solver.

Problem Solving challenges **ALL** students.

“How many times do I have to tell you you’re not supposed to read ahead.”

Problems of the Month

A program to foster school-wide participation in math and problem solving.

George Polya, (1887 - 1985)
Father of Problem Solving;
"How to Solve It", 1945

Mathematics, you see, is not a spectator sport. To understand mathematics means to be able to do mathematics. And what does it mean doing mathematics? In the first place it means to be able to solve mathematical problems.

Why a Problem of the Month?

- George Polya, said, “A problem is not a problem if you can solve it in 24 hours.”
- Doing math is solving non-routine problems.
- Perseverance and learning from mistakes are important attributes of good mathematicians.

CCSS Mathematical Practices

OVERARCHING HABITS OF MIND

1. Make sense of problems and persevere in solving them
6. Attend to precision

REASONING AND EXPLAINING

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

4. Model with mathematics
5. Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

How are the POM be used?

- The POM are used school wide to promote problem solving.
- Each problem is divided into five levels, A-E, to meet the learning development needs of all students.
- A great tool for *Differentiated Instruction*.
- Students, teachers and parents learn to ask questions and persevere in solving non-routine problems.
- The whole school celebrates doing mathematics at school.

It's better to solve one problem five different ways than to solve five different problems.

George Polya

Problem of the Month

The Wheel Shop

Level A:

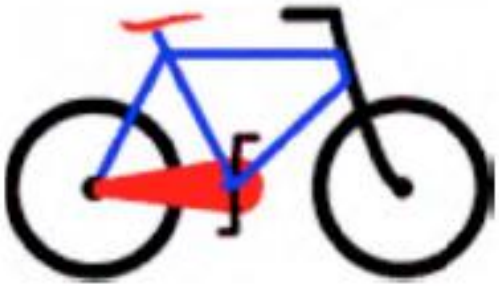
You go to a shop that sells tricycles. There are 18 wheels in the Wheel Shop.

How many tricycles are in the shop?

Explain how you know.

Level B:

The Wheel Shop sells other kinds of vehicles. There are bicycles and go-carts in a different room of the shop. Each bicycle has only one seat and each go-cart has only one seat. There are a total of 21 seats and 54 wheels in that room.



How many are bicycles and how many are go-carts?

Explain how you figured it out.

Level C:

Three months later some vehicles have sold and new models have been brought into the Wheel Shop. Now, there are a different number of bicycles, tandem bicycles, and tricycles in the shop. There are a total of 135 seats, 118 front handlebars (that steer the bike), and 269 wheels.

How many bicycles, tandem bicycles and tricycles are there in the Wheel Shop?

Level D:

In the back stockroom at the Wheel Shop, the number of seats and horns equaled the number of wheels. The number seats and handlebars equaled the number of horns. Twice the number of wheels is equal to three times number of handlebars. Determine the relationship of horns to seats.

Level E:

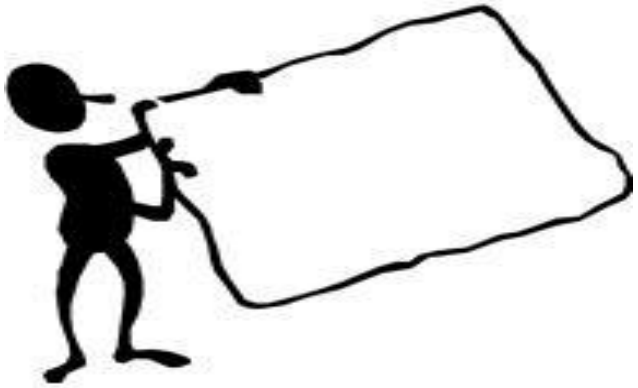
The repair department of the bicycle shop repairs three things: flat tires, bent handle bars and ripped seats. Today in the repair department, 25% of the bikes had flat tires only, 5% had bent handlebars only, and 10% had ripped seats only. Just 1/12th of the bikes had all three repairs to do: flat tires, bent handlebars and ripped seats. No bikes were completely fixed and there are a total of 101 repairs to be made. How many bikes are in the repair department? How many bikes need two repairs? If less than half of all the bikes have a ripped seat, what is the range of bikes that need both the tires and handlebars repaired without needing to fix the seat?

“A problem is not a problem if you can solve it in 24 hours.”

George Polya

Creating a Poster

Your concluding thoughts on an explanation poster for a level you feel you have completed



AND

Your current thoughts on a status poster for a level you are still exploring.

Break

Explanation Poster: *The focus of your poster should be on how your findings can be justified mathematically and how your findings make sense. Include words and visuals (such as drawings) as a part of your justification.*

Status Poster: *The focus of your poster should be on your processes so far and where you think you want to go next and/or questions/wonderings you have about this level. Include words and visuals as a part of your justification.*

*Remember to justify or explain your processes you have used so far and why they make mathematical

Gallery Walk

- Each group will display their poster.
- Each group selects a group member to be the docent to answer questions or provide clarifications/explanations.
- The other group members examines, explores, reviews the other groups' posters.
- There will be time for your group to re-assemble and discuss the information shared in the groups' posters.
- Please mind gallery walk norms and be respectful of the work and information shared.

Guidelines for Problem of the Month

Why Problem of the Month?

Problem Solving is the cornerstone of doing mathematics. George Polya, a famous mathematician from Stanford, once said, "a problem is not a problem if you can solve it in 24 hours." His point was that a problem that you can solve in less than a day, is usually a problem that is similar to one that you have solved before or at least recognized that a certain approach will lead to the solution. But in real life a problem is a situation that confronts you and you don't have an idea of where to even start. Mathematics is the toolbox that solves so many problems. Whether it is calculating an estimate measure, modeling a complex situation, determining the probability of a chance event, transforming a graphical image or proving a case using deductive reasoning, mathematics is used. If we want our students to be problem solvers and mathematically powerful, we must model perseverance and challenge students with non-routine problems.

How should the Problem of the Month be used?

The Problems of the Month are designed to be used school wide to promote a problem-solving theme at your school. The problem is divided into five levels, Level A through Level E, to allow access and scaffolding for the students into different aspects of the problem and to stretch students to go deeper into mathematical complexity. The goal is for all students to have the experience of attacking and solving non-routine problems and developing their mathematical reasoning skills. Although obtaining and justifying solutions to the problems is the objective, the process of learning to problem solve is even more important. Administrators, teachers and parents should facilitate and support students in the process of attacking and reasoning about the problems. The self-analysis by students of how they went about approaching, exploring and solving the problems is a critical step in the development of becoming a strong problem solver.

Dear Parents,

Problem solving is a fundamental goal of any strong mathematics program. Our school is committed to making each of our students a problem solver. It is never too early or too late to develop the real life learning skills of problem solving. Therefore, our school is embarking on a Problem of the Month program to help achieve this important goal.

Each month we will present our students with a non-routine problem for them to attack and solve. The problem will have several levels so that all students at our school will be able to work on a part of the problem appropriate to their learning development. All students should start with level A and work through the different tasks. It is understood that some students will not get too far into the problem. The process of attacking and struggling on a non-routine problem is important to learn. When your child has reached the maximum level of his/her understanding please celebrate their progress.

Trials, errors, and retries are key attributes of good problem solvers. We ask you to encourage your students to persevere. Many students might want to initially give up. The best support for your student is encouragement through good questions. Some good questions are: *What have you tried?, Why do you think it doesn't work?, Have you tried to make the problem simpler?, What do you need to know to be able to solve the problem?*. There are many other good questions; however leading or guiding questions are not helpful. The process of finding and understanding a solution outweighs the benefit of having a correct answer. Doing the problem for the student actually hurts the problem solving process. Many students will receive the hidden message that they can't solve problems by themselves and, will learn to stop and wait for someone else to answer.

You will play an important role in supporting your child's work on these problems. Once students have reached their level of understanding, they are asked to complete a write-up of their findings. Students should communicate how they went about solving the problem as well as the solution they found. This write up helps students understand how they think and approach new problems. We look forward to a partnership with you around problem solving. Thank you for supporting your child.

Sincerely,

Teacher's Notes
Problem of the Month: **Squirreling It Away**

Overview:

In the Problem of the Month, *Squirreling It Away,* students use number operations, organized lists and counting methods to solve problems. The mathematical topics that underlie this POM are knowledge of number sense, comparison subtraction, division, factors and divisibility, counting principles, systematic charting and closed-form equations. The mathematics that includes counting principles and systemic charting is often referred to as discrete mathematics.

In the first level of the POM, students are presented with a situation that involves making sense totals and comparison differences. Their task involves making number story about giving acorns to two difference size groups of squirrels and then determining how many were left over from of the original total of acorns. In level B, students start to examine how the acorns can be partitioned into different sets. The students are told that different squirrels can carry different amounts of acorns on given trips. Students are asked to find the number of trips it takes to carry the acorn for each type of squirrel. In level C, students are asked how many ways can three different type of squirrels carry away 24 acorns. In level D, the student determines the number of ways 24 acorns can be divided between three specific squirrels. In level E, students are asked to find and justify a closed form equation that will determine the number of ways that n acorns can be divided between three squirrels.

Mathematical Concepts:

Discrete mathematics is the study of the sets of and operations on discrete objects. Discrete means distinct from others, separate or discontinuous. Basically discrete is not continuous mathematics. Traditionally, K-12 mathematics starts with discrete topics in lower grades, but by secondary mathematics the topics usually focus on continuous mathematics as a preparation for calculus. The real number line is an example of a continuous set of points. But, discrete mathematics plays important role in higher mathematics courses. Much of the mathematics that come from real life come in the form of distinct objects and values. Making sense of this distinct information and quantities is important, thus discrete mathematics has numerous applications to real life. A more formal definition of discrete mathematics may be stated as the study of sets of distinct data. Tasks usually performed while doing discrete mathematics

Findings are shared through Group Collaboration, Individual Write-Up, Gallery Walks, and/or Presentations

Problem of Month Write-up

- **Problem Statement**
  In your own words, state the general overall problem clearly enough that someone unfamiliar with the problem could pick up your paper and understand what you are asked to do.

- **Process**
  Describe in detail how you attempted to solve this problem. You may want to consider some of the following questions. You should also include things that didn't work.
  How did you get started?
  What approaches did you try?
  Where did you get stuck?
  Did you talk to anyone about the problem?
  Did talking to someone help or hinder you?
  What drawing, chart, graph, or model did you use?

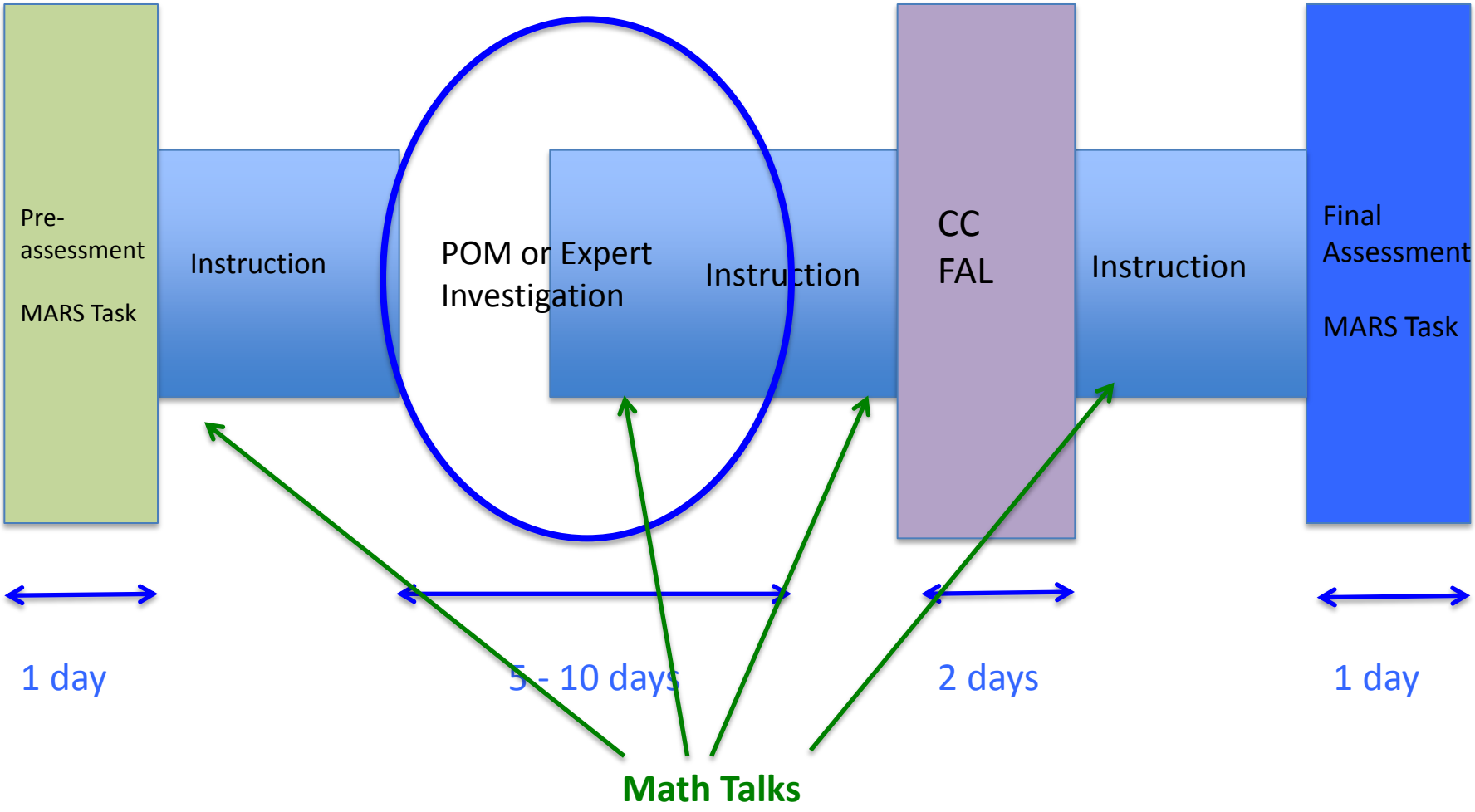
- **Solution**
  State your solutions as clearly as you can. Include any charts, graphs, and lists and so on that you used to help you. If you were able to generalize the solution, include your results. Defend why you believe your solution is correct or the best possible answer. Your explanations should be written in a way that will be convincing to someone else.

- **Learning**
  Reflect on the problem. What did you learn? What mathematics did you use?

LUNCH BREAK

Unit of Study

Around a Big Idea



Formative Assessment Lessons

Mathematics Assessment Project

Formative Assessment Lesson Materials

Solving Linear Equations in Two Variables

MARS Shell Center
University of Nottingham & UC Berkeley
Beta Version

If you encounter errors or other issues in this version, please send details to the MAP team c/o map.feedback@mathshell.org.

Solving Linear Equations in Two Variables

Mathematical goals

This lesson unit is intended to help you assess how well students are able to formulate and solve problems using algebra and, in particular, to identify and help students who have the following difficulties:

- Solving a problem using two linear equations with two variables.
- Interpreting the meaning of algebraic expressions.

Common Core State Standards

This lesson involves *mathematical content* in the standards from across the grades, with emphasis on:

A-CED: Create equations that describe numbers or relationships.
A-REI: Solve systems of equations.

This lesson involves a range of *mathematical practices*, with emphasis on:

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Introduction

This lesson is structured in the following way:

- Before the lesson, students work individually on the assessment task *Notebooks and Pens*. You then review their work and create questions for students to answer in order to improve their solutions.
- During the lesson, students work individually on a task that requires them to interpret and solve two equations in two variables. Students then compare and discuss their solutions in small groups.
- In the same small groups, students evaluate some sample solutions of the same task.
- In a whole-class discussion, students explain and compare the alternative solution strategies they have seen and used.
- Finally, students use what they have learned to revise their work on *Notebooks and Pens*.

Materials required

- Each individual student will need two copies of the assessment task *Notebooks and Pens,* and a copy of the lesson task *Cash Registers*.
- Each small group of students will need a blank sheet of paper, and copies of the four sheets *Sample Student Work*.
- Graph paper should be kept in reserve and used only when requested.
- Projector resources are provided to support the whole-class discussion.

Time needed

Approximately fifteen minutes before the lesson, a one-hour lesson, and ten minutes in a follow-up lesson (or for homework). Timings given are only approximate. Exact timings will depend on the needs of the class.

Notebooks and Pens

A store sells pens at \$2 and notebooks at \$5.

n = number of notebooks sold.

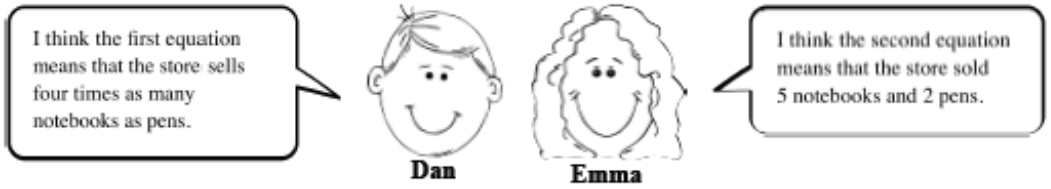
p = number of pens sold.

The following equations are true:

$4n = p$

$5n + 2p = 39$

Here is what Dan and Emma think the equations mean:



Are Dan and Emma correct?

.....

If you think Dan is wrong, explain the mistake and explain what *you* think the equation means.

.....

.....

If you think Emma is wrong, explain the mistake and explain what *you* think the equation means.

.....

.....

Figure out for yourself the number of pens and the number of notebooks sold in the store.

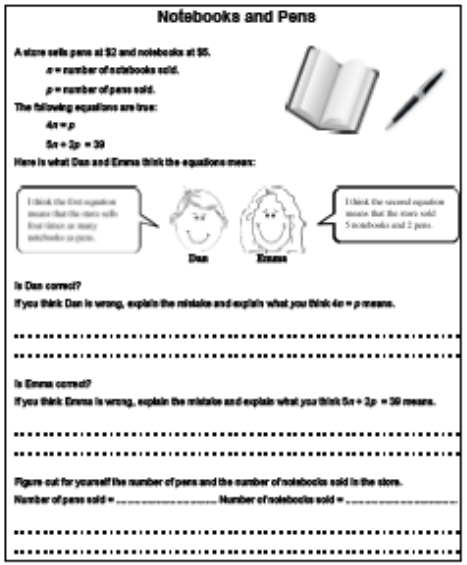
.....

.....

.....

Before the lesson

Assessment task: *Notebooks and Pens* (15 minutes)


Notebooks and Pens

A store sells pens at $2 and notebooks at $5.

n = number of notebooks sold.

p = number of pens sold.

The following equations are true:

$4n = p$

$5n + 2p = 39$

Here is what Dan and Emma think the equations mean:

I think the first equation means that the store sells four times as many notebooks as pens.

Dan

Emma

I think the second equation means that the store sold 5 notebooks and 2 pens.

Is Dan correct?

If you think Dan is wrong, explain the mistake and explain what you think $4n = p$ means.

Is Emma correct?

If you think Emma is wrong, explain the mistake and explain what you think $5n + 2p = 39$ means.

Figure out for yourself the number of pens and the number of notebooks sold in the store.

Number of pens sold = ... Number of notebooks sold = ...

Have the students do this task in class or for homework a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Notebooks and Pens*.

Introduce the task briefly and help the class to understand the problem and its context.

> *Read through the questions, and try to answer them as carefully as you can.*
>
> *Show all your work, so that I can understand your reasoning.*

It is important that students are allowed to answer the questions without assistance, as far as possible.

Students should not worry too much if they cannot understand or do everything, because there will be a lesson using a similar task, which should help them. Explain to students that by the end of the next lesson, they should expect to answer questions such as these confidently. This is their goal.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of issues that will arise during the lesson itself, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it encourages students to compare their scores and distracts their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this lesson unit.

We suggest that you write a list of your own questions, based on your students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the end of the lesson.

Common issues:	Suggested questions and prompts:
Student assumes that the letter stands for an object not a number For example: The student says that the statements are correct. Or: The student realizes the equations are incorrect, but is unable to explain why.	• *What does the letter **p** represent?* • *Write the equation as a sentence. Does your sentence match what Dan/Emma said?* • *If **n** = 3, what would **p** equal in the first equation? Which is greater: **n** or **p**?* • *Are there more notebooks than pens? How do you know?*
Student only uses one equation For example: The student finds a value or values for n and p that fits one equation but not the other, such as $n = 1$ and $p = 4$ for the first equation.	• *For this equation, is there another pair of values for **n** and **p**? And another? How do you know which value is correct?* • *How can you check that your values for **n** and **p** work for both equations?*
Student produces unsystematic guess and check work For example: The student works out three or four seemingly unconnected combinations of values for n and p.	• *What is a sensible value to try for **n** (or **p**)? Why?* • *Can you organize your work in a table?*
Student provides poor explanation For example: The student presents the work as a series of unexplained numbers and/or calculations.	• *Would someone unfamiliar with your type of solution easily understand your work?* • *Have you explained how you arrived at your answer?*
Student makes algebraic mistakes For example: The student makes a mistake when manipulating the algebra in the equations.	• *How can you check that your answer is correct?*
Student solves the two equations correctly Student needs an extension task.	• *Can you now use a different method, for example, a table, a graph, or algebra?* • *Is this method better than your original one? Why?*

Cash Registers

The drawer of a cash register contains some quarters and some dollar bills.

x = the number of quarter coins in the cash register.
y = the number of dollar bills in the cash register.

The following two equations are true:

$$3x = y$$
$$4x + y = 70$$

1. Explain in words the meaning of each equation.

2. Find two pairs of values for x and y that satisfy the first equation.

3. Find two pairs of values for x and y that satisfy the second equation.

4. Find pairs of values for x and y that satisfy both equations *simultaneously*.

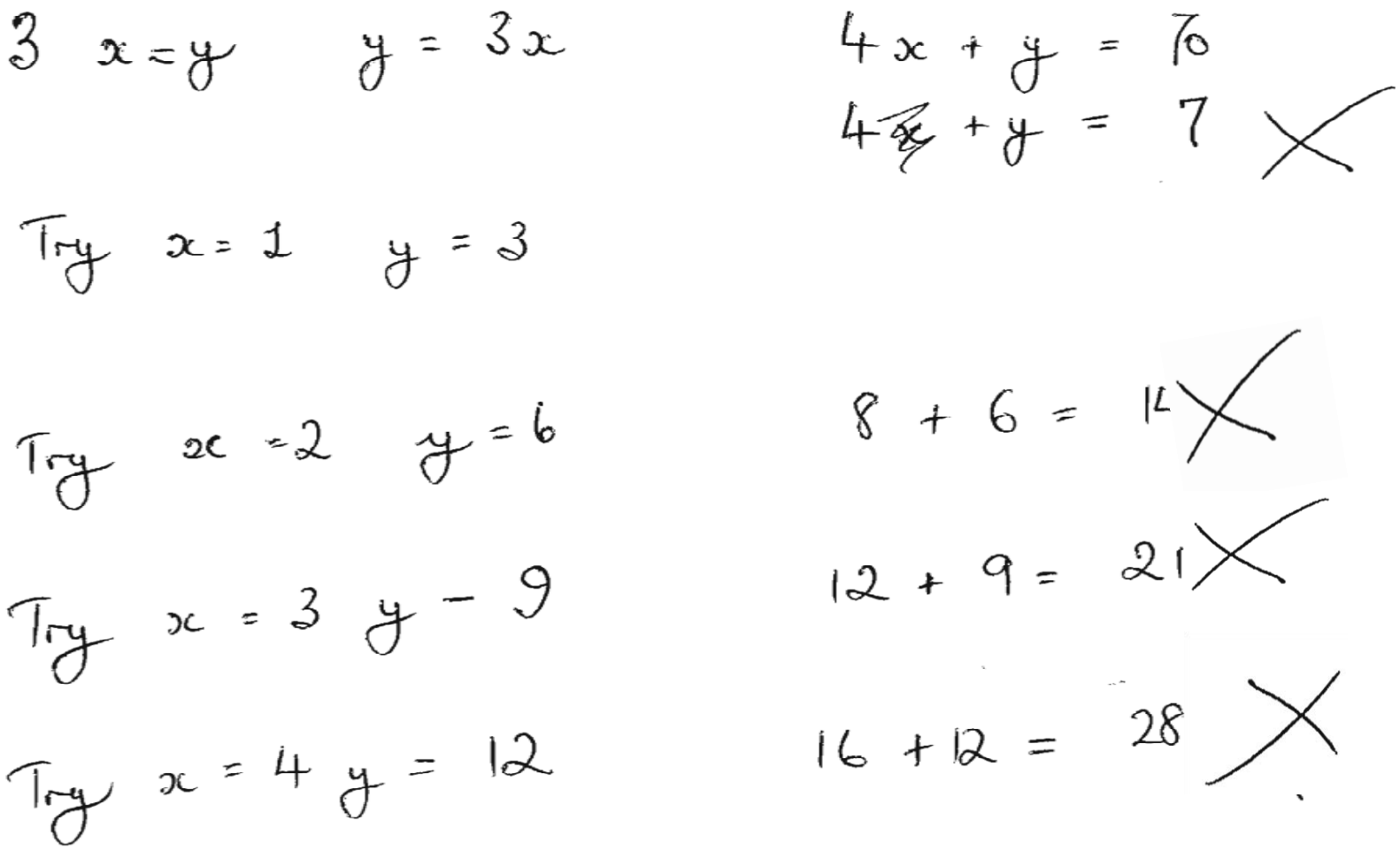
Assessing Sample Student Work

You are the teacher and have to assess this work.

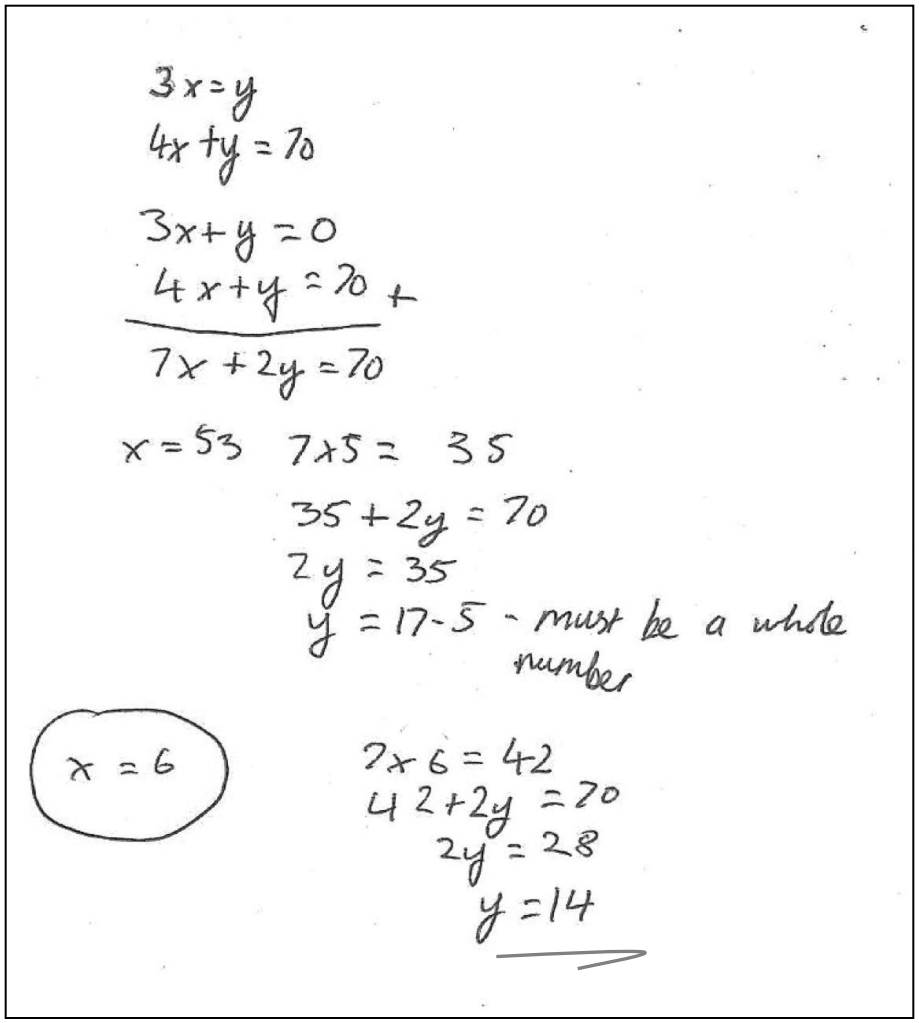
Correct the work and write comments on the accuracy and organization of each response.

- What do you like about this student's work?
- What method did the student use?
  Is it clear? Is it accurate? Is it efficient?
- What errors did the student make?
- How might the work be improved?

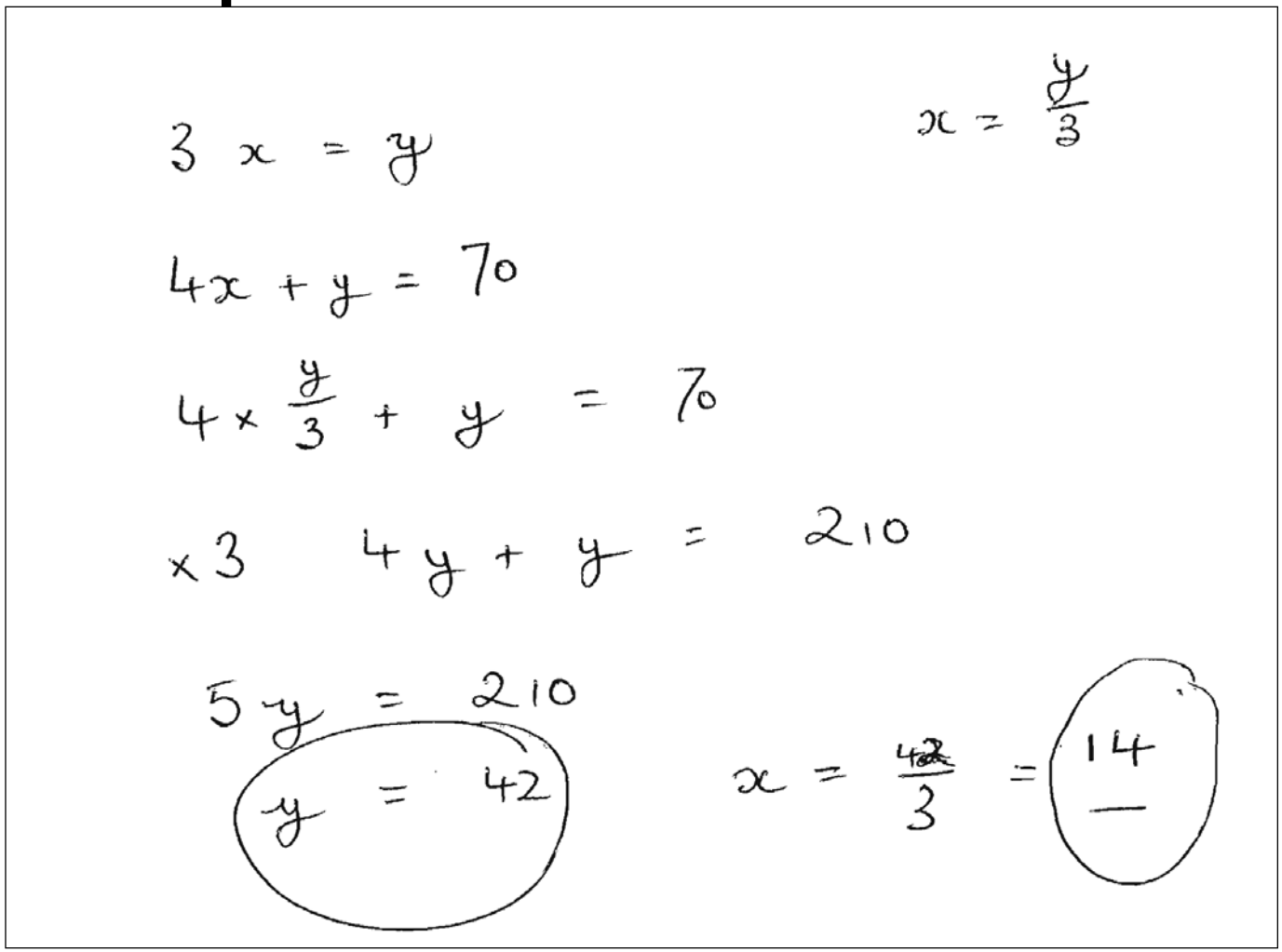
Sample Student Work: Ava



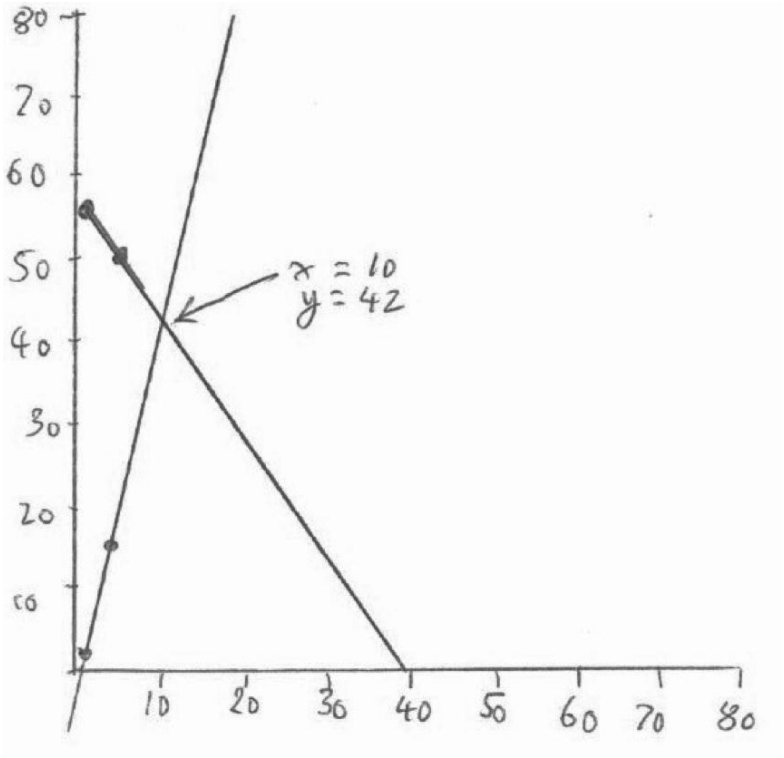
Sample Student Work: Ethan



Sample Student Work: Joe



Sample Student Work: Mia



$3x = y$

x	y
1	3
20	60
5	15

$4x + y = 70$

x	y
1	56
20	~~10~~
5	50

$x = 1 \quad 4 + y \; 70$

$y = 66$

Wrong $x = 20 \quad 80 + y = 70$

$y = 10$

$x = 5$

$20 + y = 70 \quad y = 50$

Notebooks and Pens

A store sells pens at \$2 and notebooks at \$5.

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The following equations are true:

$4n = p$

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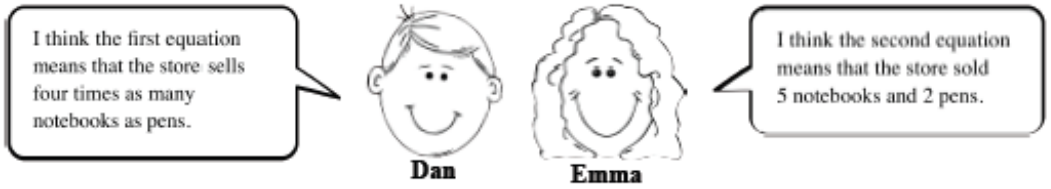
Here is what Dan and Emma think the equations mean:



Are Dan and Emma correct?

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.....

.....

If you think Emma is wrong, explain the mistake and explain what *you* think the equation means.

.....

.....

Figure out for yourself the number of pens and the number of notebooks sold in the store.

.....

.....

.....

Suggested lesson outline

Individual work: *Cash Registers* (10 minutes)

Give each student the task sheet *Cash Registers*. Help students to understand the problem, and explain the context of the task briefly.

Spend ten minutes on your own answering these questions.

What does "simultaneously" mean?

Show all your work on the sheet.

Cash Registers

The drawer of a cash register contains some quarters and some dollar bills.

x = the number of quarter coins in the cash register.
y = the number of dollar bills in the cash register.

The following two equations are true:

$3x = y$
$4x + y = 70$

1. Explain in words the meaning of each equation.
2. Find two pairs of values for x and y that satisfy the first equation.
3. Find two pairs of values for x and y that satisfy the second equation.
4. Find pairs of values for x and y that satisfy both equations simultaneously.

Students who sit together often produce similar answers and, when they come to compare their work, they have little to discuss.

For this reason we suggest that, when students do this task individually, you ask them to move to different seats. Then, for the collaborative task, allow them to return to their usual places. Experience has shown that this produces more profitable discussions.

Collaborative small-group work: *Cash Registers* (10 minutes)

Organize the class into small groups of two or three students and hand out a fresh sheet of paper to each group. Students should now have another go at the task, but this time they will combine their ideas.

I want you now to work together in your groups.

Your task is to produce an answer together that is better than your individual ones.

Throughout this activity, encourage students to articulate their reasoning, justify their choices mathematically, and question the choices put forward by others.

As students work you have two tasks, to note student approaches to their work, and to support their thinking.

Note student approaches to their work

How do students choose to tackle this task? Notice the variety in approaches. Notice any common errors. You can use this information to focus your questioning in the whole-class discussion towards the end of the lesson.

Support student thinking

Try not to make suggestions that prompt students towards a particular answer. Instead, ask questions to help students clarify their thinking.

You may find that some students interpret the letters as "quarters" and "dollars" rather than the number of quarters and number of dollars. For example, they may say things like:

"$3x = y$ means three times as many quarters as dollars."
"$4x + y = 70$ means 4 quarters plus dollars equals 70."
"There is \$70 in the till."

The following questions and prompts may be helpful for both students struggling with the task and those making quick progress:

What do the letters x and y represent?

Replace x and y in this equation by words and now say what the equation means.

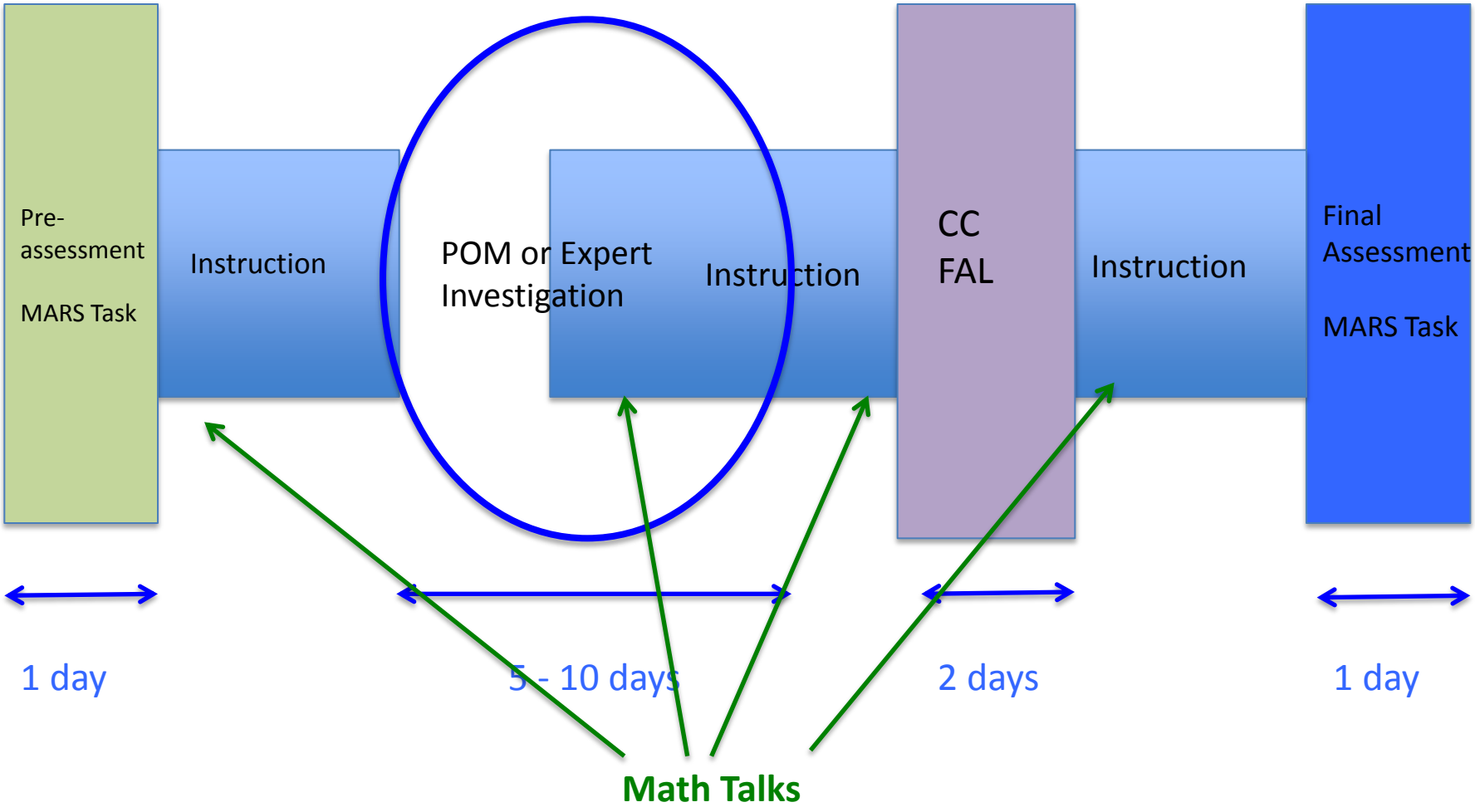
Are there more dollar bills or more quarters in the cash register? How do you know?

Do you have any values for x and y that work for the first equation? How can you check to see if they also work for the second one? If these don't fit, what other values for x and y can you use?

Why have you chosen these values for x and y?

Unit of Study

Around a Big Idea



The Trip

This problem gives you the chance to:
- form and solve equations

A group of students is planning a trip using cars and minibuses.

Each car can hold 5 students and each minibus can hold 14 students.

There will be 53 students on the trip.

Let x be the number of cars and y the number of minibuses they will need.

1. Write an equation in x and y for the number of students on the trip.

_____

The estimate for gas costs is \$4 for each car and \$6 for each minibus.

The total costs for gas will be \$32.

2. Write an equation in x and y for the gas costs.

_____

3. Use both of your equations to find the number of cars and minibuses the students will use. Show your work clearly.

$x =$ _____ $y =$ _____

Common Core Standards:

A New Direction linking Instruction and Assessment

Four More Years to Implemented the CCSSM

Domains K–8

Counting & Cardinality						Ratios & Proportional Relationships		
Operations and Algebraic Thinking						The Number System		
Number and Operations in Base Ten						Expressions and Equations		
			Fractions					Functions
Measurement and Data								
Geometry						Geometry		
						Statistics and Probability		
K	1	2	3	4	5	6	7	8

Two Mathematics Pathways

Two Regular Sequences:

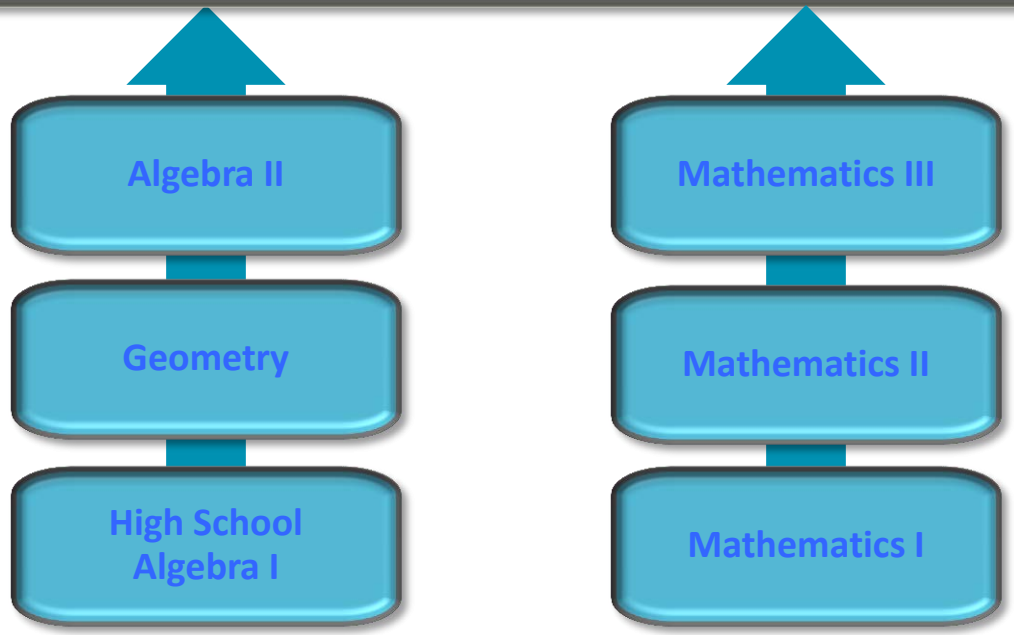
Traditional Pathway

- 2 Algebra courses,1 Geometry course, with Probability and Statistics interwoven

Integrated Pathway

- 3 courses that attend to Algebra, Geometry, and Probability and Statistics each year

Courses in higher level mathematics: Precalculus, Calculus*, Advanced Statistics, Discrete Mathematics, Advanced Quantitative Reasoning, or courses designed for career technical programs of study.



Traditional Pathway
Typical in U.S.

Integrated Pathway
Typical outside of U.S.

Algebra Forever vs CCSSM

Arnold Schwarzenegger
July 8, 2008

"We have made significant gains in enrolling students in Algebra I in eighth grade in recent years, surpassing other state in the U.S. But we must set our goal higher."

We have also made more significant gains in FAILING students in Algebra I in eighth grade in recent years, surpassing other state in the U.S.

3 out of 4 failed in 2008

California Adopted the CCSSM on August 2, 2010 with an addition 15% of a traditional Algebra 1 course and other added standards. We selected PARCC as the assessment to complete the Race to the Top application that we never won.

State Senator Hancock

"Several implementation issues arise by California adopting a different set of grade 8 math standards from other participating states. For example, instructional materials for use in California would need to be different from those used by other states - the unique additional standards may increase the costs of those materials for our local school districts. In addition, assessment consortia will be developing assessment aligned to the common core standards and not the variation adopted in California. This may result in issues with our Algebra standards and curriculum not being aligned with our assessment and accountability system."

CALIFORNIA DEPARTMENT OF EDUCATION
NEWS RELEASE

TOM TORLAKSON
State Superintendent
of Public Instruction

California Adopts Modified Math Standards to Restore Local Decision Making

Required by Legislation, Move Allows Progress Toward Common Core

The move rescinds action by the prior Board in 2010, which adopted standards that contained a unique Grade 8 Algebra I course inconsistent with the published *Common Core State Standards for Mathematics*.

Torlakson recommended the unique Grade 8 Algebra I course be replaced with Algebra I and Mathematics I courses based upon the *Common Core State Standards for Mathematics*.

Date: Wed, 16 Jan 2013


A FLU NT
My Child is in **PRE-NATAL**
Algebra $3x^2 - y = \pi$

The California Algebra Experiment

- In 2012, 59% of all eighth grade students took the CST Algebra 1 exam and more than half were not successful. Even more will repeated the class again in high school.
- In 9th grade, 49% of the students took CST Algebra 1 exam and 75% of those students did not pass.
- Research studies indicate nearly 65% of the students who were placed in Algebra in eighth grade are placed in the same level of Algebra in ninth grade.
- About 46% of the students who were successful in Algebra in the eighth grade (B- grade and Proficient) and who were placed again in Algebra in ninth grade were less successful in their second experience.

It is Algebra Forever not Algebra for All

The word *Algebra* in the title of the course is a major hurdle to improving student learning in mathematics.

It leads to misplacement of students, tracking, and traditional beliefs that only some students can success at mathematics.

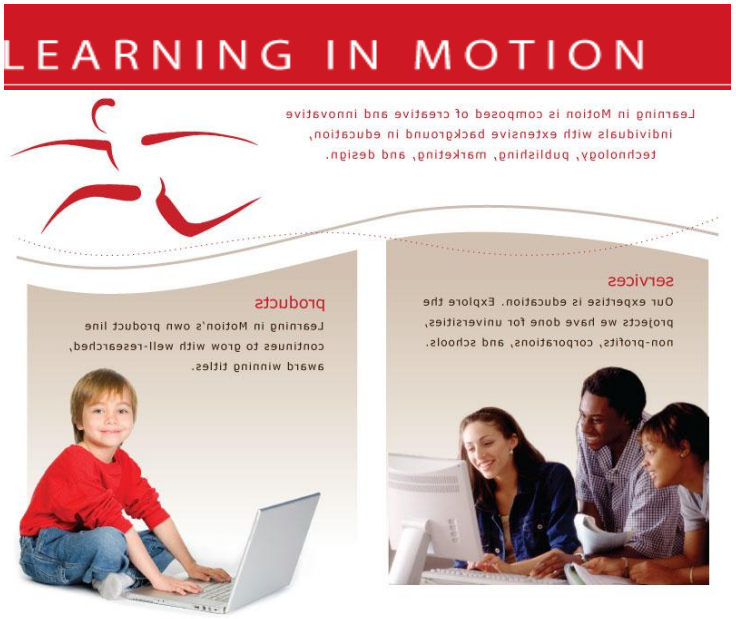
What's in a Name?

- Algebra is the greatest gatekeeper to higher education. (ETS 1988)
- There is no official Algebra 1 course, it differs greatly from state to state although they are all use the same title.
- No other course a has such a failure rate. Teachers believe the Algebra 1 course must be difficult .
- There is no mathematical reason why for a course to be called Algebra (it is a misnomer). Algebra is a strand of mathematics.
- Failure in the course Algebra 1 is the single greatest reasons cited for dropping out in CA (Los Angeles Times 2009).
- Algebra as a course is all about status.

New K-12 Math Curriculum Inspired by The Common Core State Standards

BILL & MELINDA GATES *foundation*

PEARSON FOUNDATION

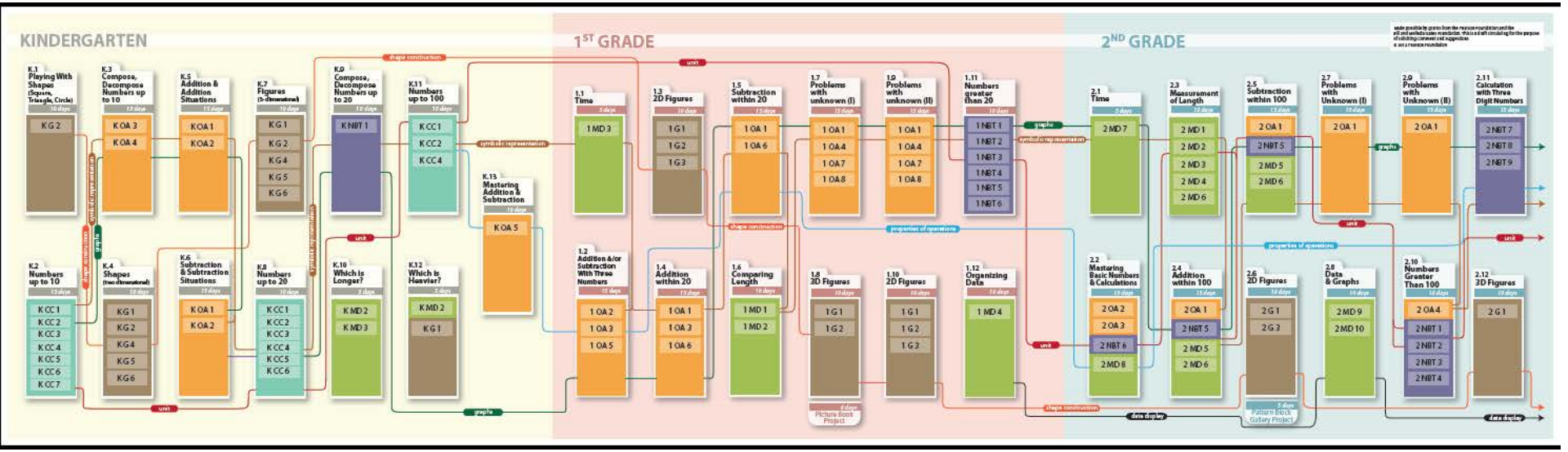


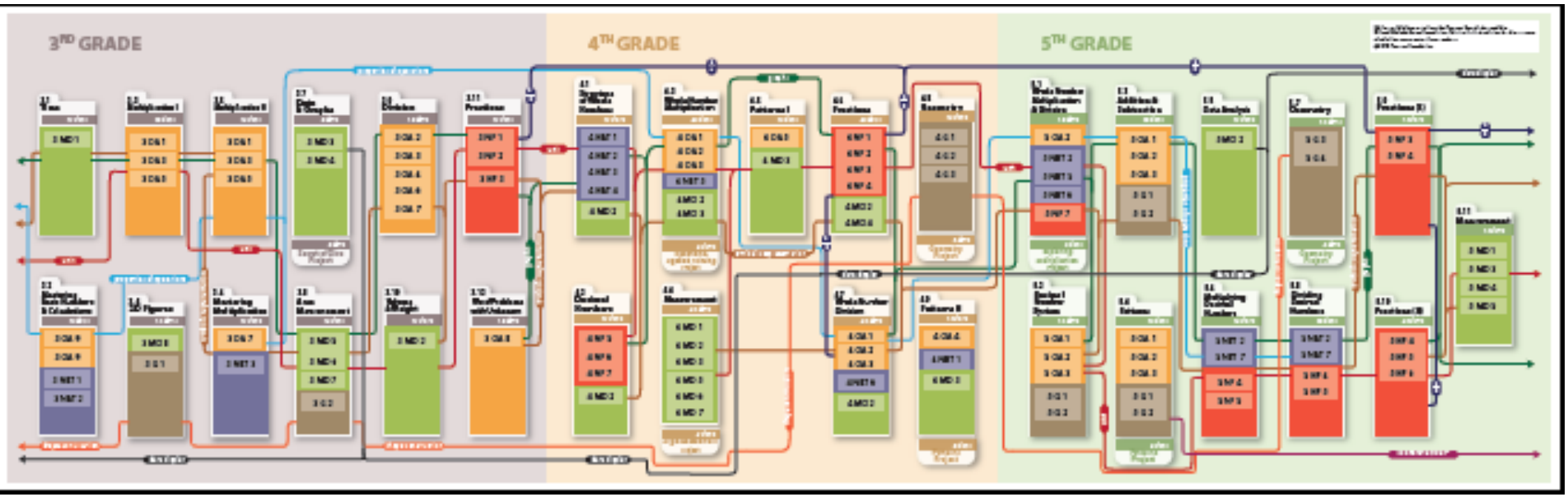
The Gates Foundation and the Pearson Foundation are funding a large scale project to create a system of courses to support the ELA and Mathematics CCSS. These will be a modular, electronic curriculum spanning all grade levels. A Santa Cruz based company, Learning In Motion, is working to write the lessons.

Think in Terms of Units

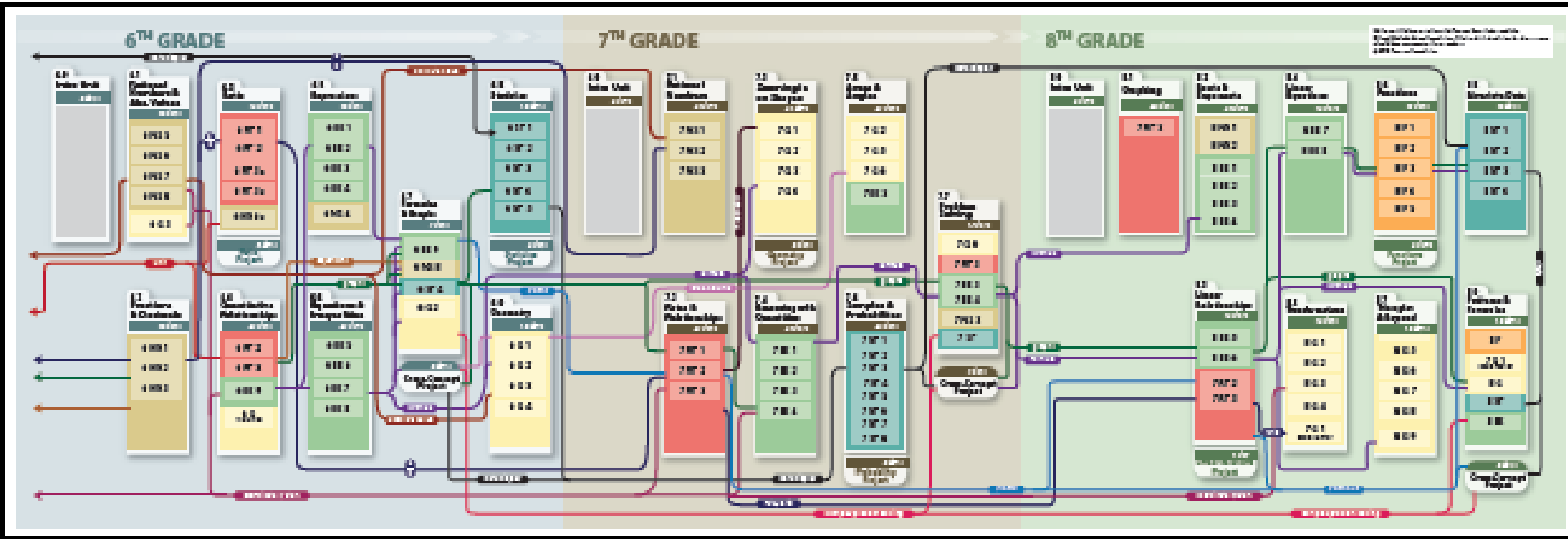
Phil Daro has suggested that it is not the lesson or activity, but rather the **unit** that is the "optimal grain-size for the learning of mathematics". Hence that was the starting point for our Scope and Sequence.

Developers of High School:
Patrick Callahan, Dick Stanley,
David Foster, Brad Findell,
Phil Daro, and Marge Cappo


KINDERGARTEN
1ST GRADE
2ND GRADE
K.1 Playing With Shapes (Square, Triangle, Circle)
K.2 Numbers up to 10
K.3 Compose, Decompose Numbers up to 10
K.4 Shapes (two dimensional)
K.5 Addition & Addition Situations
K.6 Subtraction & Subtraction Situations
K.7 Figures (3-dimensional)
K.8 Numbers up to 20
K.9 Compose, Decompose Numbers up to 20
K.10 Which is Longer?
K.11 Numbers up to 100
K.12 Which is Heavier?
K.13 Mastering Addition & Subtraction
1.1 Time
1.2 Addition &/or Subtraction With Three Numbers
1.3 2D Figures
1.4 Addition within 20
1.5 Subtraction within 20
1.6 Comparing Length
1.7 Problems with unknown (I)
1.8 3D Figures
1.9 Problems with unknown (II)
1.10 2D Figures
1.11 Numbers greater than 20
1.12 Organizing Data
2.1 Time
2.2 Mastering Basic Numbers & Calculations
2.3 Measurement of Length
2.4 Addition within 100
2.5 Subtraction within 100
2.6 2D Figures
2.7 Problems with Unknown (I)
2.8 Data & Graphs
2.9 Problems with Unknown (II)
2.10 Numbers Greater Than 100
2.11 Calculation with Three Digit Numbers
2.12 3D Figures
Picture Book Project
Pattern Block Gallery Project


3RD GRADE
4TH GRADE
5TH GRADE

Middle School Curriculum



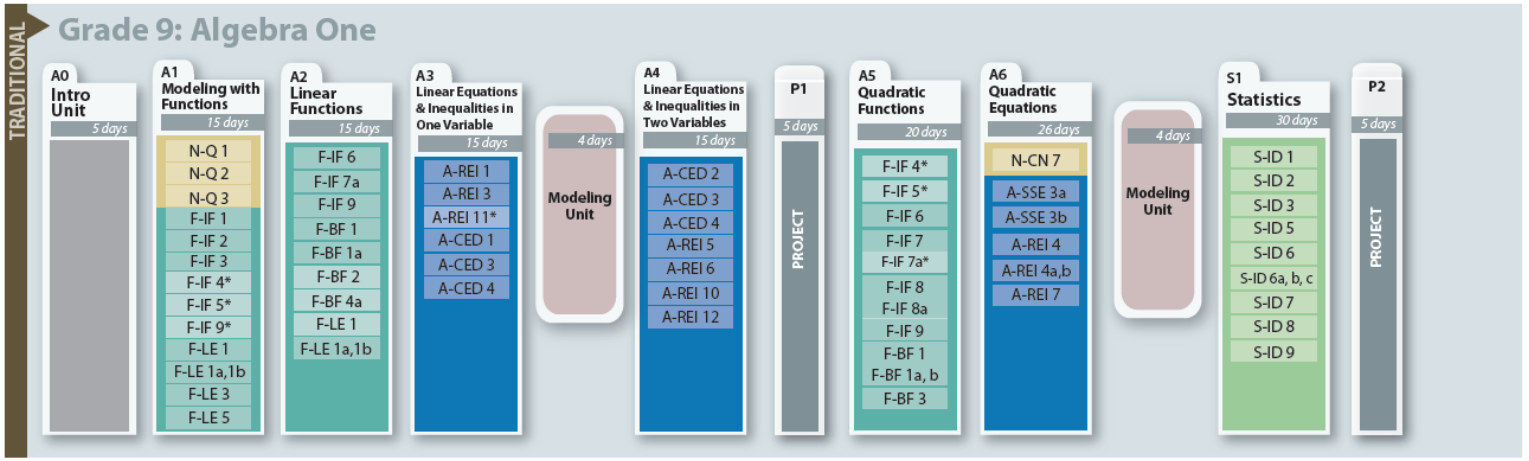
CCSS High School Units

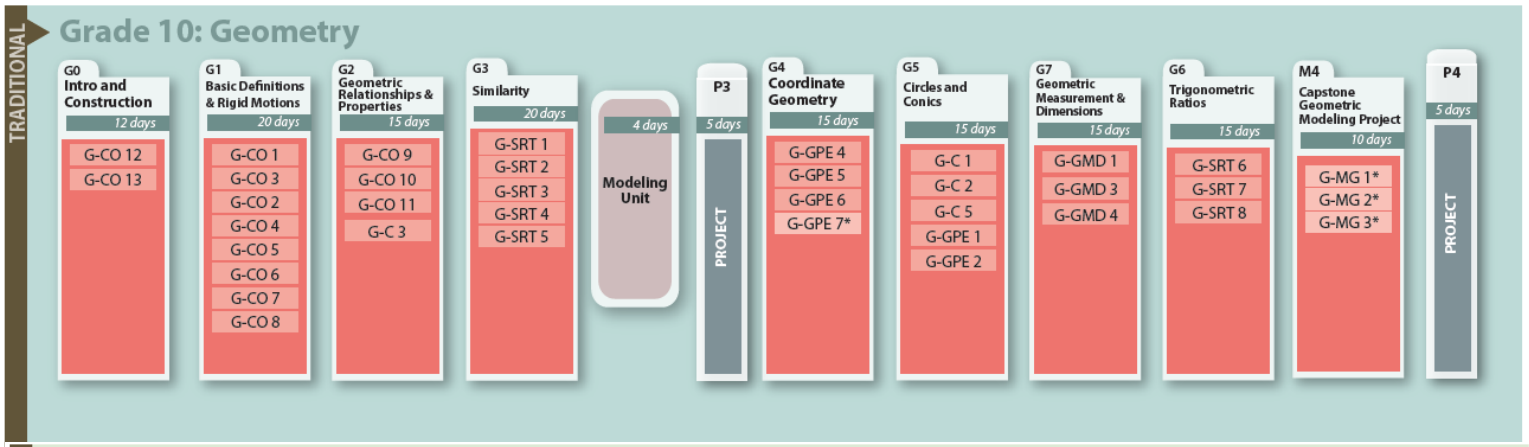
High School Algebra Units:
A0 Introductory Unit
A1 Modeling with Functions
A2 Linear Functions
A3 Linear Equations and Ineq in One Var
A4 Linear Equations and Ineq in Two Var
A5 Quadratic Functions
A6 Quadratic Equations
A7 Exponential Functions
A8 Trigonometric Functions
A9 Functions
A10 Rational and Polynomial Expressions

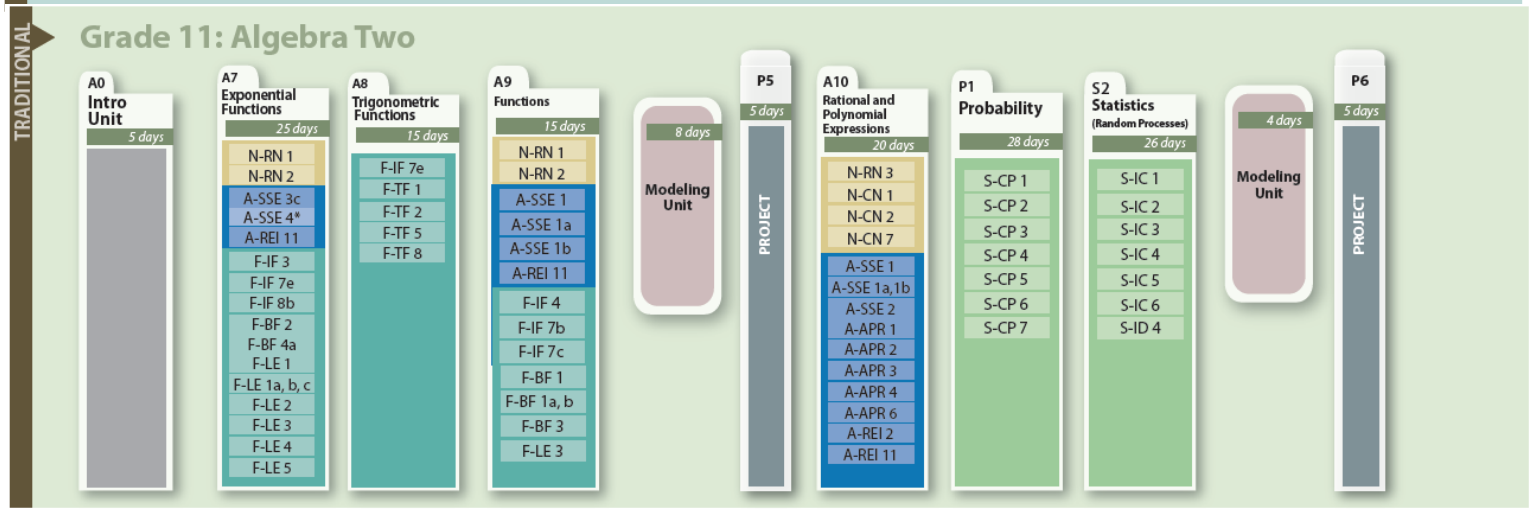
High School Geometry Units:
G0 Introduction and Construction
G1 Basic Definitions and Rigid Motions
G2 Geometric Relationships and Properties
G3 Similarity
G4 Coordinate Geometry
G5 Circle and Conics
G6 Trigonometric Ratios
G7 Geometric Measurement and Dimension
M4 Capstone Geometric Modeling Project

High School Prob & Stat Units:
P1 Probability
S1 Statistics
S2 Statistics (Random Process)

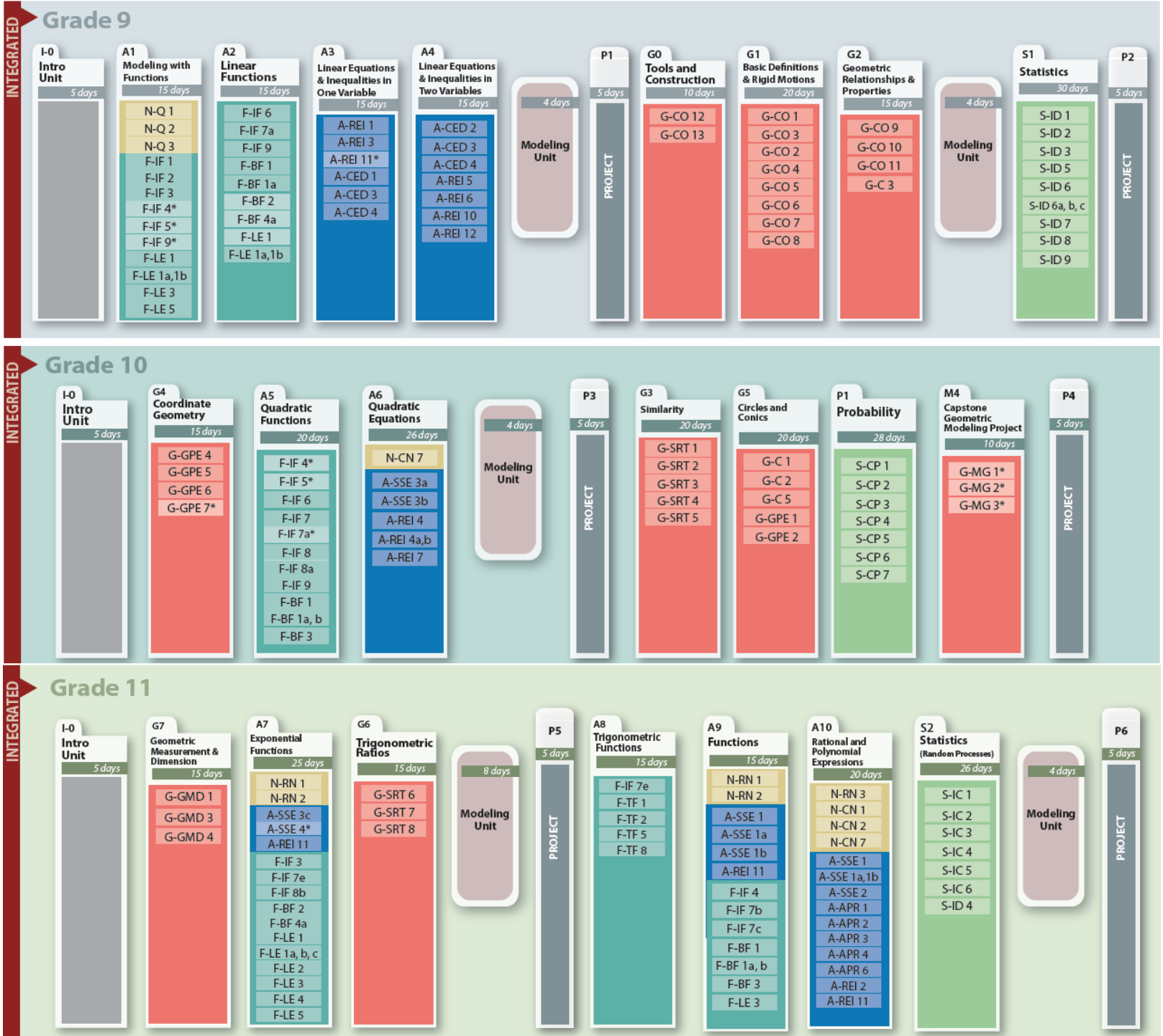
TRADITIONAL



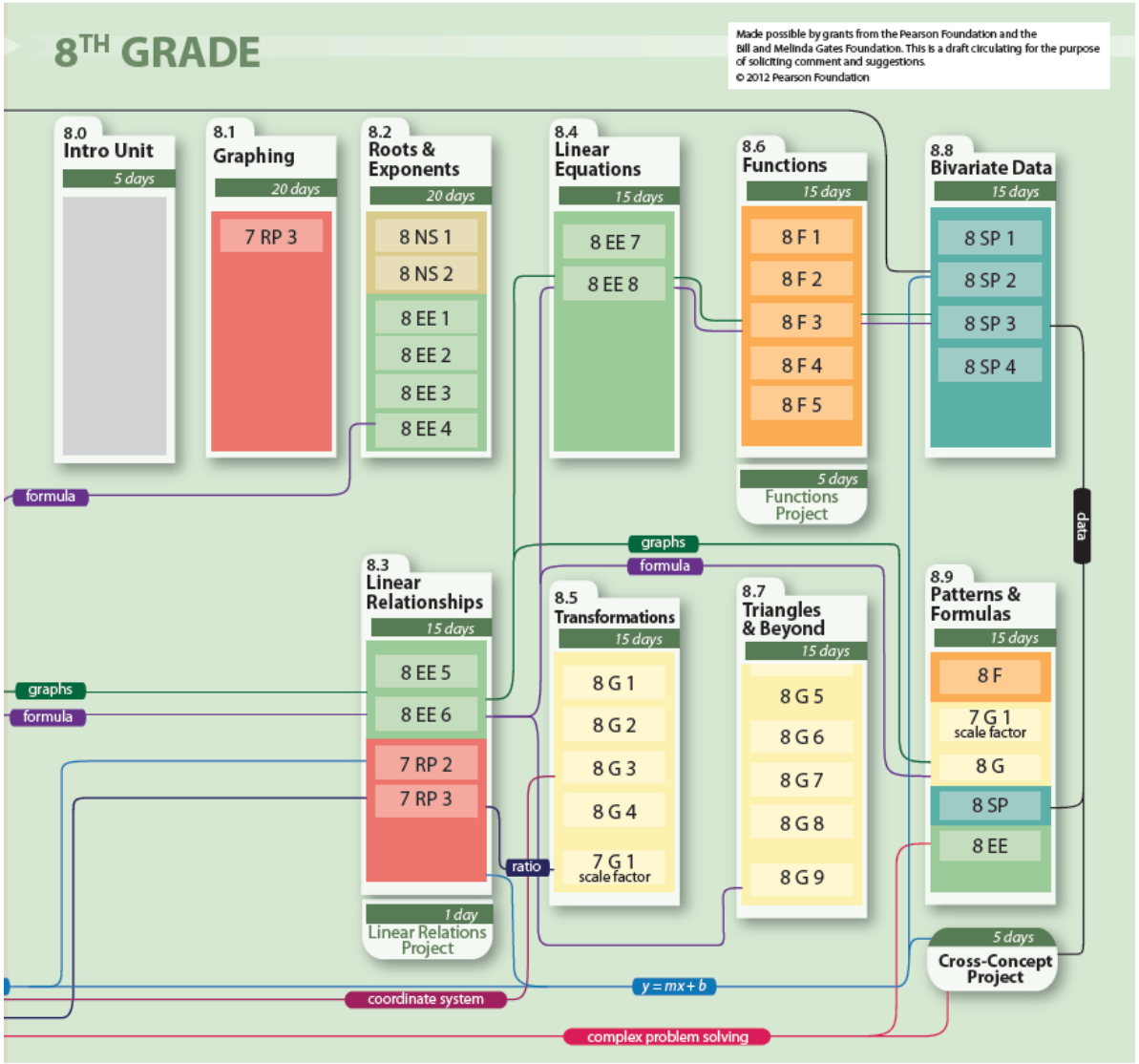




INTERNATIONAL



CCSSM 8th Grade are HS Standards



- Algebra/Functions 67%
- Geometry (Transformations and Triangle Proofs) 20%
- Bivariate Data 10%
- Cross-Concept Project 3%

When do we Accelerate?????

Where to Accelerate?

Can we live without understanding….

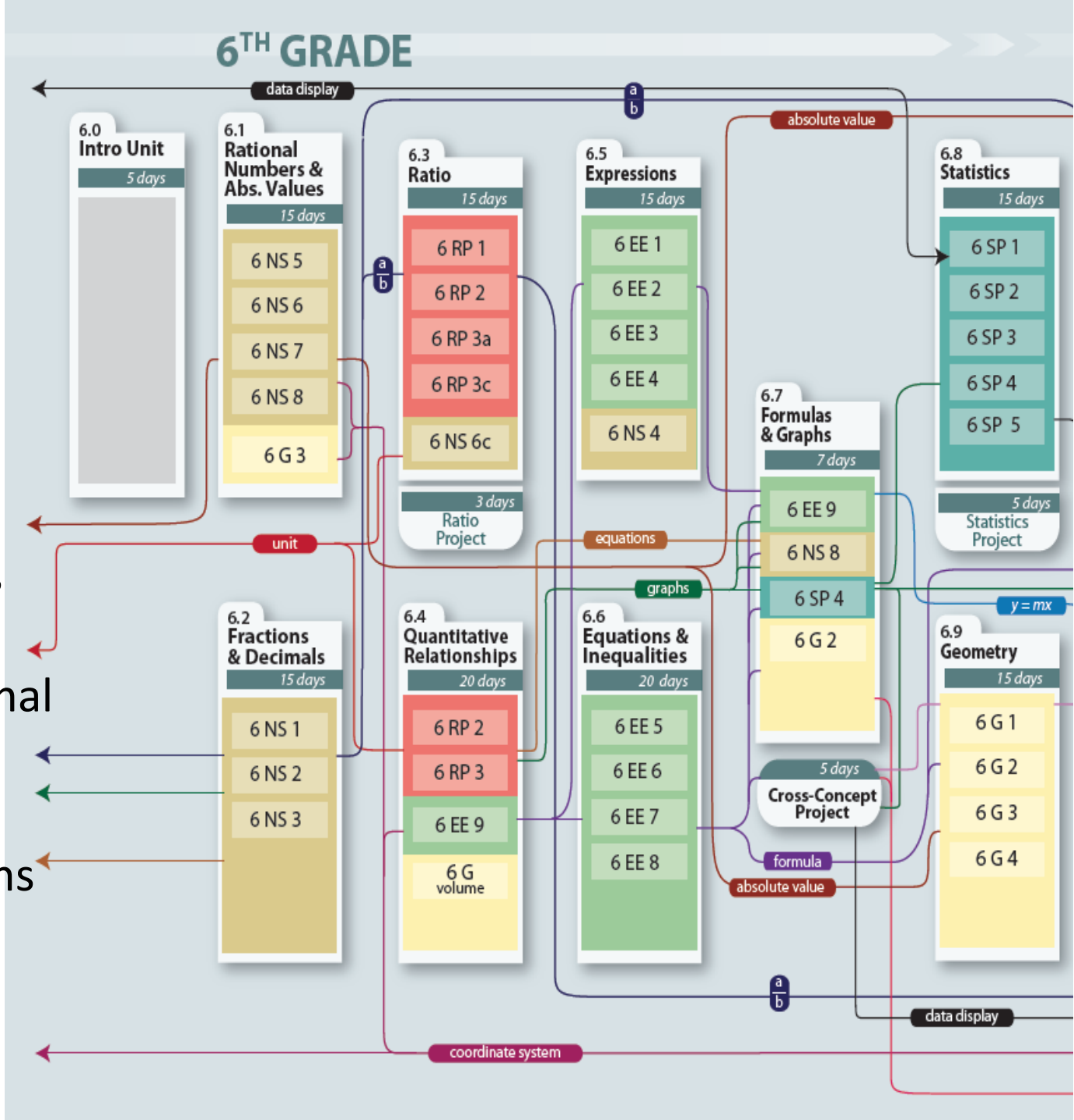
Integer and their operations

Division of Fractions

Ratio and proportional reasoning

Expression, Equations and Inequalities

Statistics



Where to Accelerate?

Can we live without understanding….

Properties of rational numbers, percents, discounts, markups, etc.

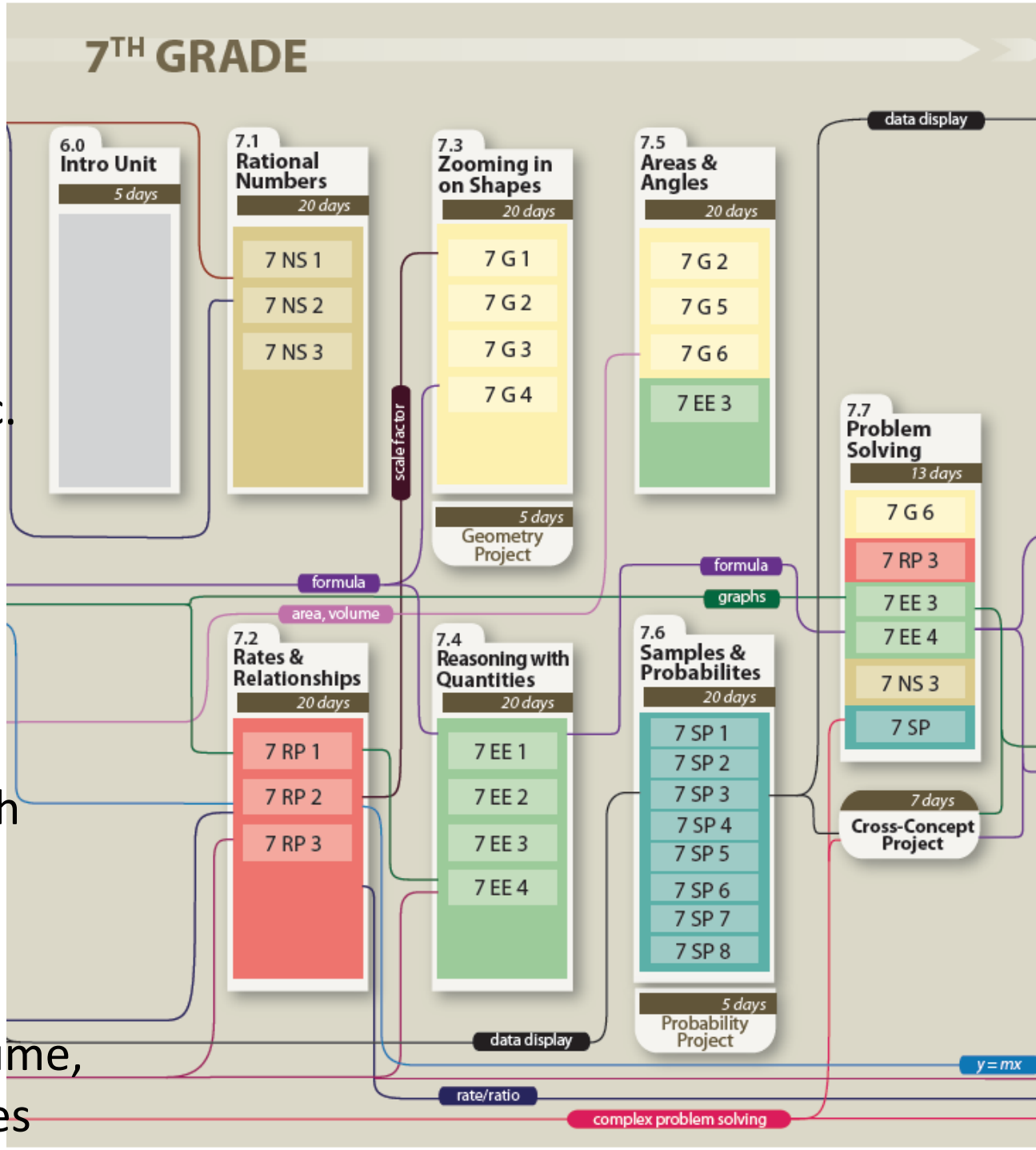
Rate and problems solving using rate

Similarity, proportional reasoning

Algebraic Modeling with Equations

Probability

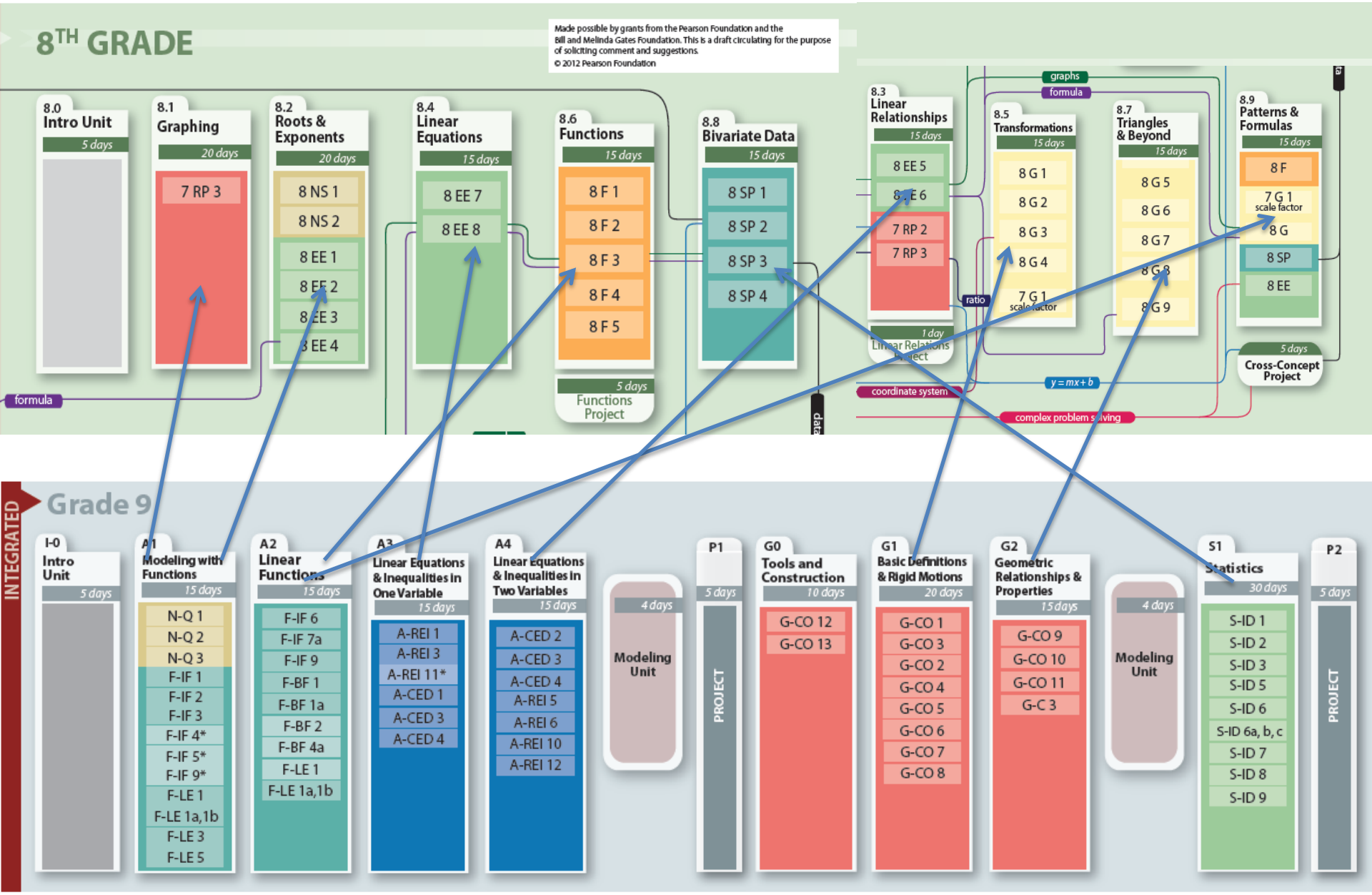
Geometry: Angles, Volume, Surface Area, 3-D shapes



When do they Accelerate in Japan?

After 8th Grade!!!!!!!

Where to Accelerate????



When do we Accelerate?????

The Only Reasonable Answer for Learning: 9th Grade!!!!

This site gets 10,000 hits each week. 6,000 to download POMs.

INSIDE +=X÷ MATHEMATICS

INSIDE MATHEMATICS

HOME | TOUR OF INSIDE MATHEMATICS | TOO EDUC

Home

Welcome to the Inside Mathematics Website

Welcome to **Inside Mathematics**, a professional resource for educators passionate about improving students' mathematics learning and performance. This site features classroom examples of innovative teaching methods and insights into student learning, tools for mathematics instruction that teachers can use immediately, and video tours of the ideas and materials on the site.

We are glad you're here and look forward to learning with you!

News - Inside Mathematics is aligning its resources with the Common Core State Standards for Mathematics.

A PROFESSIONAL RESOURCE FOR EDUCATORS

Number Talks

Number talks were developed for classroom teachers to engage students in "mental math" through gra with interesting mathematics problems. Educators can use number talks regularly as introductions to th mathematical practice, as "warm ups" for other lessons, or as stand-alone extended engagements with mathematical concepts.

2nd grade: Number of the Day

In this Number Talk, Stephanie Letson engages her 2nd grade students in finding multiple ways to write equations that result in the number 170. She introduces her students with some norms for their learning, such as listening, sharing, writing. She introduces the day's number by asking them what they notice about the number, and what those noticings (multiple of ten, even number) might mean for their equations. Students work individually, then share in small groups, then share as a whole group. Letson facilitates their discussion and follows student insights when they are offered.

Play

2nd grade: ¿Cómo se suma?/ How do we add?

In this Number Talk, Katie Arrillaga engages her 2nd grade Spanish bilingual students in a discussion about strategies for mentally solving a problem involving two-digit addition with regrouping. The students share their strategies and evaluate each other's approaches for solving the problem.

Play

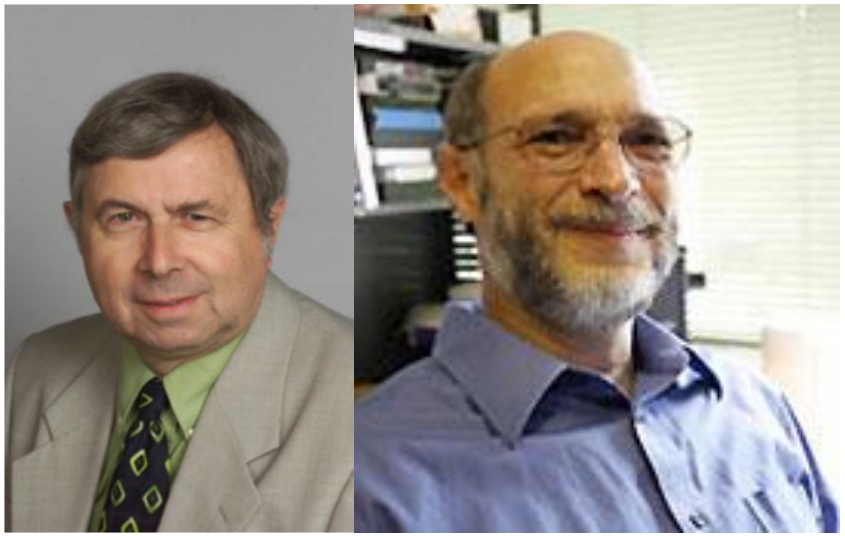
4th grade: Can This Be True?

In this Number Talk, Elysha Passeggi engages her 4th grade students in a discussion about whether or not the sum of two two-digit numbers can be 238. Students share their responses, defend their thinking, and formulate declarative statements about why the problem is definitively false.

Play

Curriculum inspired by the CCSS

MAP's Formative Assessment Lessons and Professional Development Modules



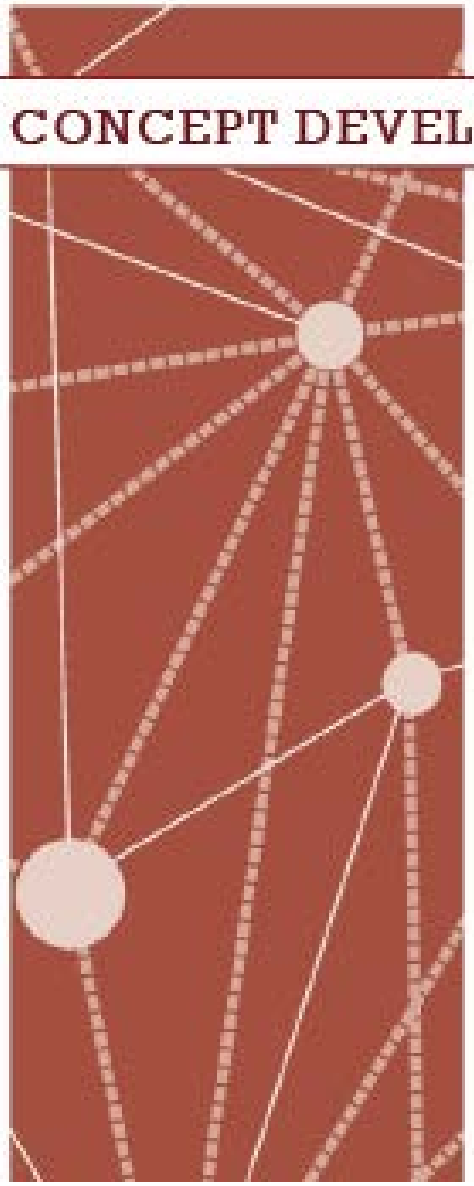
BILL & MELINDA GATES foundation

MARS Team
Mathematics Assessment Resource Service

Assessment For Learning

Formative Assessment Lessons (2 days) for High School and Middle School

CONCEPT DEVELOPMENT



Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Lines and Linear Equations

Inside Mathematics Website

http://www.insidemathematics.org

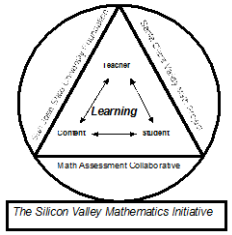
Mathematics Assessment Pro

UC Berkeley & Shell Centre for Mathematical Education

MARS Team
Mathematics Assessment Resource Service

http://map.mathshell.org/materials/lessons.php

Silicon Valley Mathematics Initiative



http://www.svmimac.org