

Name:

Teacher:

Period:

2013 –

MATH 7

2014

Probability

UNIT 7A

P1-1 Are You Psychic?

Experiment #1

Directions:

Player 1 holds card on his/her forehead

Player 2, without speaking, concentrates on the color of the card (no hints!)

Player 1 tries to read Player's 2 mind, and guesses the color on his/her forehead

Players tally correct and incorrect guesses below:

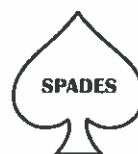
- play 10-20 games per person
- replace card after each turn

	<u>Correct Guesses</u>	<u>Incorrect Guesses</u>	<u>Total Guesses</u>	<u>Correct Guesses as a Fraction</u>
Player 1 Name:			10	$\frac{4}{10}$
Player 2 Name:			10	$\frac{5}{10}$

Record your data on class chart #1.

Remember class tally.

Do you or your partner appear to be psychic? Explain why or why not?



Experiment #2

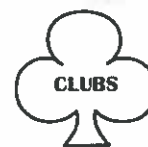
Directions:

Player 1 holds card on his/her forehead

Player 2, without speaking, concentrates on the suit of the card (no hints!)

Player 1 tries to read Player's 2 mind, and guesses the suit on his/her forehead

Players tally correct and incorrect guesses below:



	<u>Correct Guesses</u>	<u>Incorrect Guesses</u>	<u>Total Guesses</u>	<u>Correct Guesses as a Fraction</u>
Player 1 Name:			10	$\frac{3}{10}$
Player 2 Name:	I		10	$\frac{1}{10}$

Record your data on class chart #2.

Do you or your partner now appear to be psychic? Explain why or why not?

Experiment #3

How to play:

Player 1 predicts what number will be rolled.

Player 2 rolls a die.

Players tally numbers as they are rolled.

Players also tally whether their predictions were correct or incorrect.

Outcome tallies:

1	2	3	4	5	6

Your Correct Tallies

|||

Your Wrong Tallies

|||| ||

Total correct as a fraction
(Simplify if possible.)

$\frac{3}{10}$

Partner Correct Tallies

Partner Wrong Tallies

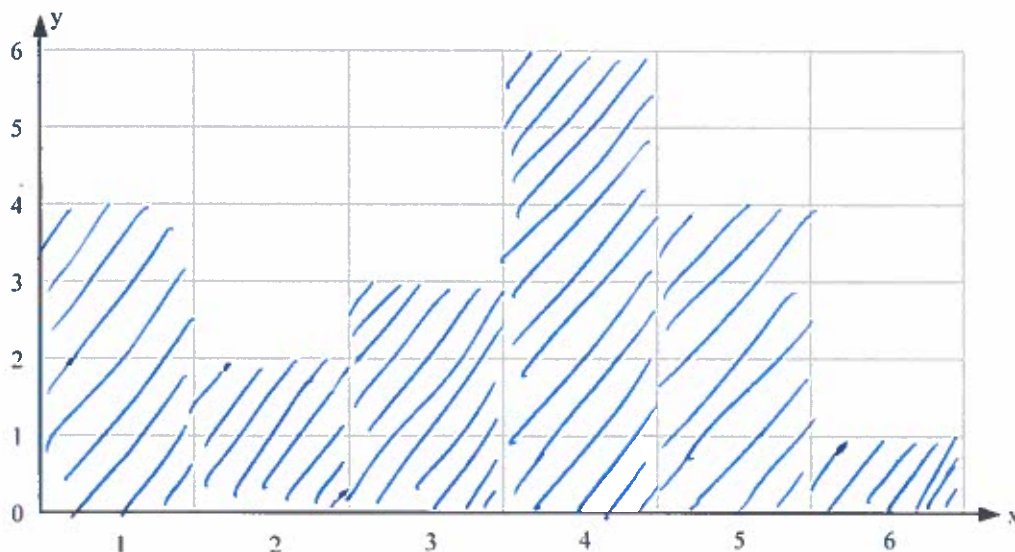
|||| |||

Total correct as a fraction
(Simplify if possible.)

$\frac{0}{10}$

Do you or your partner appear to be psychic? Why or why not?

Make a bar graph of your outcome tallies.



P1-2 Probability Investigation: Guess & Draw

Examine the tallies from the Guess and Draw activity.

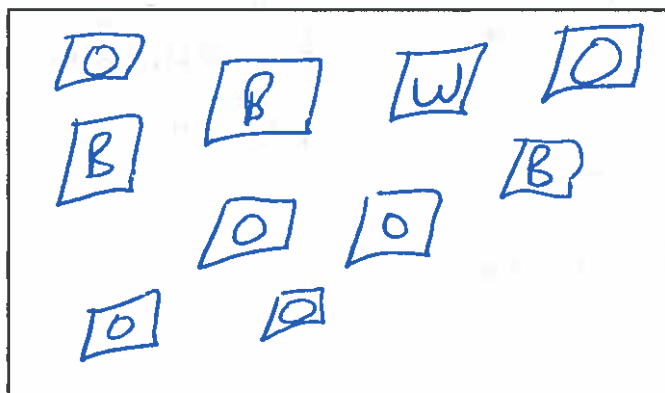
Partner Talk:

- Based on our classroom tally, why do you think some of the colors were drawn out more than others?
- If you want to correctly guess the color of the object, what color do you think is the BEST choice for your guess?

Record your ideas here:

- need a bag or some way to conceal objects (ie, a cup)

Sketch the contents of the container here:



There are 6 orange cubes.

There are 3 blue cubes.

There are 1 white cube.

There are 0 green cubes.

There are 10 cubes in total.

The probability of pulling out a orange is $\frac{6}{10}$ or $P(\text{orange}) = \frac{6}{10} = \frac{3}{5}$

The probability of pulling out a blue is $\frac{3}{10}$ or $P(\text{blue}) = \frac{3}{10}$

The probability of pulling out a white is $\frac{1}{10}$ or $P(\text{white}) = \frac{1}{10}$

The probability of pulling out a green is $\frac{0}{10}$ or $P(\text{green}) = \frac{0}{10}$

Probability written as a ratio:

want
total possible

← the number of chances
for what we want or
expect to happen

← the number of total
possible outcomes

PROBABILITY MEANS

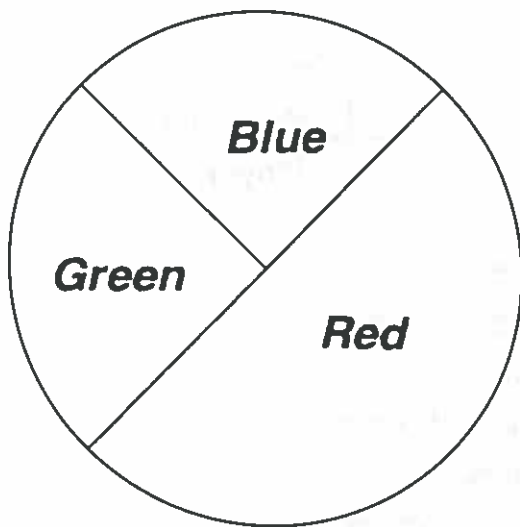
Partner Talk:

- Was the Guess and Draw game the same for everyone who played?
- If you could play Guess and Draw again, would you prefer to make your guess first, last, or in the middle of the game? Explain your thoughts?

Record your ideas here: _____

P1-3 Probability Investigation: Predict & Spin

Color this spinner so that the two smaller parts are blue and green. Make the largest part red. Your team will spin this spinner 25 times. **Before** spinning, make predictions about how many blue, green and red you will get. Record your predictions on the chart, then spin and record the actual results.



Color	Prediction	Actual
Blue	6	1
Green	6	
Red	13	1
Total	25	25

Who in your team had the closest prediction?

Carlos

Look at the spinner. Record the probability for each color here:

P(Blue) = $\frac{1}{4}$

P(Green) = $\frac{1}{4}$

P(Red) = $\frac{2}{4}$ or $\frac{1}{2}$

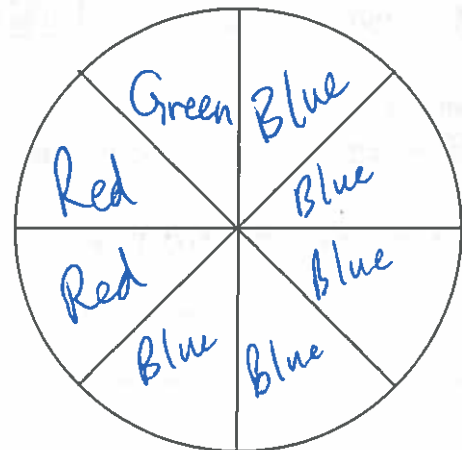
Partner Talk: How do you read this? (write it out below)

The probability of spinning blue is one out of four.

For the spinner below, choose colors for the parts of the spinner, but every person at your team needs to color the spinner the same. You must use exactly 3 different colors.

Again, make predictions **before** you spin 25 times.

Color	Prediction	Actual
Blue	15	1
Green	3	
Red	7	1
Total	25	25



Who had the closest prediction? Carlos Again!

P(Blue) = $\frac{5}{8}$

P(Green) = $\frac{1}{8}$

P(Red) = $\frac{2}{8} = \frac{1}{4}$

P1-4 A Two-Coin Game for Three People

Today you will play a game with two other students. In this game, you will flip two coins at the same time.

Each person will take one of these outcomes as their own:

Both Heads

Christian

Player 1

Both Tails

Nadia

Player 2

One of Each

Dayana

Player 3

Directions: Player 1 flips the two coins 10 times
Each player records a tally for the outcome
Player 2 flips the two coins 10 times
Each player records a tally for the outcome
Player 3 flips the two coins 10 times
Each player records a tally for the outcome

	Player 1 wins (both heads)	Player 2 wins (both tails)	Player 3 wins (one of each)
Tallied Wins			

Who won the game? Player 3

Partner Talk:

If you could play the game again which player would you want to be? Why?

Record your thoughts below: _____

How many events are taking place when the coins are flipped? two events

A "compound event" represents when two or more events are taking place at the same time.

See Bottom of page if you wish to teach "Sample Space" at this time. This lists all possible outcomes.

Possibilities

First coin:

Second Coin:

	H	T
H	<u>Player 1</u> (H) (H)	(T) (H)
T	(H) (T)	<u>Player 2</u> (T) (T)

Handwritten notes on the table:
- A large blue oval encircles the top-left cell (HH) and is labeled "Player 1".
- A large blue oval encircles the bottom-right cell (TT) and is labeled "Player 2".
- A large blue oval encircles the entire 2x2 grid and is labeled "Player 3!".

$$P(\text{Both Heads}) = \frac{1}{4}$$

$$P(\text{Both Tails}) = \frac{1}{4}$$

$$P(\text{One of each}) = \frac{2}{4} = \frac{1}{2}$$

↑ Could also be $P(\text{Heads "and" Tails})$

* There are four possible outcomes.

Sample Space = {HH, TH, HT, TT}

P2-1 Investigating Dice

Should be "P1-5."



A pair of dice will be rolled and the two numbers will be added.

What is the largest possible total you can get with two dice?

12

What is the smallest possible total you can get with two dice?

2

Directions:

- 1) List all of the possible sums from smallest to largest on the "sum" line.
- 2) Each person chooses a sum and circles it as his/her own.
- 3) Roll the dice 20 times and tally the sum for each roll.
- 4) The person with the most tallies is the winner!

Sum:	2	3	4	5	6	7	8	9	10	11	12	
Tallies:												

Angel

Noe

Maria

Who won the game? Noe

Compile a class tally table before Partner Talk

Partner Talk:

If you were to play the game again, which sum would you choose? Explain your reasoning.

A "compound event" represents when two or more events are taking place at the same time.

		First Die					
		1	2	3	4	5	6
Second Die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

How many *total* possibilities did you find from the table? 36
 Based on this information what would everyone choose and why?

$$P(2) = \frac{1}{36}$$

$$P(12) = \frac{1}{36}$$

$$P(3) = \frac{2}{36}$$

$$P(11) = \frac{2}{36}$$

$$P(4) = \frac{2}{36}$$

$$P(10) = \frac{3}{36}$$

$$P(5) = \frac{4}{36}$$

$$P(9) = \frac{4}{36}$$

$$P(6) = \frac{5}{36}$$

$$P(8) = \frac{5}{36}$$

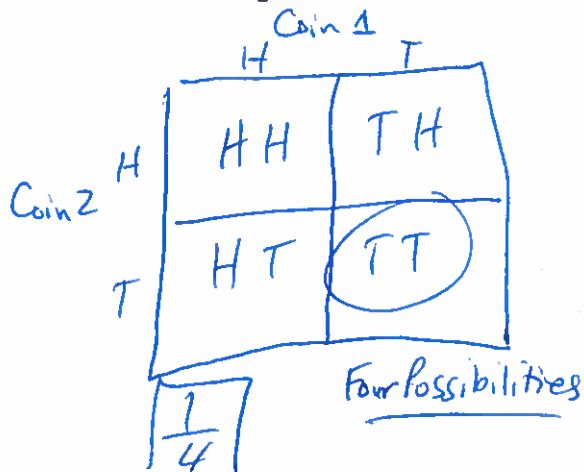
$$P(7) = \frac{6}{36}$$

DO NOT REDUCE!!
(yet...)

Sample Space: { 2 3 3 4 4 4 5 5 5 5 6 6 6 6 6 7 7 7 7 7 7 8 8 8 8 8 9 9 9 9 10 10 10 11 11 11 12 }

P2-2 Compound Probability - Chart Strategy

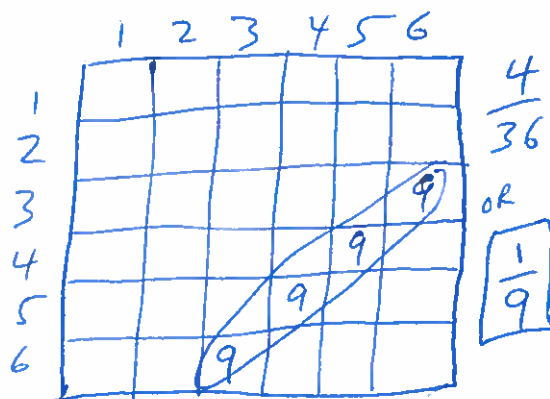
Shirley flipped two coins.
What is the probability of both coins landing on "tails"?



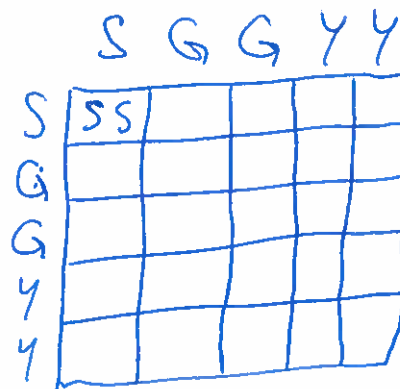
Jordan had a bag of marbles. In the bag were 3 blue marbles, 2 purple marbles, and 1 green marble. What is the probability of pulling a blue marble, putting it back in, and pulling out a green marble?



Brian rolled two dice.
What is the probability of Brian getting a sum of 9?



Jesus has one striped shirt, two green shirts, and two yellow shirts. He also has two pairs of jeans and a pair of shorts. How many different outfits can Jesus make?



Do we need to fill this in to find the number of possibilities?

Compound probability means the probability of two or more events happening.

Whenever you do a Compound probability problem, one strategy is to use a chart.

P2-3 The Order Game

If you had the numbers A, B, C, and D in a bag, how many tries would it take for you to pull them out *in order*?

Make a prediction: It will take 7 tries

Directions:

- 1) Pull cards from the bag one at a time.
- 2) Do not replace the card that is pulled.
- 3) Pull the other cards out of the bag *one at a time*.
- 4) Record your results.

Record your results each time:

A

Card 1

A

Card 1

B

Card 1

Card 1

Card 1

Card 1

Card 1

Card 1

Card 1

C

Card 2

D

Card 2

D

Card 2

Card 2

Card 2

Card 2

Card 2

Card 2

Card 2

D

Card 3

C

Card 3

A

Card 3

Card 3

Card 3

Card 3

Card 3

Card 3

Card 3

B

Card 4

B

Card 4

C

Card 4

Card 4

Card 4

Card 4

Card 4

Card 4

Card 4

X

X

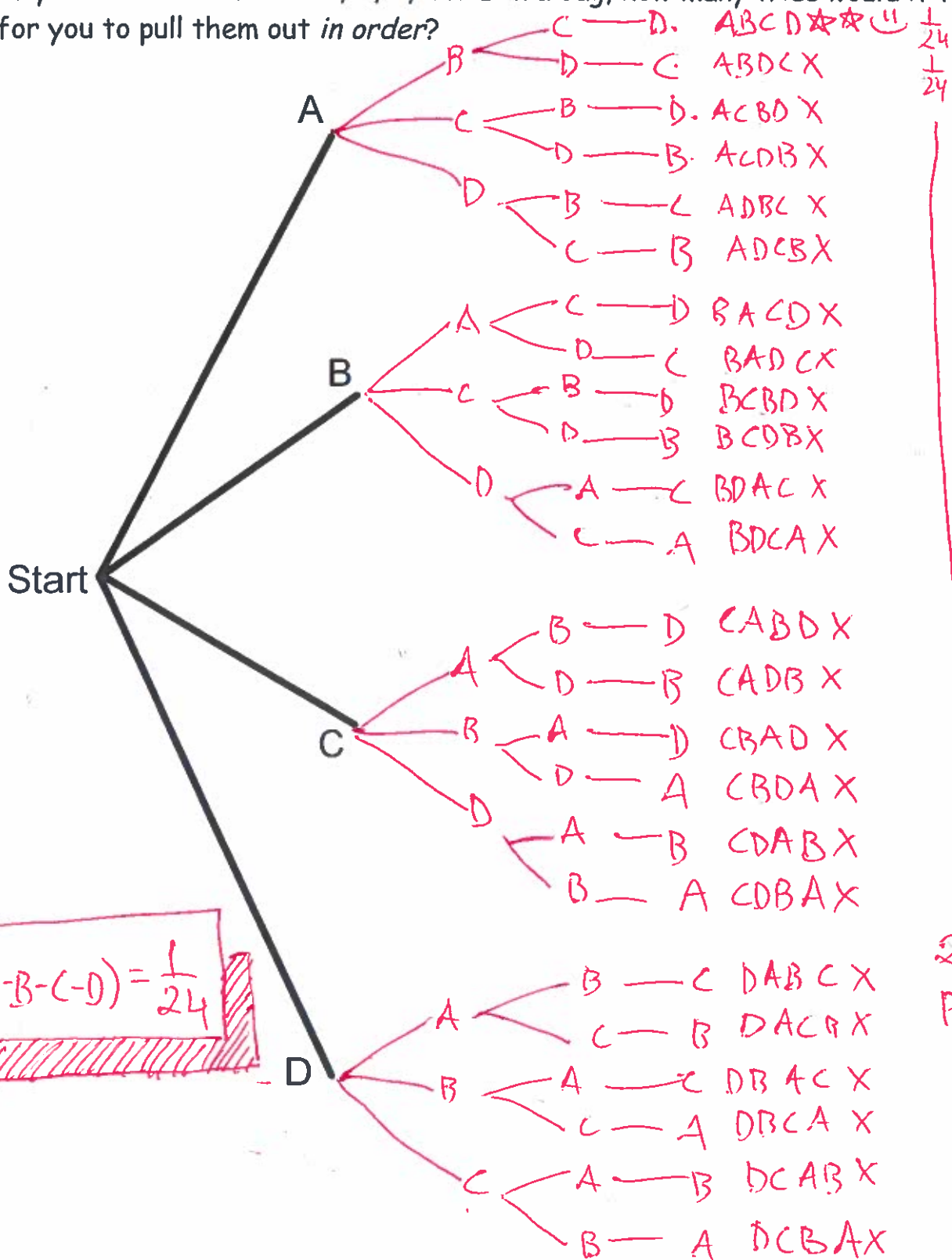
X

↓

etc.

P2-3 The Order Game - Tree Strategy

If you had the numbers A, B, C, and D in a bag, how many tries would it take for you to pull them out *in order*?



P2-4 What is the probability?

Directions: Some things are *certain* to happen. If today is Monday, tomorrow will be Tuesday. Some things are *impossible*. You can't roll a 7 using only one regular die. Some things may or may not happen. *Maybe* it will rain, maybe it won't.

Part A:

Mark each of the following statements with one of the following:

C for *certain*, *I* for *impossible*, or *M* for *maybe*.

- I 1. There is a live dinosaur in the zoo.
- M 2. You will get tails if you flip a coin.
- M 3. It will rain on Sunday.
- M/C 4. Superman will always beat the bad guys.
- M 5. Someone will win a state lottery twice in one year.
- M/I 6. When you grow up, you will be 10 feet tall.
- C/M 7. Outside at night, you can see the stars.
- M 8. Your little league team will win its next game.
- C 9. The Earth revolves around the sun.
- M 10. You will be in school tomorrow.
- M 11. In a new box of crayons, at least one will be red.
- I 12. The next time you throw an ordinary ball up into the air, it will keep going into space.

Part B:

Now write 2 more statements that fit into each category. Include your answers.

Certain


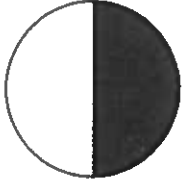

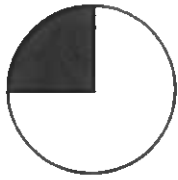
1. Death
2. Taxes

Possible

3. I will own my own home some day
4. I will buy a Prius and become a yuppie

Impossible

5. I will learn to fly
6. I will transform into a dragon

No Way	Maybe	Good Chance	For Sure
Unlikely	Likely	Equally likely	Small Chance
Even Chance	Probable	Equally unlikely	0.25
25%	50%	75%	90%
100%	0%	1/4	1/2
3/4	.75	.50	10%
1/6	5/6	.30	.8
The Braves will win the World Series.	You will have two birthdays this year.	If today is Friday, tomorrow will be Saturday.	At least one student will be absent tomorrow.
It will rain this week.	There will be an earthquake in California this month	The sun will rise in the morning.	You will be in 8th grade next year.
Add your own word:	Add another word	Add another decimal	Make up your own sentence
			

P2-5 Number Line Activity

Directions:

1. Draw a line down the length of a separate piece of paper
2. Label the line as indicated below.
3. Cut out the Probability cards (previous page) and glue them onto the appropriate place on the probability scale.
4. Add additional words to the scale.

Probability Scale	
0 Impossible	1 Certain

P3-1 Let's Practice!



What is the probability of choosing a green marble from a jar containing 5 red, 6 green and 4 blue marbles?

Chart:

R R R R R G G G G G G B B B B

Tree diagram:

START

My answer:

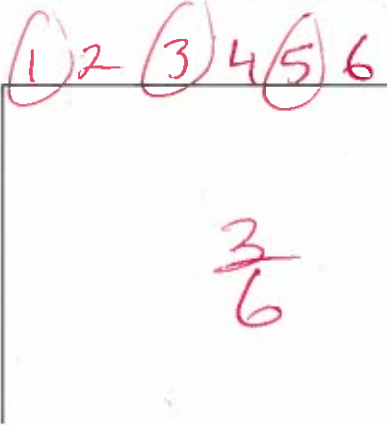
$$\frac{6}{15} \text{ (or } \frac{2}{5} \text{) chance}$$
$$P(\text{green}) = \frac{2}{5}$$

P3-2 Let's Practice!



What is the probability of getting an odd number when rolling a single six-sided die?

Chart:



Tree diagram:

START

My answer: $P(\text{odd}) = \frac{3}{6}$ or $\frac{1}{2}$

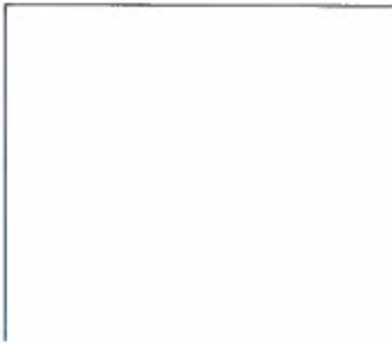
↑
not necessary to write
the answers in this manner,
accept written responses
as well

P3-3 Let's Practice!

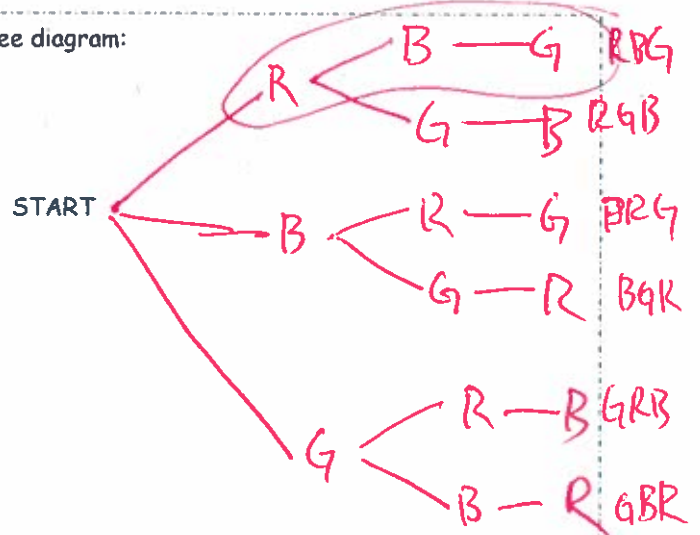


What is the probability of picking a red marble, a blue marble and a green marble in order (red, blue, green) from a bag containing only three

Chart:



Tree diagram:



My answer:

$$p(\text{red, blue, green}) = \frac{1}{6}$$

P3-4 Let's Practice!

i

What is the probability of choosing the letter *i* from the word probability?

Choose a strategy! Show your work:

p r o b a b (i) (i) t y

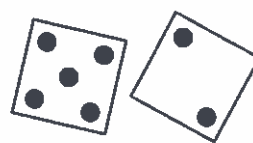
$$\frac{2}{11}$$

My answer:

$$P(i) = \frac{2}{11}$$

P3-5 Let's Practice!

What is the probability of getting a sum of 11 when two dice are rolled at the same time?



Choose a strategy! Show your work:

Students can repeat
chart from previous
activity

My answer:

$$p(11) = \frac{2}{36} \text{ or } \frac{1}{18}$$

P3-6 Let's Practice!

Out of 72 apples at the store, 12 are rotten and half are bruised. What is the probability that Michael will choose a perfect apple?



Choose a strategy! Show your work:

$$\begin{array}{r} 72 \\ - 12 \\ \hline 60 \\ - 36 \\ \hline 24 \end{array} \rightarrow \frac{24}{72}$$

My answer:

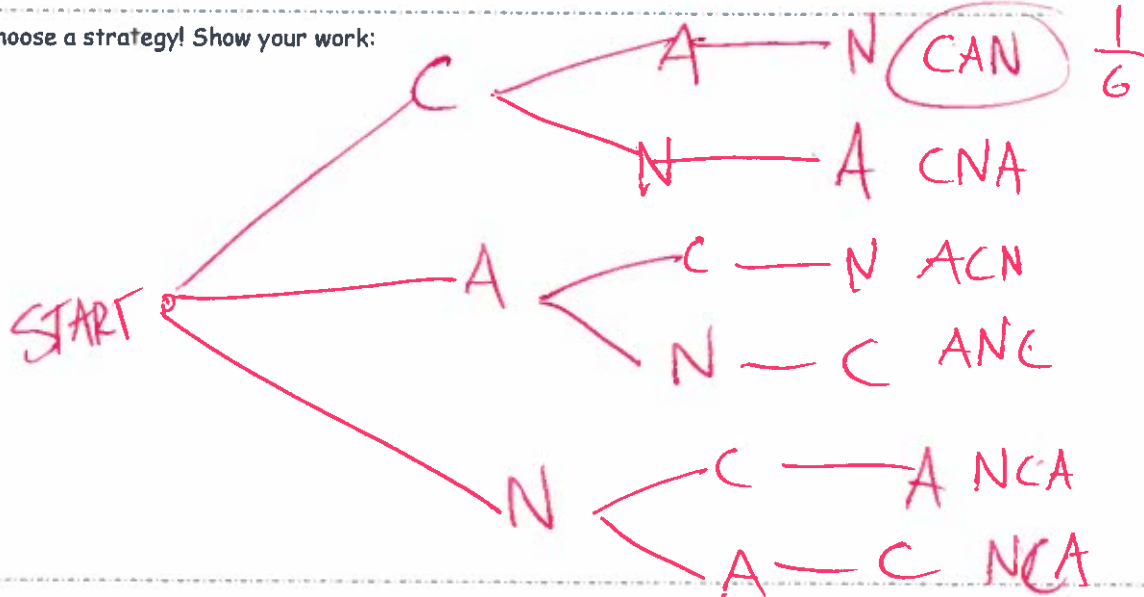
$$p(\text{perfect}) = \frac{24}{72} \text{ or } \frac{1}{3}$$

P3-7 Let's Practice!



What is the probability of selecting cards labeled C-A-N in order from a bag containing only the cards C, A, and N?

Choose a strategy! Show your work:



My answer:

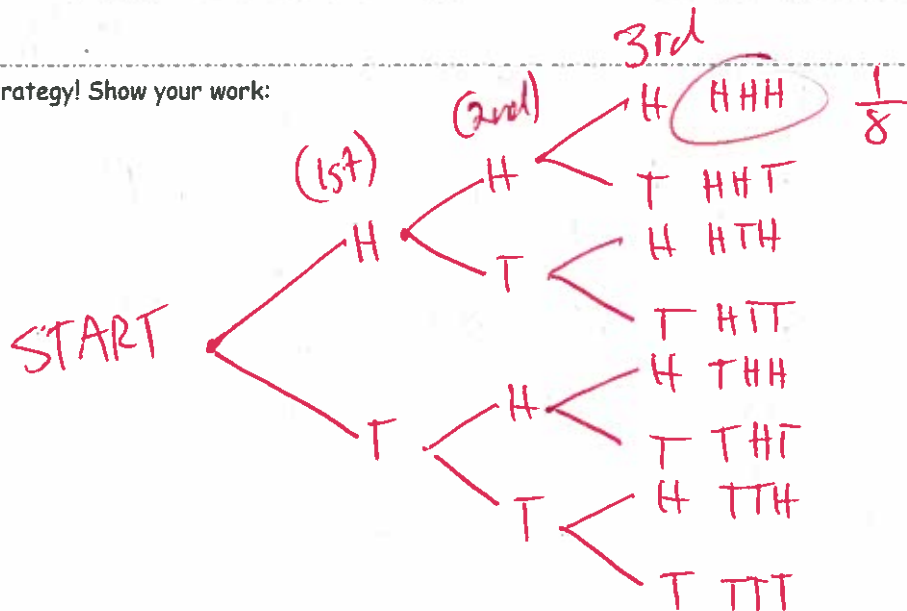
$$P(\text{CAN}) = \frac{1}{6}$$

P3-8 Let's Practice!

What is the probability of flipping three coins at the same time and getting "heads" on all three?



Choose a strategy! Show your work:



My answer:

$$P(\text{HHH}) = \frac{1}{8}$$

P3-9 Let's Practice!

* SUPER CHALLENGE PROBLEM!

Janice has 2 dimes and 3 nickels in her pocket. What's the probability that she will have 15 cents by picking two coins at random?



Choose a strategy! Show your work:

6 correct choices

10 possibilities w/o duplicates

DUPLICATES

	D ₁	D ₂	N ₁	N ₂	N ₃
D ₁	20	20	15	15	15
D ₂	20	20	15	15	15
N ₁	15	15	10	10	10
N ₂	15	15	10	10	10
N ₃	15	15	10	10	10

My answer:

$$p(15 \text{ cents}) = \frac{6}{10} \text{ or } \frac{3}{5}$$

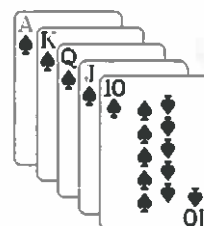
EASY VERSION:

James AND Janice each have 2 dimes, and 3 nickels. What's the probability that, if they randomly pull one coin, the total will be 15 cents?

$$P(15 \text{ CENTS}) = \frac{12}{25}$$

P3-10 Let's Practice!

**SUPER SUPER
CHALLENGE
PROBLEM*



Two playing cards are flipped at random. What's the probability that they both have the same suit (assume that the Joker cards have been

removed)?

Choose a strategy! Show your work:

choose Hearts...

$$P(\text{HEARTS}) = \frac{1}{4}$$

$$P(\text{HEARTS} \text{ -- AGAIN}) = \frac{12}{51}$$

$$P(\text{HEARTS}) \cdot P(\text{HEARTS} \text{ -- AGAIN}) = \frac{1}{4} \cdot \frac{12}{51} = \frac{12}{204} = \frac{1}{17}$$

My answer:

$$P(\text{PICKING SAME SUIT}) = \frac{1}{17}$$

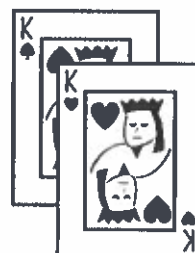
OR! change the problem to "draw one card each from two decks of cards".

$$P(\text{SAME SUIT}) = \frac{1}{16}$$

easier version

P3-11 Let's Practice

**SUPER-ULTRA CHALLENGE!*



What is the probability of selecting two Kings by picking two cards at random?

Choose a strategy! Show your work:

$$P(\text{KING}) = \frac{4}{52} = \frac{1}{13}$$

$$P(\text{KING-AGAIN}) = \frac{3}{51}$$

$$\frac{1}{13} \cdot \frac{3}{51} = \frac{3}{663} = \frac{1}{221}$$

My answer:

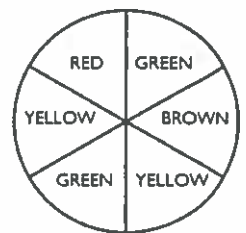
$$P(\text{KING TWICE}) = \frac{1}{221}$$

EASY VERSION: 2 separate decks,
pull one card randomly from
each, find the probability
that they're both kings

$$P(\text{BOTH KINGS}) = \frac{1}{169}$$

P3-12 Let's Practice!

What is the probability of landing on "green"?

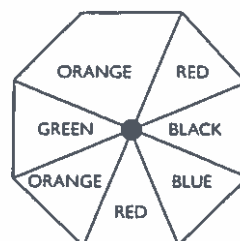


Choose a strategy! Show your work:

My answer: $P(\text{green}) = \frac{2}{6}$ or $\frac{1}{3}$

P3-13 Let's Practice!

What is the probability of landing on "Orange"?



Choose a strategy! Show your work:

My answer:

$$P(\text{orange}) = \frac{3}{8}$$

P3-14 Let's Practice!



What is the probability of landing on an odd number after spinning a spinner with 7 equal sectors numbered 1 through 7?

Choose a strategy! Show your work:

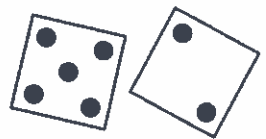
① 2 ③ 4 ⑤ 6 ⑦

My answer:

$$P(\text{odd}) = \frac{4}{7}$$

P3-15 Let's Practice!

Alex is rolling two dice and counting the number of times he gets "doubles". After 100 times, he's rolled "doubles" 19 times, is that close to



What you would expect?

Choose a strategy! Show your work:

	1	2	3	4	5	6
1	✓	X	X	X	X	X
2	X	✓	X	X	X	X
3	X	X	✓	X	X	X
4	X	X	X	✓	X	X
5	X	X	X	X	✓	X
6	X	X	X	X	X	✓

$$P(\text{DOUBLES}) = \frac{6}{36}$$

$$\frac{6}{36} = \frac{x}{100}$$

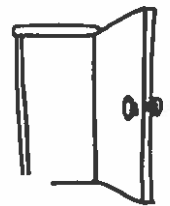
$$x \approx 17$$

My answer:

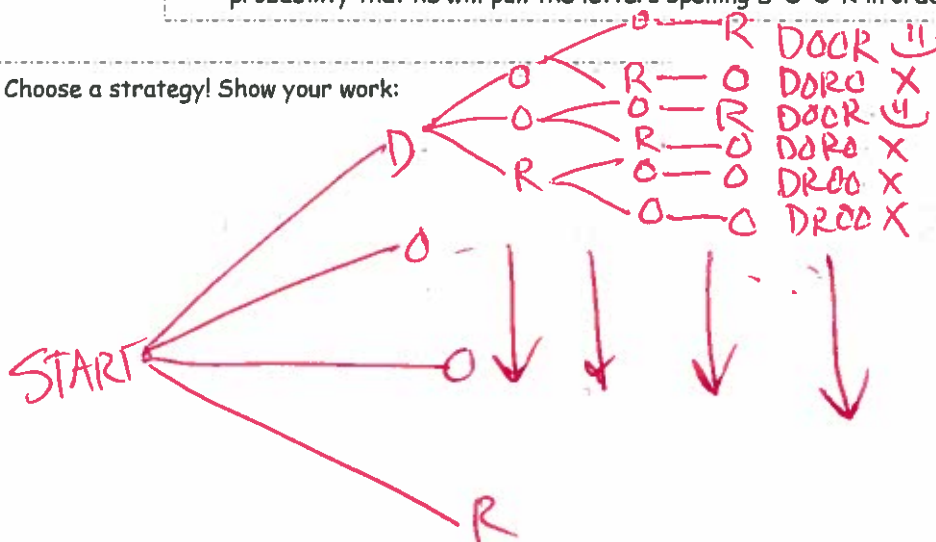
Yes, it would be really close to $\frac{17}{100}$

P3-16 Let's Practice!

James has a bag containing the letters D, O, O, and R. What is the probability that he will pull the letters spelling D-O-O-R in order?



Choose a strategy! Show your work:



My answer:

$$P(D-O-O-R) = \frac{2}{24} \text{ or } \frac{1}{12}$$

P3-17

Let's Practice!

There are 10 counters in a bag: 3 are red, 2 are blue and 5 are green. If all 10 counters are mixed in a bag, what is the probability that Maxine will NOT pick a red counter?



Choose a strategy! Show your work:

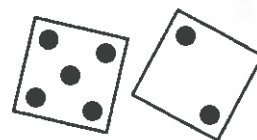
~~X~~ ~~X~~ ~~X~~ B B G G G G G
7
10

My answer:

$$P(\text{not RED}) = \frac{7}{10}$$

P3-18 Let's Practice!

Two dice are thrown together. What is the probability that both numbers are less than five?



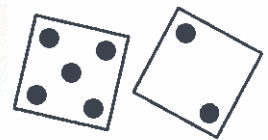
Choose a strategy! Show your work:

	1	2	3	4	5	6
1	✓	✓	✓	✓	X	X
2	✓	✓	✓	✓	X	X
3	✓	✓	✓	✓	X	X
4	✓	✓	✓	✓	X	X
5	X	X	X	X	X	X
6	X	X	X	X	X	X

My answer: $P(\text{Both} < 5) = \frac{16}{36}$ or $\frac{4}{9}$

P3-19 Let's Practice!

Two dice are rolled at the same time. What is the probability that their product is a multiple of 4?



Choose a strategy! Show your work:

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

My answer: $P(\text{multiple of 4}) = \frac{15}{36}$ or $\frac{5}{12}$

P3-20 Let's Practice!



If $P(\text{red}) = \frac{1}{4}$, $P(\text{green}) = \frac{1}{5}$, $P(\text{blue}) = \frac{1}{2}$, find $P(\text{purple})$ if there are 20 total marbles in a bag.

Choose a strategy! Show your work:

$$\begin{aligned} P(\text{RED}) &= \frac{1}{4} \Rightarrow \frac{5}{20} \\ P(\text{GREEN}) &= \frac{1}{5} \Rightarrow \frac{4}{20} \\ P(\text{BLUE}) &= \frac{1}{2} \Rightarrow \frac{10}{20} \\ &\quad \underline{\quad\quad\quad} \\ &\quad\quad 19 \\ &\quad\quad \underline{\quad\quad} \\ &\quad\quad 20 \end{aligned}$$
$$\therefore P(\text{PURPLE}) = \frac{1}{20}$$

My answer:

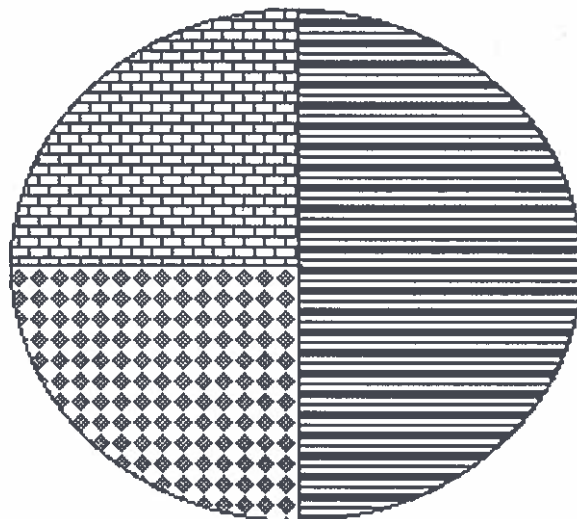
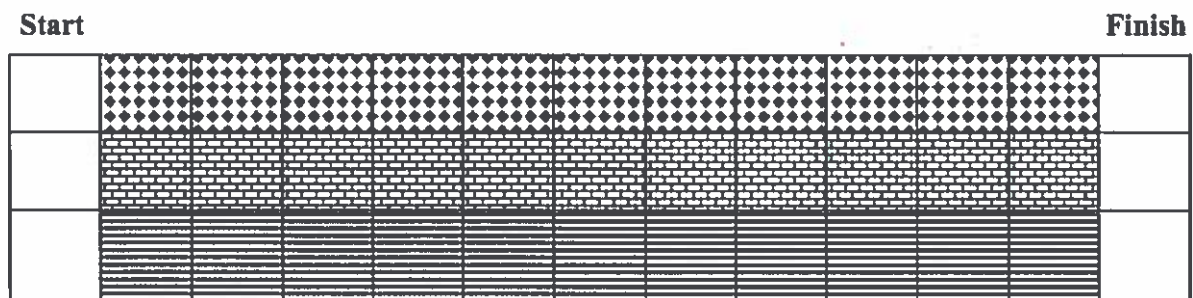
$$P(\text{PURPLE}) = \frac{1}{20}$$

Problem of the Month: "Fair Games"

Level A: The Race

Rules:

There are three players: Brick, Stripe, and Diamond. Each player puts a token on the Start square of their pattern path. The players take turns by spinning the spinner. The player whose pattern comes up on the spinner moves their token one space on the game board. The other players do not move their tokens. The game continues as each player takes turns with the spinner and a move is made, until one player reaches the Finish Line. That player wins.



Problem of the Month: "Fair Games"

Level A (continued): The Race

What is a fair game? Explain.

In groups of three, play The Race game five times. Keep track of who won each game, who came in second and who came in third.

Which player won the most?

How many times?

Which player came in second the most?

Is this a fair game? Explain why this game is fair or not fair.

How could you change this game to make it better?

Problem of the Month: "Fair Games"

Level B: The Race

Play The Race game five times. This time keep track of each spin, listing who moved each time.

Draw a bar graph showing the outcome of the spins. Label the horizontal axis by pattern and the vertical axis by the number of spins that occurred.

How many more times did Stripe move than Brick?

Between Brick and Diamond, which player moved more times in all 10 games?

How much of the spinner is a stripe pattern?

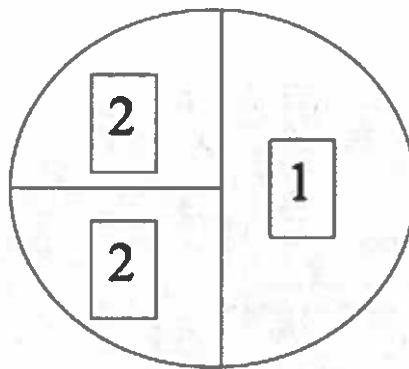
How much of the spinner is a diamond pattern?

How much of the spinner is a brick pattern? How could you change the spinner to make the game more fair?

Problem of the Month: "Fair Games"

Level C: Alex's Version

Your friend Alex says that he can make the game more fair. He makes a second spinner with numbers on it. He says the number stand for the number of spaces a token is moved. He modified the rules as follows: First the pattern spinner is spun to find out who moves. Then the number spinner is spun to find out how many spaces the token is moved. Below is the spinner he made.



What is the probability that Diamond comes up on the first spinner? Explain.

What is the probability that 1 comes up on the second spinner? Explain.

What is the probability that Stripe moves 1 space on any turn? Explain.

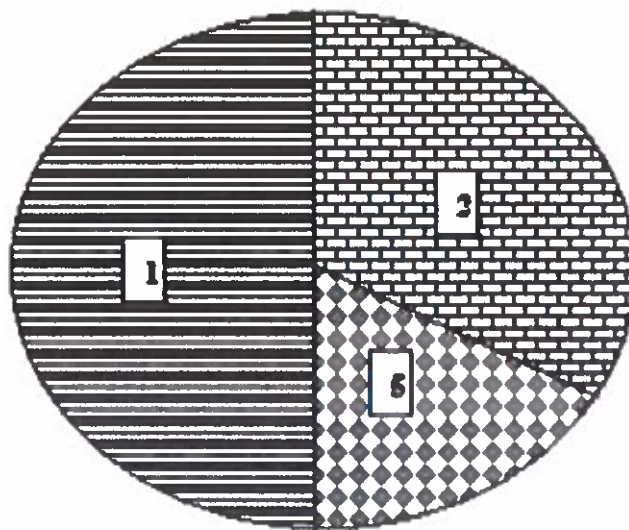
What is the probability that Brick moves 2 spaces on any turn? Explain.

Alex says that the new spinner makes the game fair because Stripe will mostly move 1 space and the Brick and Diamond players will mostly move 2 spaces. Explain why you either agree or disagree with Alex.

Problem of the Month: "Fair Games"

Level D: Dolores' Version

Another friend, Dolores, says that she made the game fair by drawing just one new spinner, but using both patterns and numbers on the same spinner. When the spinner is spun, the player with that pattern moves the number of spaces indicated in that sector of the spinner. She said she made the spinner by first drawing the diameter and then making the central angle of the brick sector 120 degrees.



What is the probability of Brick moving on any spin?

How many times would Diamond have to move to win a game?

How much bigger in area is the Stripe sector than the Diamond sector?

Problem of the Month: "Fair Games"

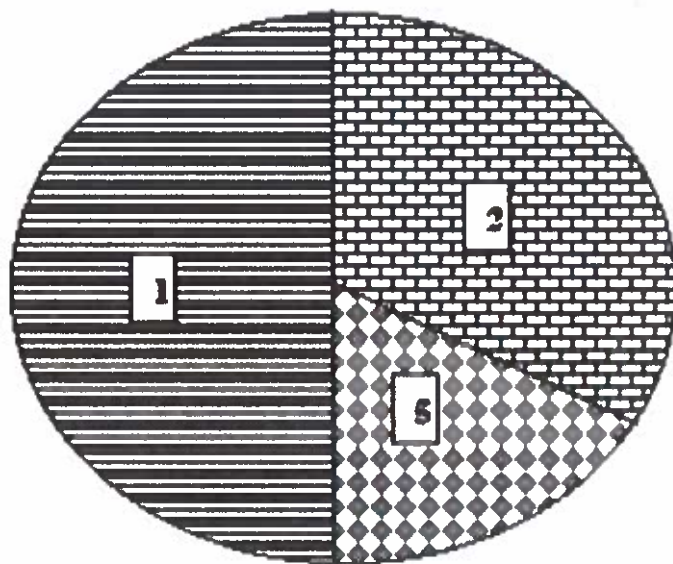
Level D (continued): Dolores' Version

Dolores' spinner makes the game unfair. Use mathematics to explain why her game is unfair.

How could you change the spinner to make her game fair?

Problem of the Month: "Fair Games"

Level E: Last-Minute Changes



Dolores has already made several copies of her spinner. You don't want to hurt her feelings by not using her spinner, but you need to make sure the game is fair. You decide to make a new track in the shape of an oval where racers near the inside of the track have fewer spaces to travel than racers near the outside of the track. Design an oval track that can be used with Dolores' spinner. Design the game board track so that the game will be fair to all players, but Dolores' spinner isn't changed.

Use mathematics to justify why your game board makes the game fair to all players.