

AP Calculus BC

WS 78 - Sequences & Series Review 2

$$1) \sum_{k=0}^{\infty} \frac{3}{10^{k+1}} = \sum_{k=0}^{\infty} 3 \left(\frac{1}{10}\right)^{k+1}$$

$$S = \frac{\frac{3}{10}}{1 - \frac{1}{10}} = \frac{\frac{3}{10}}{\frac{9}{10}} = \frac{1}{3} \quad \text{(B)}$$

$$3) 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$$

$$a_n = \frac{(-1)^{n+1}}{2n-1} \quad \text{(E)}$$

$$5) \int_1^{\infty} \frac{e^{1/x}}{x^2} dx$$

$$\lim_{b \rightarrow \infty} \int_1^b \frac{e^{1/x}}{x^2} dx = \lim_{b \rightarrow \infty} [-e^{1/x}]_1^b$$

$$\lim_{b \rightarrow \infty} [-e^{1/b} + e] = \boxed{e-1} \quad \text{(B)}$$

- 7) (i)  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges by harmonic
- (ii)  $\sum_{n=1}^{\infty} \frac{1}{e^n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n = S$   
 $r = \frac{1}{e}$ ,  $S$  converges by GST
- (iii)  $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}} = S$   
 $p = \frac{1}{2} < 1$ ,  $S$  diverges by p-series
- (iv)  $\sum_{n=1}^{\infty} n = S$   $\lim_{n \rightarrow \infty} n = \infty$   
 $S$  diverges by  $n^{\text{th}}$  term test

$$2) a_1 = 1; a_2 = 1; a_n = a_{n-1} + a_{n-2}$$

$$a_3 = 2; a_4 = 3; a_5 = 5; a_6 = 8$$

(A)

$$4) \sum_{n=1}^{\infty} \frac{1}{(\sqrt{5}-1)^n} = \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{5}-1}\right)^n = S$$

$$r = \frac{1}{\sqrt{5}-1} < 1, S \text{ converges by GST}$$

- 6) (i)  $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = S$   $r = \frac{1}{2} < 1$ ,  $S$  converges by GST
- (ii)  $\sum_{n=1}^{\infty} \frac{1}{n}$  diverges by harmonic series
- (iii)  $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$   $p = \frac{3}{2} > 1$ , converges by p-series
- (iv)  $\sum_{n=1}^{\infty} \frac{1}{n^2}$   $p = 2 > 1$ , converges by p-series
- (A)

$$8) \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^{n-1}$$

$$S = \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$9) \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^{n-1}} = \sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^{n-1}$$

$$S = \frac{1}{1 + \frac{1}{3}} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$