

AP Calculus BC

$$1) \sum_{k=1}^{\infty} \frac{1}{17^k} = S$$

$$\text{Geometric: } a_n = \left(\frac{1}{17}\right)^k$$

$$S_1 = \frac{1}{17}$$

$$r = \frac{1}{17} < 1$$

$$S_2 = \frac{1}{17} + \frac{1}{289}$$

S converges

$$S_3 = \frac{1}{17} + \frac{1}{289} + \frac{1}{4913}$$

$$S = \frac{\frac{1}{17}}{1 - \frac{1}{17}} = \frac{\frac{1}{17}}{\frac{16}{17}} = \frac{1}{16}$$

$$3) \sum_{n=0}^{\infty} \frac{5^n}{9^{n+1}} = \sum_{n=0}^{\infty} \frac{1}{9} \cdot \left(\frac{5}{9}\right)^n = S$$

$$r = \frac{5}{9} < 1, S \text{ converges by GST}$$

$$S = \frac{a_0}{1 - r} = \frac{\frac{1}{9}}{1 - \frac{5}{9}} = \frac{\frac{1}{9}}{\frac{4}{9}} = \frac{1}{4}$$

$$5) \sum_{k=0}^{\infty} \frac{3}{900 + 3k} = S$$

$$\lim_{k \rightarrow \infty} \frac{3}{900 + 3k} = 0$$

n^{th} term test is inconclusive

$$7) \sum_{k=3}^{\infty} \frac{1}{(k-2)^4} = \sum_{k=1}^{\infty} \frac{1}{k^4} = S$$

$p = 4 > 1$, S converges by p-series

WS 75 - p-series & Integral Test

$$2) 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots + \left(\frac{1}{2}\right)^n$$

$$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n = S$$

$$r = \frac{1}{2} < 1, S \text{ converges by GST}$$

$$S = \frac{1}{1 - \frac{1}{2}} = 2$$

$$4) \sum_{n=0}^{\infty} \frac{n}{5n+1} = S$$

$$\lim_{n \rightarrow \infty} \frac{n}{5n+1} = \frac{1}{5} \neq 0$$

S diverges by n^{th} term test

$$6) \sum_{k=2}^{\infty} \frac{1}{(k-1)^3}$$

$$\sum_{k=1}^{\infty} \frac{1}{k^3} = S, p = 3 > 1, S \text{ converges by p-series test}$$

$$8) \sum_{n=1}^{\infty} \frac{12}{n^{1/2}} = S$$

$p = \frac{1}{2} < 1$, S diverges by p-series test

$$9) \sum_{k=2}^{\infty} \frac{1}{6e^k} = S$$

$$f(x) = \frac{1}{6e^x}$$

$f(x)$ is dec, cont, & positive.

$$\int_2^{\infty} \frac{1}{6} e^{-x} dx$$

$$\frac{1}{6} \cdot \lim_{b \rightarrow \infty} \int_2^b e^{-x} dx$$

$$\frac{1}{6} \left[\lim_{b \rightarrow \infty} [-e^{-x}]_2^b \right]$$

$$\frac{1}{6} \left[\lim_{b \rightarrow \infty} \left[-\frac{1}{b} + \frac{1}{e^2} \right] \right] = \frac{1}{6e^2}$$

S converges by Integral Test

$$10) \sum_{k=0}^{\infty} \frac{6}{\sqrt{k+3}} = S$$

$$f(x) = \frac{6}{\sqrt{x+3}}$$

$f(x)$ is cont, dec, pos.

$$\int_0^{\infty} 6(x+3)^{-1/2} dx$$

$$6 \cdot \lim_{b \rightarrow \infty} \int_0^b (x+3)^{-1/2} dx$$

$$6 \cdot \lim_{b \rightarrow \infty} \left[2(x+3)^{1/2} \right]_0^b$$

$$6 \cdot \lim_{b \rightarrow \infty} \left[2(b+3)^{1/2} - 2(3)^{1/2} \right] = \infty$$

S diverges by Integral Test