

# AP Calculus BC

## Unit 9 - Sequences and Series

1	Find the first four terms and the 50 <sup>th</sup> term for the sequence: $a_n = \frac{n}{n+7}$
2	Find the first six terms and the 50 <sup>th</sup> term for the sequence: $d_n = n^2 - 2n$ .
3	Find the first four terms and the 8 <sup>th</sup> term for the sequence: $a_1 = 1$ ; $a_n = 2a_{n-1}$ for all $n \geq 2$ .
4	Find the first four terms and the 8 <sup>th</sup> term for the sequence: $u_1 = 1$ ; $u_2 = 2$ ; $u_n = u_{n-1} + u_{n-2}$ for all $n \geq 3$ .
5	Determine the convergence or divergence of $a_n = \frac{2n+1}{n}$ . If the sequence converges, find its limit.
6	Determine the convergence or divergence of $a_n = (-1)^n \frac{n-1}{n+1}$ . If the sequence converges, find its limit.
7	Determine the convergence or divergence of $a_n = 5 + (0.9)^n$ . If the sequence converges, find its limit.
8	Determine the convergence or divergence of $a_n = n \sin\left(\frac{7}{n}\right)$ . If the sequence converges, find its limit.
9	Find the limit of the sequence $\left\{ \frac{3n^4}{n^4 + 1} \right\}$ or state that it does not exist.
10	Find the limit of the sequence $\left\{ \frac{5e^n + 1}{e^n} \right\}$ or state that it does not exist.

Evaluate each limit or state that it does not exist.

1) $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^{11x}$	2) $\lim_{x \rightarrow 1} \frac{\int_1^x \cos(5t) dt}{x^2 - 1}$	3) $\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^n$
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Find the limit of each sequence or determine that the limit does not exist.

4) $a_n = \sqrt{\left(1 - \frac{1}{10n}\right)^n}$	5) $a_n = \left(1 + \frac{9}{n}\right)^{8n}$	6) $a_n = \frac{n}{e^n + 11n}$
7) $a_n = \left(\frac{10}{n}\right)^{10/n}$	8) $a_n = \ln(n^3 + 5) - \ln(7n^3 + 11n)$	9) $a_n = \left(n \sin \frac{17}{n}\right)$
10) $a_n = \frac{(-1)^n n}{2n + 3}$	11) $a_n = \cos\left(\frac{n\pi}{12}\right)$	12) $a_n = \frac{(-1)^{n+1}}{4n + 1}$
13) $a_n = \frac{3^n}{3^n + 5^n}$	14) $a_n = 2 + \cos \frac{5}{n}$	15) $a_n = \frac{\ln n}{n^{1.6}}$
16) $a_n = \frac{8^n + 5^n}{8^n + n^{110}}$	17) $a_n = \frac{9^n}{n^9 7^n}$	

Answers

1) $e^{-11}$	2) $\frac{\cos 5}{2}$	3) $e^2$	4) $e^{-1/20}$	5) $e^{72}$	6) 0
7) 1	8) $-\ln 7$	9) 17	10) DNE	11) DNE	12) 0
13) 0	14) 3	15) 0	16) 1	17) DNE	



Determine if each series converges or diverges. If it converges, find the sum.

1) $\frac{1}{17} + \frac{1}{289} + \frac{1}{4913} + \dots + \frac{1}{17^k} + \dots$	2) $1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots + \left(\frac{1}{2}\right)^n + \dots$	3) $\sum_{n=0}^{\infty} \frac{5^n}{9^{n+1}}$
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Use the divergence test ( $n^{\text{th}}$  term test) to determine whether each series diverges or state that the test is inconclusive.

4) $\sum_{n=0}^{\infty} \frac{n}{5n+1}$	5) $\sum_{k=0}^{\infty} \frac{3}{900+3k}$
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Determine the convergence or divergence of each series.

6) $\sum_{k=2}^{\infty} \frac{1}{(k-1)^3}$	7) $\sum_{k=3}^{\infty} \frac{1}{(k-2)^4}$	8) $\sum_{n=1}^{\infty} \frac{12}{n^{1/2}}$
9) $\sum_{k=2}^{\infty} \frac{1}{6e^k}$	10) $\sum_{k=0}^{\infty} \frac{6}{\sqrt{k+3}}$	

### Answers

1) Converges; $\frac{1}{16}$	2) Converges; 2	3) Converges; $\frac{1}{4}$	4) Diverges by nth term test	5) Inconclusive
6) Converges by p-series test	7) Converges by p-series test	8) Diverges by p-series test	9) Converges by Integral test	10) Diverges by integral test

Write out the first four terms of the sequence of partial sums for each geometric series. Then find the sum of the infinite series.

1. $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^n}$	2. $\sum_{n=1}^{\infty} \frac{7}{4^n}$	3. $\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$	4. $\sum_{n=0}^{\infty} \left( \frac{2^{n+1}}{5^n} \right)$
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Determine whether each series converges or diverges. Give a reason for your answer. If the series converges, find its sum, if possible.

5. $\sum_{n=0}^{\infty} \left( \frac{1}{\sqrt{2}} \right)^n$	6. $\sum_{n=0}^{\infty} \cos(n\pi)$	7. $\sum_{n=1}^{\infty} \ln \frac{1}{n}$	8. $\sum_{n=0}^{\infty} \frac{n!}{1000^n}$
9. $\sum_{n=0}^{\infty} \left( \frac{e}{\pi} \right)^n$	10. $\sum_{n=1}^{\infty} \left( \frac{1}{10} \right)^n$	11. $\sum_{n=1}^{\infty} \frac{n}{n+1}$	12. $\sum_{n=2}^{\infty} \frac{5}{n+1}$
13. $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$	14. $\sum_{n=1}^{\infty} \frac{-2}{n\sqrt{n}}$	15. $\sum_{n=1}^{\infty} \left( -\frac{1}{8^n} \right)$	16. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$
17. $\sum_{n=1}^{\infty} \frac{2^n}{3^n}$	18. $\sum_{n=0}^{\infty} \frac{-2}{n+1}$	19. $\sum_{n=1}^{\infty} \left( 1 + \frac{1}{n} \right)^n$	20. $\sum_{n=1}^{\infty} \frac{e^n}{1 + e^{2n}}$

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1.  $\sum_{k=0}^{\infty} \frac{2}{6^{k+1}} =$  (A)  $\frac{1}{3}$  (B)  $\frac{12}{5}$  (C)  $\frac{2}{5}$  (D)  $\frac{3}{5}$  (E) none of these

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2. The first six terms in the sequence defined recursively by  $a_1 = 1$ ,  $a_2 = 3$ ,  $a_n = 2a_{n-1} + 3$ , for  $n \geq 3$ , are  
 (A)  $\{1,3,6,9,12,15\}$  (B)  $\{1,3,5,7,9,11\}$  (C)  $\{1,3,5,8,10,13\}$  (D)  $\{1,3,9,21,45,93\}$   
 (E) None of these
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3. The formula for the  $n$ -th term of the series  $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \dots$  is

(A)  $\frac{-1}{2n+1}$  (B)  $\frac{1}{2n-1}$  (C)  $\frac{(-1)^{n+1}}{n+2}$  (D)  $\frac{(-1)^{n+1}}{4n-1}$  (E)  $\frac{(-1)^{n+1}}{2n-1}$

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4. Does the following converge or diverge?  $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{3}-1)^n}$
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5. The integral test confirms that the series  $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$  converges. What is  $\int_1^{\infty} \frac{e^{1/x}}{x^2} dx$  ?

(A)  $e+1$  (B)  $e-1$  (C)  $\sqrt{e}-1$  (D)  $e^2$  (E)  $e^{1/4}-1$

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6. **How many** of the series shown diverge? (i)  $\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^n$  (ii)  $\sum_{n=1}^{\infty} \frac{2}{n}$  (iii)  $\sum_{n=1}^{\infty} \frac{1}{n^{5/2}}$  (iv)  $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

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7. **How many** of the series shown converge? (i)  $\sum_{n=1}^{\infty} \frac{1}{n}$  (ii)  $\sum_{n=1}^{\infty} \frac{1}{e^n}$  (iii)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  (iv)  $\sum_{n=1}^{\infty} n$

(A) 1 (B) 2 (C) 3 (D) 4 (E) none of them converge

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8. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{3^{(n-1)}}$ .
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9. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2^{n-1}}$ .
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**Answers: 1. C 2. D 3. B 4. diverge 5. B 6. B 7. A 8. 3/2 9. 2/3**

1.  $\sum_{k=0}^{\infty} \frac{3}{10^{k+1}} =$  (A)  $\frac{3}{10}$  (B)  $\frac{1}{3}$  (C)  $\frac{3}{7}$  (D)  $\frac{20}{33}$  (E)  $\frac{6}{11}$

2. The first six terms in the sequence defined recursively by  $a_1 = 1, a_2 = 1, a_n = a_{n-1} + a_{n-2}$  are  
 (A)  $\{1,1,2,3,5,8\}$  (B)  $\{1,1,2,4,6,8\}$  (C)  $\{1,1,4,5,9,14\}$  (D)  $\{1,1,0,1,0,1\}$   
 (E) None of these

3. The formula for the n-th term of the series  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \dots$  is

(A)  $\frac{1}{2n+1}$  (B)  $\frac{(-1)^n}{2n-1}$  (C)  $\frac{(-1)^{n+1}}{n+2}$  (D)  $\frac{(-1)^{n+1}}{4n-1}$  (E)  $\frac{(-1)^{n+1}}{2n-1}$

4. Does the following converge or diverge?  $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{5}-1)^n}$

5. The integral test confirms that the series  $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$  converges. What is  $\int_1^{\infty} \frac{e^{1/x}}{x^2} dx$  ?

(A)  $e+1$  (B)  $e-1$  (C)  $\sqrt{e}-1$  (D)  $e^2$  (E)  $e^{1/4}-1$

6. **How many** of the series shown diverge? (i)  $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$  (ii)  $\sum_{n=1}^{\infty} \frac{1}{n}$  (iii)  $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$  (iv)  $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

7. **How many** of the series shown converge? (i)  $\sum_{n=1}^{\infty} \frac{1}{n}$  (ii)  $\sum_{n=1}^{\infty} \frac{1}{e^n}$  (iii)  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  (iv)  $\sum_{n=1}^{\infty} n$

(A) 1 (B) 2 (C) 3 (D) 4 (E) none of them converge

8. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{2^{(n-1)}}$  .

9. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^{n-1}}$  .

**Answers: 1. B 2. A 3. E 4. converge 5. B 6. A 7. A 8. 2 9. 3/4**