Unit 9 Outline – Sequences and Series

Friday 1/26	**Today's Topic:** Introduction to Infinite Sequences and their limits; Investigate recursive sequences

Key Idea:

Let $\{a_n\}$ be a sequence of real numbers.

Possibilities:

1) If $\lim_{n\to\infty} a_n = \infty$, then $\{a_n\}$ diverges to infinity
2) If $\lim_{n\to\infty} a_n = -\infty$, then $\{a_n\}$ diverges to negative infinity
3) If $\lim_{n\to\infty} a_n = c$, an finite real number, then $\{a_n\}$ converges to c
4) If $\lim_{n\to\infty} a_n$ oscillates between two fixed numbers, then $\{a_n\}$ diverges by oscillation

In-Class Examples: Write out the first 4 terms of the following sequences and, if they converge, find the limit:

1. $a_n = \frac{1}{n}$ 2. $a_n = \frac{n+1}{2n+1}$ 3. $a_n = \frac{n-1}{n}$ 4. $a_n = 3$ 5. $a_n = (-1)^{n+1}\frac{1}{n}$ 6. $a_n = (-1)^{n+1}\left(\frac{n-1}{n}\right)$

Write the first 6 terms of the sequences:

7. $x_1 = 1;\ x_n = x_{n-1} + (2n+1)$ 8. $a_1 = 1;\ a_n = a_{n-1} + \left(\frac{1}{2}\right)n$

Homework: Worksheet 72

Monday 1/29	**Today's Topic:** Find the convergence or divergence of an infinite sequence; To find the limit of a convergent sequence

In-Class Examples: If $a_n \to L$ as $n \to \infty$, then $f(a_n) \to f(L)$

Write out the first 4 terms of the following sequences and, if they converge, find the limit:

1. $a_n = \frac{-1}{n}$ 2. $a_n = \frac{4-7n^6}{n^6+3}$ 3. $a_n = \frac{n^3+5n}{n^4-6}$ 4. $a_n = \frac{n^2-5}{n+1}$ 5. $a_n = \frac{5n}{2^n}$ 6. $a_n = \frac{n!}{(n+2)!}$

Common Limits

1. $\lim_{n\to\infty} \frac{\ln n}{n} = 0$
2. $\lim_{n\to\infty} \sqrt[n]{n} = 1$
3. $\lim_{n\to\infty} x^{1/n} = 1 \quad (x > 0)$
4. $\lim_{n\to\infty} x^n = 0 \quad (|x| < 1)$
5. $\lim_{n\to\infty} \left(1 + \frac{x}{n}\right)^n = e^x \quad (\text{Any } x)$
6. $\lim_{n\to\infty} \frac{x^n}{n!} = 0 \quad (\text{Any } x)$

In formulas (3)–(6), x remains fixed as $n \to \infty$.

Homework: Worksheet 73

Tuesday 1/30 (Block)	Today's Topic: An introduction to infinite series

In-Class Examples:

1. Given the series $\sum_{n=1}^{\infty} \frac{3}{2^n}$, write the first 6 terms of the **sequence of partial sums**. Do you think this series converges or diverges?

2. Given the series $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$, write the first 6 terms of the **sequence of partial sums**. Do you think this series converges or diverges?

3. Given the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$, write the first 6 terms of the **sequence of partial sums**. Do you think this series converges or diverges?

Homework: None

Tuesday 1/30 (Block)	Today's Topic: Compute the sum of an infinite geometric series; Use the Divergence (n^{th} term) Test to determine if a series diverges.

Key Idea: A series converges if its sequence of partial sums converges.

Geometric Series Test (GST)

A geometric series is in the form $\sum_{n=0}^{\infty} a \cdot r^n$ or $\sum_{n=1}^{\infty} a \cdot r^{n-1}$, $a \neq 0$

The geometric series **diverge**s if $|r| \geq 1$.

If $|r| < 1$, the series **converge**s to the sum $S = \frac{a_1}{1-r}$.

Where a_1 is the first term, regardless of where n starts, and r is the common ration.

***n*th Term Test for Divergence (ONLY)**

If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.

(think about it, it should make perfect sense!)

In-Class Examples:

1. Use the Geometric Series Test, to determine whether the following series converge or diverge. If they converge, find the sum.

a) $\sum_{n=1}^{\infty} \frac{1}{2^{n-1}}$ b) $\sum_{n=1}^{\infty} \frac{3}{2^n}$ c) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$ d) $\sum_{n=1}^{\infty} 3\left(-\frac{1}{2}\right)^n$

1. Determine if the following series converge or diverge. If they converge, find the sum, if possible.

a) $\sum_{n=1}^{\infty} \frac{1}{2^n}$ b) $\sum_{n=1}^{\infty} \frac{n+1}{n}$ c) $\sum_{n=1}^{\infty} \frac{5(-1)^n}{4^n}$ d) $\sum_{n=1}^{\infty} \frac{n}{2n+5}$

e) $\sum_{n=1}^{\infty} \frac{1}{n}$ f) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ g) $\sum_{n=1}^{\infty} n^2$ h) $\sum_{n=1}^{\infty} (-1)^{n+1}$

Homework: Worksheet 74

Thursday 2/1	**Today's Topic:** Learn about the *p*-series and its convergence and divergence.

Key Idea:

***p*-series**

A series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots$ is called a *p*-series, where *p* is a positive constant.

For $p = 1$, the series $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} + \cdots$ is called the **harmonic series**.

***p*-series Test:** $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$, diverges if $p \le 1$

In-Class Examples: 1. $\sum_{n=1}^{\infty} \frac{1}{n^2}$ 2. $\sum_{n=1}^{\infty} \frac{1}{n}$ (harmonic series)

3. Determine if the following series converge or diverge.

a) $\sum_{n=1}^{\infty} n^3$ b) $\sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n$ c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ d) $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}}\right)^3$ e) $\sum_{n=1}^{\infty} \frac{3n^2}{2n^2 + 7}$

Homework: None

Friday 2/2	**Today's Topic:** Use the Integral Test for convergence of an infinite series;

Integral Test

If *f* is **D**ecreasing, **C**ontinuous, and **P**ositive for $x \ge 1$ AND $a_n = f(x)$, then $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\,dx$ either BOTH converge or diverge.

Note 1: This does not mean that the series converges to the value of the definite integral.
Note 2: The function need only be decreasing for all $x > k$ for some $k \ge 1$.

If the series converges to *S*, then the remainder, $R_n = |S - S_n|$ is bounded by $0 \le R_n \le \int_n^{\infty} f(x)\,dx$. This means that $S_n \le S \le S_n + R_n$.

In-Class Examples: Determine if each series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$ 2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$ 3. $\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ 4. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$

Homework: Worksheet 75

Monday 2/5	**Today's Topic:** Review for Test
In-Class Examples: None	
Homework: Worksheet 76	

Tuesday 2/6	**Today's Topic:** Review for Test
In-Class Examples: None	
Homework: Worksheet 77	

Wednesday 2/7	**Today's Topic:** Review for Test
In-Class Examples: None	
Homework: Worksheet 78	

Thursday 2/8	**Today's Topic:** Our first test on Sequences and Series
In-Class Examples: None	
Homework: Exam	