AP Calculus BC

Parametric & Polar Equations

1) $x(t) = t^2 - 2 \quad y(t) = \frac{2}{3}t^3$

a) $x'(t) = 2t \quad y'(t) = 2t^2$

$||v(2)|| = \sqrt{16 + 64}$
$= \sqrt{80}$

b) $\int_0^4 \sqrt{(2t)^2 + (2t^2)^2}\, dt$

c) $\frac{dy}{dx} = \frac{2t^2}{2t} = t$

d) on y-axis → $x(t) = 0$
$t^2 - 2 = 0$
$t = \sqrt{2}$
$a(t) = \langle 2, 4t \rangle$

$a(\sqrt{2}) = \langle 2, 4\sqrt{2} \rangle$

2) $s(t) = \langle 2t + 3\sin t,\ t^2 + 2\cos t \rangle$

$t^2 + 2\cos t = 7$ ← FUNCTION MODE

$t = 2.996 \to A$

$v(A) = \langle -0.968, 5.7038 \rangle$

3) $v(t) = \langle \cos(e^t), \sin(e^t) \rangle$

a) point: $(3, 2)$
$\left.\frac{dy}{dx}\right|_{t=1} = -0.4505$

$y - 2 = -0.4505(x - 3)$

b) $||v(1)|| = 1$

c) $\int_0^2 \sqrt{(\cos(e^t))^2 + (\sin(e^t))^2}\, dt$

$= 2$

d) $x(2) = 3 + \int_1^2 \cos(e^t)\, dt = 2.8957$

$y(2) = 2 + \int_1^2 \sin(e^t)\, dt = 1.6759$

$s(2) = (2.8957, 1.6759)$

4) $r = \frac{\theta}{2} \quad \theta = k$

$\text{Area} = \frac{1}{2}\int_0^k \left(\frac{\theta}{2}\right)^2 d\theta$

$= \frac{1}{2}\int_0^k \frac{1}{4}\theta^2 d\theta$

$= \frac{1}{24}\theta^3 \Big|_0^k$

$= \boxed{\frac{k^3}{24}}$

5) $r = 2\cos 4\theta \qquad r' = -8\sin 4\theta$

$x = 2\cos 4\theta \cos\theta \qquad\qquad y = 2\cos 4\theta \sin\theta$

$\frac{dx}{d\theta} = -2\cos 4\theta \sin\theta - 8\sin 4\theta \cos\theta$

$\frac{dy}{d\theta} = 2\cos 4\theta \cos\theta - 8\sin 4\theta \sin\theta$

$$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = \frac{2(-1)(\frac{\sqrt{2}}{2}) - 8(0)(\frac{\sqrt{2}}{2})}{-2(-1)(\frac{\sqrt{2}}{2}) - 8(0)\frac{\sqrt{2}}{2}} = -1$$

6) $\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta \qquad \frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$

$$\frac{dy}{dx} = \frac{\sin\theta + \theta\cos\theta}{\cos\theta - \theta\sin\theta}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=6} = 2.073$$

7) a) $2\theta\cos\theta = 0$

$2\theta = 0 \qquad \cos\theta = 0$

$\theta = 0 \qquad \theta = \frac{\pi}{2}$

$\text{Area} = \frac{1}{2}\int_0^{\pi/2} (2\theta\cos\theta)^2 d\theta = 0.5065$

b) $\left.\frac{dr}{d\theta}\right|_{\theta=\frac{\pi}{2}} = -3.14159 < 0$

r is decreasing

c) $\frac{dr}{d\theta} = 2\cos\theta - 2\theta\sin\theta = 0$

$\theta = 0.860 \qquad \theta = 3.4256$

θ	r
0	0
0.860	1.1222
3.4256	-6.577
$3\pi/2$	0

The greatest distance from the origin is 6.577

d) $\frac{dy}{dx} = \frac{2}{2-\pi}$

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\frac{1}{2}}{\frac{dx}{d\theta}} = \frac{2}{2-\pi}$$

$2\frac{dx}{d\theta} = \frac{1}{2}(2-\pi)$

$$\boxed{\frac{dx}{d\theta} = \frac{1}{4}(2-\pi)}$$