AP Calculus BC

Vector Calculus 1

1) $\vec{s} = \langle \ln(t^2+5t), 3t^2 \rangle$

$\vec{V} = \left\langle \frac{2t+5}{t^2+5t}, 6t \right\rangle$

$\vec{V}(2) = \left\langle \frac{9}{14}, 12 \right\rangle$

2) $x = t^5 - 1 \quad y = 3t^4 - 2t^3$

$\vec{V} = \langle 5t^4, 12t^3 - 6t^2 \rangle$

$\vec{a} = \langle 20t^3, 36t^2 - 12t \rangle$

$\vec{a}(1) = \langle 20, 24 \rangle$

3) $\vec{s} = \langle \sin(3t - \frac{\pi}{2}), 3t^2 \rangle$

$\vec{V} = \langle 3\cos(3t - \frac{\pi}{2}), 6t \rangle$

$\vec{V}(\frac{\pi}{2}) = \langle -3, 3\pi \rangle$

4) $x'(t) = t+1 \quad y = \ln x$

$x(t) = \frac{1}{2}t^2 + t + C$

$1 = 0 + 0 + C \quad y = \ln 5/2$

$1 = C$

$x(t) = \frac{1}{2}t^2 + t + 1 \quad \vec{s}(1) = \langle \frac{5}{2}, \ln \frac{5}{2} \rangle$

$x(1) = \frac{1}{2} + 1 + 1$

$= 5/2$

5) $\vec{V} = \langle 1+t, t^3 \rangle$

$\vec{s}(0) = \langle 5, 0 \rangle$

$x(t) = t + \frac{1}{2}t^2 + C \quad y(t) = \frac{1}{4}t^4 + C$

$5 = 0 + 0 + C \quad 0 = 0 + C$

$x(t) = t + \frac{1}{2}t^2 + 5 \quad y(t) = \frac{1}{4}t^4$

$\vec{s}(2) = \langle 9, 4 \rangle$

6) $x = t^3 - \frac{3}{2}t^2 - 18t + 5$

At rest: $\frac{dx}{dt} = \frac{dy}{dt} = 0$

$\frac{dx}{dt} = 3t^2 - 3t - 18 = 0$

$= t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$t = 3 \quad t = -2$

$\boxed{t = 3}$

$y = t^3 - 6t^2 + 9t$

$\frac{dy}{dt} = 3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t=1)$

$t = 3$

7) $x(t) = 5t + 3\sin t \quad y(t) = (8-t)(1-\cos t)$

$x(t) = 25$ @ $t = 5.445 \to A$

$x'(A) = 7.008 \quad \vec{v}(A) = \langle 7.008, -2.228 \rangle$

$y'(A) = -2.228$

8) $x(t) = t^2 - 3 \quad y(t) = \frac{2}{3}t^3$

a) $x'(t) = 2t \quad y'(t) = 2t^2$

$x'(5) = 10 \quad y'(5) = 50$

$\|\vec{v}(5)\| = \sqrt{100 + 2500}$

$= \sqrt{2600}$

$= 10\sqrt{26}$

b) $D = \int_0^5 \sqrt{(2t)^2 + (2t^2)^2}\, dt$

$= \int_0^5 \sqrt{4t^2 + 4t^4}\, dt$

$= \int_0^5 \sqrt{4t^2(1+t^2)}\, dt$

$= \int_0^5 2t \underbrace{(1+t^2)}^{1/2}\, dt$

$\frac{2}{3}(1+t^2)^{3/2} + C \Big|_0^5$

$\boxed{\frac{2}{3}(26)^{3/2} - \frac{2}{3}}$

c) $\frac{dy}{dx} = \frac{2t^2}{2t} = t$

$x = t^2 - 3$

$x + 3 = t^2$

$t = \sqrt{x+3}$

$\frac{dy}{dx} = \sqrt{x+3}$

9) $y = 2x \quad x'(t) = 3t^2 + 1$

$x(0) = 2 \quad y(0) = 4$

$x(1) = 2 + \int_0^1 (3t^2 + 1)\, dt$

$= 2 + \left[t^3 + t\right]\Big|_0^1$

$x(1) = 4$

$y(1) = 8 \quad \boxed{(4, 8)}$