

## AP Calculus BC

## Parametric Equations (64)

1)  $x = \ln t, y = t, t > 0$

$$t = e^x \quad y = e^x \quad (C)$$

2)  $xy = 10, x = 2 \quad \frac{dy}{dt} = 3$

$$x \frac{dy}{dt} + y \frac{dx}{dt} = 0$$

$$2(3) + 5 \frac{dx}{dt} = 0$$

$$\frac{dx}{dt} = -\frac{6}{5}$$

3)  $\frac{dx}{dt} = \frac{1}{t+1} \quad \frac{dy}{dt} = 2t$

a)  $x = \int \frac{1}{t+1} dt$

$$x = \ln|t+1| + C$$

$$\ln 2 = \ln 2 + C$$

$$0 = C$$

$$x = \ln(t+1)$$

$$y = \int 2t dt$$

$$y = t^2 + C$$

$$0 = 1 + C$$

$$-1 = C$$

$$y = t^2 - 1$$

b)

$$x = \ln(t+1)$$

$$e^x = t+1$$

$$e^x - 1 = t$$

$$y = (e^x - 1)^2 - 1$$

$$y = e^{2x} - 2e^x, x > 0$$

c)  $\frac{\Delta y}{\Delta x} = \frac{y(4) - y(0)}{x(4) - x(0)}$

$$= \frac{15 - (-1)}{\ln 5 - \ln 1}$$

$$= \frac{16}{\ln 5}$$

d)  $\frac{dy}{dx} = \frac{2t}{1/t+1} = 2t(t+1)$

$$\left. \frac{dy}{dx} \right|_{t=1} = 4$$

4)  $x = 3t+1, y = 9-4t$

$$\frac{dx}{dt} = 3 \quad \frac{dy}{dt} = -4$$

$$L = \int_0^2 \sqrt{3^2 + (-4)^2} dt$$

$$= \int_0^2 \sqrt{25} dt$$

$$= 10$$

5)  $x = 2t^2, y = 3t^2 - 1$

$$\frac{dx}{dt} = 4t \quad \frac{dy}{dt} = 6t$$

$$L = \int_0^4 \sqrt{16t^2 + 36t^2} dt$$

$$= \int_0^4 \sqrt{52t^2} dt$$

$$= \int_0^4 t\sqrt{52} dt$$

$$= \left. \frac{\sqrt{52}}{2} t^2 \right|_0^4$$

$$= 8\sqrt{52}$$

$$= \boxed{16\sqrt{13}}$$

$$6) x = \sin 3t \quad y = \cos 3t$$

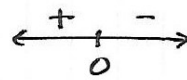
$$\frac{dx}{dt} = 3 \cos 3t \quad \frac{dy}{dt} = -3 \sin 3t$$

$$\begin{aligned} L &= \int_0^\pi \sqrt{9 \cos^2 3t + 9 \sin^2 3t} dt \\ &= \int_0^\pi \sqrt{9(\cos^2 3t + \sin^2 3t)} dt \\ &= \int_0^\pi 3 dt \\ &= 3\pi \end{aligned}$$

$$7) x = t^9, \quad y = 4 - t^2$$

$$\frac{dy}{dt} = -2t = 0$$

$$t = 0$$



MAX  
for y-value  
when  $t=0$

$$\boxed{y(0) = 4}$$

$$8) x = \cos^3 t \quad y = \sin^3 t$$

$$\frac{dx}{dt} = -3 \cos^2 t \sin t$$

$$\frac{dy}{dt} = 3 \sin^2 t \cos t$$

$$\begin{aligned} L &= \int_0^{2\pi} \sqrt{9 \cos^4 t \sin^2 t + 9 \sin^4 t \cos^2 t} dt \\ &= \int_0^{2\pi} \sqrt{9 \cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)} dt \\ &= \int_0^{2\pi} \sqrt{9 \cos^2 t \sin^2 t} dt = 6 \end{aligned}$$

CALCULATOR!

$$10) x = 2 \sin t \quad y = \cos t$$

rightmost - abs max x-value

$$\frac{dx}{dt} = 2 \cos t = 0$$

t	x
0	0
$\frac{\pi}{2}$	2
$\pi$	0

point: (2, 0)

$$9) x = t + 1 \quad y = t^2 + t$$

lowest  $\rightarrow$  Abs min y-value

$$\frac{dy}{dt} = 2t + 1 = 0$$

$$t = -\frac{1}{2}$$

t	y
-2	2
0	0
2	6

$\leftarrow$  lowest

$$x(0) = 1$$

point: (1, 0)

$$11) x = t^2 + 2t \quad y = t^2 - 2t + 3$$

leftmost  $\rightarrow$  abs min x-value

$$\frac{dx}{dt} = 2t + 2 = 0$$

$$t = -1$$

t	x
-2	0
-1	-1
3	15

$$y(-1) = 6$$

point: (-1, 6)