AP Calculus BC

Unit 8 – Parametric and Polar Equations

AP Calculus BC – Worksheet 63 **Parametric Equations**

1	Sketch the parametric curves. Find an equation that relates *x* and *y* directly. a) $x = 2t + 3$ and $y = 4t - 3$ for t in the interval $[0, 3]$ b) $x = \sin t$ and $y = 2\cos t$ for t in the interval $[0, \pi]$
2	Find (a) $\frac{dy}{dx}$ and (b) $\frac{d^2y}{dx^2}$ in terms of *t*. a) $x = 4\sin t,\ y = 2\cos t$ b) $x = t^2 - 3t,\ y = t^3$ c) $x = \ln(2t),\ y = \ln(3t)^4$ d) $x = \ln(5t),\ y = e^{5t}$
3	If $x = t^2 - 1$ and $y = e^{t^3}$, find $\frac{dy}{dx}$.
4	A curve *C* is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write an equation of the line that is tangent to the graph of *C* at the point $(8, -4)$.
5	Consider the curve *C* given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$. (a) Find $\frac{dy}{dx}$ as a function of *t*. (b) Find an equation of the tangent line at the point where $t = \frac{\pi}{4}$. (c) (Calculator) The curve *C* intersects the *y*-axis twice. Approximate the length of the curve between the two *y*-intercepts.
6	A parametric curve is defined by $x = \sin t$ and $y = \csc t$ for $0 < t < \frac{\pi}{2}$. Which of the following best describes the curve? (A) Increasing & concave up (B) increasing & concave down (C) decreasing and concave up (D) Decreasing & concave down (E) decreasing with a point of inflection

AP Calculus BC – Worksheet 64 **Parametric Equations**

1	Determine the rectangular equation for the parametric curve defined by $x = \ln t$ and $y = t$ for $t > 0$.
2	A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?
3	Point $P(x, y)$ moves in the *xy*-plane in such a way that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \geq 0$. (a) Find the coordinates of *P* in terms of *t* when $t = 1$, $x = \ln 2$, and $y = 0$. (b) Write an equation expression *y* in terms of *x*. (c) Find the average rate of change of *y* with respect to *x* as *t* varies from 0 to 4. (d) Find the instantaneous rate of change of *y* with respect to *x* when $t = 1$.
4	Given the parametric equations, $x = 3t + 1$, $y = 9 - 4t$, find the length of the path over the interval $0 \leq t \leq 2$.
5	Given the parametric equations, $x = 2t^2$, $y = 3t^2 - 1$, find the length of the path over the interval $0 \leq t \leq 4$.
6	Given the parametric equations, $x = \sin 3t$, $y = \cos 3t$, find the length of the path over the interval $0 \leq t \leq \pi$.
7	What is the maximum height of a particle whose path has the parametric equations $x = t^9$, $y = 4 - t^2$?
8	Find the length of the curve that has parametric equations $x = \cos^3 t$, $y = \sin^3 t$ on the interval $0 \leq t \leq 2\pi$.
9	Identify the lowest point on the curve that has parametric equations $x = t + 1$, $y = t^2 + t$ on the interval $-2 \leq t \leq 2$
10	Identify the rightmost point on the curve that has parametric equations $x = 2\sin t$, $y = \cos t$ on the interval $0 \leq t \leq \pi$.
11	Identify the leftmost point on the curve that has parametric equations $x = t^2 + 2t$, $y = t^2 - 2t + 3$ on the interval $-2 \leq t \leq 3$.

AP Calculus BC – Worksheet 65 **Particle Motion/Vector-Valued Equations**

No calculator, unless explicitly stated.

1	If a particle moves in the *xy*-plane so that at any time $t>0$, its position vector is $s(t)=\langle \ln(t^2+5t), 3t^2\rangle$, find its velocity vector at time $t=2$.
2	A particle moves in the *xy*-plane so that at any time *t*, its coordinates are given by $x(t)=t^5-1$, $y(t)=3t^4-2t^3$. Find its acceleration vector at $t=1$
3	If a particle moves in the *xy*-plane so that at time *t*, its position vector is $s(t)=\left\langle \sin\left(3t-\frac{\pi}{2}\right), 3t^2\right\rangle$, find the velocity vector at time $t=\frac{\pi}{2}$.
4	A particle moves on the curve $y=\ln x$ so that its *x*-coordinate has velocity $x'(t)=t+1$ for $t\geq 0$. At time $t=0$, the particle is at point $(1,0)$. Find the position of the particle at time $t=1$.
5	A particle moves in the *xy*-plane in such a way that its velocity vector is $v(t)=\langle 1+t, t^3\rangle$. If the position vector at $t=0$ is $\langle 5,0\rangle$, find the position of the particle at $t=2$.
6	The position of a particle in the *xy*-plane is given by the parametric equations $x(t)=t^3-\frac{3}{2}t^2-18t+5$ and $y(t)=t^3-6t^2+9t+4$. For what value(s) of *t* is the particle at rest?
7	A particle moves in the *xy*-plane so that the position of the particle is given by $x(t)=5t+3\sin t$ and $y(t)=(8-t)(1-\cos t)$. Find the speed of the particle at the time when the particle's horizontal position is $x=25$
8	The position of a particle at any time $t\geq 0$ is given by $x(t)=t^2-3$ and $y(t)=\frac{2}{3}t^3$. (a) Find the speed of the particle at time $t=5$. (b) Find the total distance traveled by the particle from $t=0$ to $t=5$. (c) Find an expression that would represent the slope of the path of particle in terms of *x*.
9	A particle moves on the curve $y=2x$ so that its *x*-coordinate has velocity $x'(t)=3t^2+1$ for $t\geq 0$. At time $t=0$, the particle is at point $(2,4)$. Find the position of the particle at time $t=1$.

1	If $x(t)=e^{2t}$ and $y(t)=\sin(3t)$, find $\frac{dy}{dx}$ in terms of *t*.
2	Write an integral expression to represent the length of the path described by the parametric equations $x=\cos^3 t$ and $y=\sin^2 t$ for $0\le t\le\frac{\pi}{2}$.
3	For what value(s) of *t* does the curve given by the parametric equations $x=t^3-t^2-1$ and $y=t^4+2t^2-8t$ have a vertical tangent?
4	Find the equation of the tangent line to the curve given by the parametric equations $x(t)=3t^2-4t+2$ and $y(t)=t^3-4t$ at the point on the curve where $t=1$.
5	If $x(t)=6-2t$ and $y(t)=t^3+3$ are the equations of the path of a particle moving in the *xy*-plane, in which direction is the particle moving as it passes through the point $(4,4)$?
6	A particle moves in the *xy*-plane so that its position at any time *t* is given by $x=\cos(5t)$ and $y=t^3$. What is the speed of the particle when $t=2$?
7	The position of a particle at time $t\ge 0$ is given by the vector-valued equation $s(t)=\left\langle \frac{(t-2)^3}{3}+4,t^2-4t+4\right\rangle$. a) Find the speed of the particle at $t=1$. b) Find the total distance traveled by the particle from $t=0$ to $t=1$. c) When is the particle at rest? What is its position at that time?
8	An object moving along a curve in the *xy*-plane has position given by $s(t)=\langle x(t),y(t)\rangle$ at time $t\ge 0$ with $\frac{dx}{dt}=1+\tan(t^2)$ and $\frac{dy}{dt}=3e^{\sqrt{t}}$. Find the acceleration vector and the speed of the object when $t=5$.
9	A particle moves in the *xy*-plane so that the position of the particle is given by $x(t)=t+\cot t$ and $y(t)=3t+2\sin t$ for $0\le t\le\pi$. Find the velocity vector when the particle's vertical position is $y=5$.
10	An object moving along a curve in the *xy*-plane has velocity vector $v(t)=\langle 2\sin(t^3),\cos(t^2)\rangle$ at time *t* for $0\le t\le 4$. At time $t=1$, the object is at position $(3,4)$. a) Write an equation for the line tangent to the curve at $(3,4)$. b) Find the speed of the object at time $t=2$. c) Find the total distance traveled by the object over the time interval $0\le t\le 1$. d) Find the position of the particle at time $t=2$.
11	An object moving along a curve in the *xy*-plane has velocity $v(t)=\langle\cos(t^2),\sin(t^3)\rangle$. At time $t=0$, the object is at position $(4,7)$. Where is the particle at time $t=2$?
12	The path of a particle moving in the plane is defined parametrically as a function of time *t* by $x=\sin 2t$ and $y=\cos 5t$. What is the speed of the particle at time $t=2$?
13	Find the total distance traveled by a particle from $t=0$ to $t=3$ whose position is given by the vector $s(t)=\left\langle t^2+1,\frac{4}{3}t^3\right\rangle$.

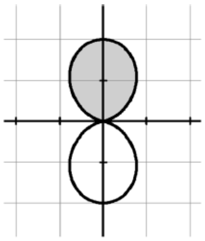
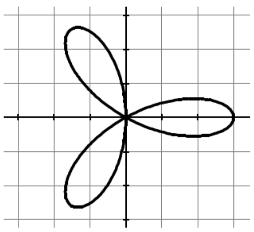
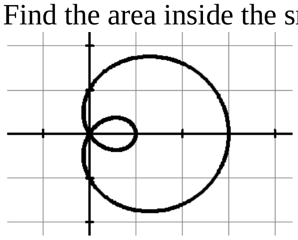
AP Calculus BC - Worksheet 67 **Polar Equations and Derivatives**

1	Convert the following equations to polar form: a) $x^2 + y^2 = 16$ b) $4x + 3y - 1 = 0$ c) $y = 7$
2	Convert the following equations to rectangular form: a) $r = 3\sec\theta$ b) $4r\cos\theta = r^2$ c) $\theta = \frac{5\pi}{6}$
3	For each of the polar functions, find $\frac{dy}{dx}$ for the given value of θ. a) $r = 1 - \sin\theta$, $\theta = 0$ b) $r = \cos\theta$, $\theta = \frac{\pi}{3}$ c) $r = 3(1 - \cos\theta)$, $\theta = \frac{\pi}{2}$
4	Find the point(s) where the polar curve given $r = 1 + \sin\theta$ has horizontal and vertical tangent lines.

AP Calculus BC - Worksheet 68 **Polar Equations and Motion**

1	For the curve $r = 3 + 3\sin(2\theta)$, find the value of $\frac{dx}{d\theta}$ at $\theta = \frac{\pi}{3}$.
2	Find $\frac{dy}{dx}$ for $r(\theta) = 3\cos\theta$ when $\theta = \frac{\pi}{4}$.
3	Find the equation of the tangent line to the curve $r = -1 + \sin\theta$ when $\theta = \pi$.
4	A particle is moving along the curve $r = 4 - \sin(3\theta)$ so that $\frac{d\theta}{dt} = 3$ for all times $t \geq 0$. Find the value of $\frac{dr}{dt}$ at $\theta = \frac{\pi}{6}$.
5	A particle moves along the polar curve $r = 4 - 2\sin\theta$ so that at time t seconds, $\theta = t^2$. a) Find the position vector in terms of t. Find the velocity vector at time $t = 1.5$ b) Find the time t in the interval $1 \leq t \leq 2$ for which the x-coordinate of the particle's position is -1.
6	For a certain polar curve, $r(\theta)$, it is known that $\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta$ and $\frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = \frac{3\pi}{2}$.

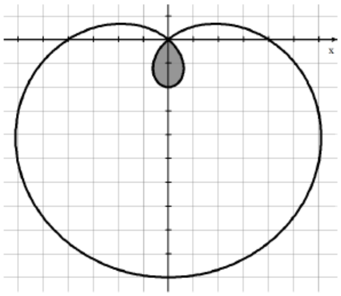
AP Calculus BC – Worksheet 69 **Polar Equations –Area**

1	Find the area bounded by $r=5\sin\theta$.
2	Find the area of the shaded region of the polar curve for $r=1-\cos 2\theta$ 
3	Find the area of one petal of the rose curve $r=3\cos(3\theta)$. 
4	Find the area of the region in the plane enclosed by the cardioid $r=4+4\sin(\theta)$.
5	Find the area inside the smaller loop of the limacon $r=1+2\cos\theta$. 
6	Find the area of one petal of the rose curve defined by $r=4\sin(6\theta)$.

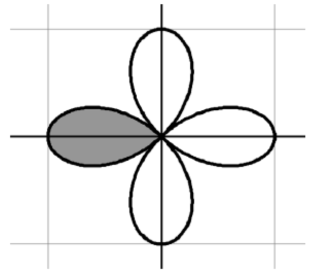
AP Calculus BC – Worksheet 70 **Polar Area**

A calculator is required for all problems.

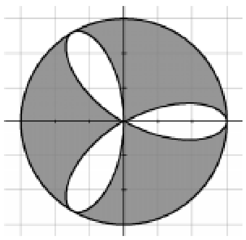
1) Find the area of the shaded region of the polar curve $r = 4 - 6\sin\theta$.



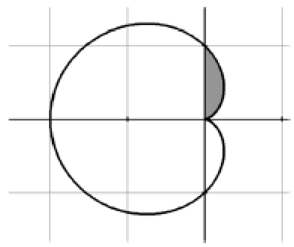
2) Find the area of the shaded region of the polar curve $r = \cos 2\theta$.



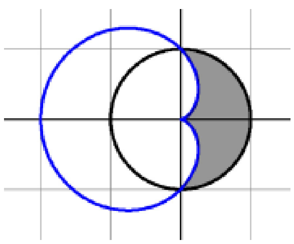
3) Find the area of the shaded region bounded by the polar curves $r = 3$ and $r = 3\cos 3\theta$ indicated in the figure below.



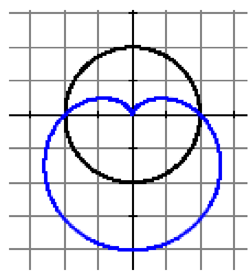
4) Find the area of the shaded region for the polar curve $r = 1 - \cos\theta$.



5) Find the area of the region bound by the two polar curves $r = 1$ and $r = 1 - \cos\theta$ as shown in the graph below.

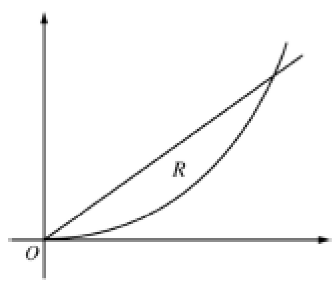


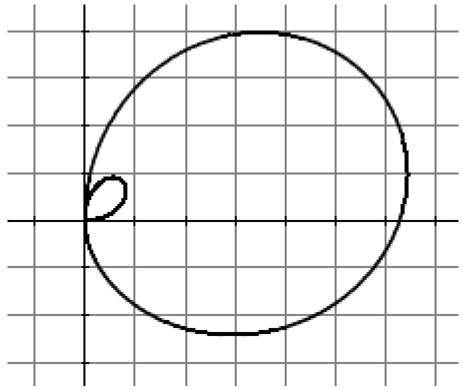
6) Find the area of the common region to the polar graphs $r = 2$ and $r = 2 - 2\sin\theta$.



7) Find the area of common interior bounded by the graphs of polar curves $r = 3\cos\theta$ and $r = 2 - \cos\theta$.

8) Find the area of the region that is inside the polar graph of $r = 1 + 2\cos\theta$ but outside the inner loop.

1	The position of a particle at any time $t \geq 0$ is given by $x(t) = t^2 - 2$ and $y(t) = \frac{2}{3}t^3$. a) Find the speed of the particle at $t = 2$. b) Set up, but do not evaluate, an integral expression that can be used to find the total distance that the particle traveled from $t = 0$ to $t = 4$. c) Find $\frac{dy}{dx}$ as a function of *x*. d) At what time *t* is the particle on the *y*-axis? Find the acceleration vector at this time.
2	A particle moving along a curve in the *xy*-plane has position vector $s(t) = \langle 2t + 3\sin t, t^2 + 2\cos t \rangle$, where $0 \leq t \leq 10$. Find the velocity vector at the time the particle's vertical position is $y = 7$.
3	An object is moving along a curve in the *xy*-plane has velocity vector $v(t) = \langle \cos(e^t), \sin(e^t) \rangle$ for $0 \leq t \leq 2$. At time $t = 1$, the object is at the point $(3,2)$. a) Find the equation of the tangent line to the curve at the point where $t = 1$. b) Find the speed of the object at $t = 1$. c) Find the total distance traveled by the object over the time interval $0 \leq t \leq 2$. d) Find the position of the particle at time $t = 2$.
4	 Let *R* be the region in the first quadrant that is bounded by the polar curves $r = \frac{\theta}{2}$ and $\theta = k$, where *k* is a constant, such that $0 < k < \frac{\pi}{2}$, as shown in the figure above. Find the area of region *R* in terms of *k*.
5	Find the slope of the tangent line to the polar curve $r = 2\cos 4\theta$ at the point where $\theta = \frac{\pi}{4}$.
6	For a certain polar curve, $r(\theta)$, it is known that $\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta$ and $\frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$. What is the value of $\frac{d^2y}{dx^2}$ at $\theta = 6$.

7	 Consider the polar curve defined by the function $r(\theta) = 2\theta\cos\theta$, where $0 \le \theta \le \frac{3\pi}{2}$. The derivative of r is given by $\frac{dr}{d\theta} = 2\cos\theta - 2\theta\sin\theta$. The figure above shows the graph of r for $0 \le \theta \le \frac{3\pi}{2}$. a) Find the area of the region enclosed by the inner loop of the curve. b) Determine if r is increasing or decreasing when $\theta = \frac{\pi}{2}$. Justify your answer. c) For $0 \le \theta \le \frac{3\pi}{2}$, find the greatest distance from any point on the graph of r to the origin. Justify your answer. d) A particle is moving along the curve. There is a point on the curve at which the slope of the line tangent to the curve is $\frac{2}{2-\pi}$. At this point, $\frac{dy}{d\theta} = \frac{1}{2}$. Find $\frac{dx}{d\theta}$ at this point. In which direction is the particle moving?