WS 8 The Definition of the Derivative

1) $f(x)=\frac{1}{x}$, $a=2$

$$f'(2)=\lim_{h\to 0}\frac{\frac{1}{2+h}-\frac{1}{2}}{h}$$
$$=\lim_{h\to 0}\frac{\frac{2}{2(2+h)}-\frac{2+h}{2(2+h)}}{h}$$
$$=\lim_{h\to 0}\frac{\frac{-h}{2(2+h)}}{h}$$
$$=\lim_{h\to 0}-\frac{1}{2(2+h)}=\boxed{-\frac{1}{4}}$$

2) $f(x)=x^2+4$, $a=1$

$$f'(1)=\lim_{h\to 0}\frac{(1+h)^2+4-(5)}{h}$$
$$=\lim_{h\to 0}\frac{1+2h+h^2+4-5}{h}$$
$$=\lim_{h\to 0}\frac{2h+h^2}{h}$$
$$=\lim_{h\to 0}(2+h)=\boxed{2}$$

3) $f(x)=x^3+x$, $a=0$

$$f'(0)=\lim_{h\to 0}\frac{(0+h)^3+(0+h)-0}{h}$$
$$=\lim_{h\to 0}\frac{h^3+h}{h}$$
$$=\lim_{h\to 0}(h^2+1)=\boxed{1}$$

4) $f(x)=x^2+4$, $a=1$

$$f'(1)=\lim_{x\to 1}\frac{x^2+4-5}{x-1}$$
$$=\lim_{x\to 1}\frac{x^2-1}{x-1}$$
$$=\lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}$$
$$=\lim_{x\to 1}(x+1)=\boxed{2}$$

5) $f(x)=\sqrt{x+1}$, $a=3$

$$f'(3)=\lim_{x\to 3}\frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$$
$$=\lim_{x\to 3}\frac{x-3}{(x-3(\sqrt{x+1}+2)}=\boxed{\frac{1}{4}}$$

6) $f(x)=2x+3$ $a=-1$

$$f'(-1)=\lim_{x\to -1}\frac{2x+3-(1)}{x+1}$$
$$=\lim_{x\to -1}\frac{2x+2}{x+1}=\boxed{2}$$

7) $f(x)=\begin{cases}3x^2-4, & x<0\\ 3x-4, & x\geq 0\end{cases}$, $x=0$

cont

$f(0)=-4$

$\lim_{x\to 0^-}(3x^2-4)=-4$ $\lim_{x\to 0^+}(3x-4)=-4$

$\lim_{x\to 0}f(x)=-4$

✓

diff

Left

$$\lim_{x\to 0^-}\frac{3x^2-4-(-4)}{x}$$
$$\lim_{x\to 0^-}\frac{3x^2}{x}=0$$

Right

$$\lim_{x\to 0^+}\frac{3x-4-(-4)}{x}$$
$$\lim_{x\to 0^+}\frac{3x}{x}=3$$

$f(x)$ is not diff. at $x=0$.

8) $g(x) = \begin{cases} (x+1)^2, & x \le 0 \\ 2x+1, & 0 < x < 5 \\ (6-x)^2, & x \ge 5 \end{cases}$

a) $x = 0$

continuity

I. $g(0) = 1$

II. $\lim_{x \to 0^-} (x+1)^2 = 1$

$\lim_{x \to 0^+} (2x+1) = 1$

$\lim_{x \to 0} g(x) = 1$

Diff

Left

$\lim_{x \to 0^-} \frac{(x+1)^2 - 1}{x}$

$\lim_{x \to 0^-} \frac{x^2+2x}{x}$

$\lim_{x \to 0^-} (x+2) = \underline{2}$

Right

$\lim_{x \to 0^+} \frac{2x+1-1}{x}$

$\lim_{x \to 0^+} \frac{2x}{x} = \underline{2}$

f(x) is diff @ $x = 0$

b) $x = 5$

continuity

I. $g(5) = 1$

II $\lim_{x \to 5^-} (2x+1) = 11$

$\lim_{x \to 5^+} (6-x)^2 = 1$

g is not cont @ $x = 5$.

g is not diff @ $x = 5$.

c) g is diff on $(-\infty, 5) \cup (5, \infty)$

9) f(x) is diff on $(-\infty, 0) \cup (0, \infty)$

$\lim_{x \to 0^-} \frac{x^2}{x} = 0 \neq \lim_{x \to 0^+} \frac{x}{x} = 1$

10) $\lim_{x \to 1^-} \frac{2-2}{x} = 0 \neq \lim_{x \to 1^+} \frac{2x-2}{x-1} = 2$

f(x) is diff on $(-\infty, 1) \cup (1, \infty)$