AP Calculus BC

WS 94 - Power Series FRQ

2015 #6

$$\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{n} x^n$$

a) Ratio Test

$$\lim_{n\to\infty} \left| \frac{(-3)^n x^{n+1}}{n+1} \cdot \frac{n}{(-3)^{n-1} x^n} \right| = |3x| < 1$$

$$-1 < 3x < 1$$

$$-\tfrac{1}{3} < x < \tfrac{1}{3}$$

$$\boxed{R = \tfrac{1}{3}}$$

b) $f(x) = x - \frac{3}{2}x^2 + 3x^3 - \frac{27}{4}x^4$

$$f'(x) = 1 - 3x + 9x^2 - 27x^3 = \frac{1}{1-(-3x)} = \frac{1}{1+3x}$$

c) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$$g(x) = e^x \cdot f(x)$$

$$= \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)\left(x - \tfrac{3}{2}x^2 + 3x^3 - \cdots\right)$$

$$T_3(x) = x - \tfrac{3}{2}x^2 + 3x^3 + x^2 - \tfrac{3}{2}x^3 + \tfrac{1}{2}x^3$$

$$T_3(x) = x + \tfrac{1}{2}x^2 + 2x^3$$

2016 #6

a) $f(1)=1$

$f'(1)=-\frac{1}{2}$

$f''(1)=(-1)^2\frac{1}{2^2}=\frac{1}{4}$

$f'''(1)=(-1)^3\frac{(3-1)!}{2^3}=-\frac{2}{8}=-\frac{1}{4}$

$$f(x)=1-\frac{1}{2}(x-1)+\frac{1/4}{2!}(x-1)^2-\frac{1/4}{3!}(x-1)^3+\dots+\frac{\frac{(-1)^n(n-1)!}{2^n}}{n!}(x-1)^n$$

b) If $R=2$, the interval $=(-1,3)$

(center @ $x=1$)

$x=-1$

$$\sum\frac{(-1)^n(n-1)!}{2^n\,n!}\cdot(-2)^n$$

$\sum\frac{1}{n}$ diverges by p-series

$x=3$

$$\sum\frac{(-1)^n(n-1)!}{2^n\,n!}\cdot 2^n$$

$\sum\frac{(-1)^n}{n}$ converges by AST

The interval of convergence : $(-1,3]$

c) $f(1.2)\approx 1-\frac{1}{2}(0.2)+\frac{1}{8}(0.2)^2=T_2(1.2)$

d) Error $=\left|f(1.2)-T_2(1.2)\right|\le\left|\frac{1}{24}(0.2)^3\right|=\left(\frac{1}{5}\right)^3\cdot\frac{1}{24}$

$=\frac{1}{3000}\le 0.001$