

$$\begin{aligned}
 1) e^{-5x} &= 1 + (-5x) + \frac{(-5x)^2}{2!} + \frac{(-5x)^3}{3!} + \dots + \frac{(-5x)^n}{n!} \\
 &= 1 - 5x + \frac{25x^2}{2} - \frac{125x^3}{6} + \dots + \frac{(-5x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n (5x)^n}{n!}
 \end{aligned}$$

$$\begin{aligned}
 2) e^{-x/2} &= 1 + (-x/2) + \frac{(-x/2)^2}{2!} + \frac{(-x/2)^3}{3!} + \dots + \frac{(-x/2)^n}{n!} \\
 &= 1 - \frac{x}{2} + \frac{x^2}{2^2 \cdot 2!} - \frac{x^3}{2^3 \cdot 3!} + \dots + \frac{(-x)^n}{2^n n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n n!}
 \end{aligned}$$

$$\begin{aligned}
 3) 5 \sin(-x) &= 5 \left[-x - \frac{(-x)^3}{3!} + \frac{(-x)^5}{5!} - \frac{(-x)^7}{7!} + \dots + \frac{(-1)^n (-x)^{2n+1}}{(2n+1)!} \right] \\
 &= 5 \left[-x + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} + \dots + \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!} \right] = 5 \sum_{n=0}^{\infty} \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}
 \end{aligned}$$

$$\begin{aligned}
 5) \cos \sqrt{x} &= 1 - \frac{(\sqrt{x})^2}{2!} + \frac{(\sqrt{x})^4}{4!} - \frac{(\sqrt{x})^6}{6!} + \dots + \frac{(-1)^n (\sqrt{x})^{2n}}{(2n)!} \\
 &= 1 - \frac{x}{2!} + \frac{x^2}{4!} - \frac{x^3}{6!} + \dots + \frac{(-1)^n x^n}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}
 \end{aligned}$$

$$\begin{aligned}
 7) x e^x &= x \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right) \\
 &= x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \dots = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}
 \end{aligned}$$

$$\begin{aligned}
 8) x^2 \sin x &= x^2 \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \right) \\
 &= x^3 - \frac{x^5}{3!} + \frac{x^7}{5!} - \frac{x^9}{7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+1)!}
 \end{aligned}$$

$$\begin{aligned}
 9) \frac{x^2}{2} - 1 + \cos x &= \frac{x^2}{2} - 1 + \left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \right) \\
 &= \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+4}}{(2n+4)!}
 \end{aligned}$$

$$\begin{aligned}
 10) \sin x - x + \frac{x^3}{3} &= \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots \right) - x + \frac{x^3}{3} \\
 &= \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+5}}{(2n+5)!}
 \end{aligned}$$

$$11) x \cos \pi x = x \left(1 - \frac{(\pi x)^2}{2!} + \frac{(\pi x)^4}{4!} - \frac{(\pi x)^6}{6!} + \dots \right)$$

$$= x - \frac{\pi^2 x^3}{2!} + \frac{\pi^4 x^5}{4!} - \frac{\pi^6 x^7}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n} x^{2n+1}}{(2n)!}$$

$$12) x^2 \cos(x^2) = x^2 \left(1 - \frac{(x^2)^2}{2!} + \frac{(x^2)^4}{4!} - \frac{(x^2)^6}{6!} + \dots \right)$$

$$= x^2 - \frac{x^6}{2!} + \frac{x^{10}}{4!} - \frac{x^{14}}{6!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n)!}$$

$$47) \lim_{x \rightarrow 0} \frac{e^x - (1+x)}{x^2} \rightarrow \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots) - 1 - x}{x^2}$$

$$\lim_{x \rightarrow 0} \frac{\frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \dots}{x^2}$$

$$\lim_{x \rightarrow 0} \left[\frac{1}{2!} + \frac{x}{3!} - \frac{x^2}{4!} + \dots \right] = \frac{1}{2}$$

$$48) \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x} \rightarrow \frac{0}{0}$$

$$\lim_{x \rightarrow 0} \frac{(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots) - (1 - x + \frac{(-x)^2}{2!} + \frac{(-x)^3}{3!} + \dots)}{x}$$

$$\lim_{x \rightarrow 0} \frac{(2x + \frac{2x^3}{3!} + \frac{2x^5}{5!} + \dots)}{x}$$

$$\lim_{x \rightarrow 0} (2 + \frac{2x^2}{3!} + \frac{2x^4}{5!} + \dots) = 2$$

$$49) \lim_{t \rightarrow 0} \frac{1 - \cos t - \frac{t^2}{2}}{t^4} \rightarrow \frac{0}{0}$$

$$\lim_{t \rightarrow 0} \frac{1 - (1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \dots) - \frac{t^2}{2}}{t^4}$$

$$\lim_{t \rightarrow 0} \frac{(-\frac{t^4}{4!} + \frac{t^6}{6!} - \frac{t^8}{8!} + \dots)}{t^4}$$

$$\lim_{t \rightarrow 0} \left[-\frac{1}{4!} + \frac{t^2}{6!} - \frac{t^4}{8!} + \dots \right] = -\frac{1}{24}$$

$$50) \lim_{\theta \rightarrow 0} \frac{\sin \theta - \theta + (\frac{\theta^3}{6})}{\theta^5} \rightarrow \frac{0}{0}$$

$$\lim_{\theta \rightarrow 0} \frac{(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots) - \theta + \frac{\theta^3}{6}}{\theta^5}$$

$$\lim_{\theta \rightarrow 0} \frac{(\frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \frac{\theta^9}{9!} - \dots)}{\theta^5}$$

$$\lim_{\theta \rightarrow 0} \left[\frac{1}{5!} - \frac{\theta^2}{7!} + \frac{\theta^4}{9!} - \dots \right] = \frac{1}{120}$$