AP Calculus BC — WS 89 - Taylor/Maclaurin Polynomials

1) $f(x) = \ln x, \; a = 1$

$f(1) = 0$

$f'(x) = \frac{1}{x} \quad f'(1) = 1$

$f''(x) = -\frac{1}{x^2} \quad f'(1) = 1$

$f''(x) = \frac{2}{x^3} \quad f''(1) = 2$

$T_3(x) = 0 + 1(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3$

$T_3(x) = (x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!}$

2) $f(x) = \sin x; \; a = \frac{\pi}{4}$

$f(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f'(x) = \cos x \quad f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}$

$f''(x) = -\sin x \quad f''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$f'''(x) = -\cos x \quad f'''(\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$

$T_3(x) = f(\frac{\pi}{4}) + f'(\frac{\pi}{4})(x - \frac{\pi}{4}) + \frac{f''(\frac{\pi}{4})}{2!}(x - \frac{\pi}{4})^2 + \frac{f'''(\frac{\pi}{4})}{3!}(x - \frac{\pi}{4})^3$

$T_3(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}(x - \frac{\pi}{4}) - \frac{\sqrt{2}/2}{2!}(x - \frac{\pi}{4})^2 - \frac{\sqrt{2}/2}{3!}(x - \frac{\pi}{4})^3$

3) $f(x) = \sqrt{x}; \; a = 4$

$f(4) = 2$

$f'(x) = \frac{1}{2\sqrt{x}} \quad f'(4) = \frac{1}{4}$

$f''(x) = -\frac{1}{4x^{3/2}} \quad f''(4) = -\frac{1}{32}$

$f'''(x) = \frac{3}{8x^{5/2}} \quad f'''(4) = \frac{3}{256}$

$T_3(x) = f(4) + f'(4)(x-4) + \frac{f''(4)}{2!}(x-4)^2 + \frac{f'''(4)}{3!}(x-4)^3$

$T_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1/32}{2!}(x-4)^2 + \frac{3/256}{3!}(x-4)^3$

$T_3(x) = 2 + \frac{(x-4)}{2^2} - \frac{(x-4)^2}{2! \cdot 2^5} + \frac{3(x-4)^3}{3! \cdot 2^8}$

4) $f(x) = e^x; \; a = 2$

$f(2) = e^2$

$f'(x) = e^x \quad f'(2) = e^2$

$f''(x) = e^x \quad f''(2) = e^2$

$f'''(x) = e^x \quad f'''(2) = e^2$

$T_3(x) = e^2 + e^2(x-2) + \frac{e^2}{2!}(x-2)^2 + \frac{e^2}{3!}(x-2)^2$

5) $f(x) = e^{-x}$ $f(0) = 1$

$f'(x) = -e^{-x}$ $f'(0) = -1$

$f''(x) = e^{-x}$ $f''(0) = 1$

$f'''(x) = -e^{-x}$ $f'''(0) = -1$

$$M_3(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!}$$

6) $f(x) = \frac{1}{1+x}$ $f(0) = 1$

$f'(x) = -(1+x)^{-2}$ $f'(0) = -1$

$f''(x) = 2(1+x)^{-3}$ $f''(0) = 2$

$f'''(x) = -6(1+x)^{-4}$ $f'''(0) = -6$

$$M_3(x) = 1 - x + \frac{2}{2!}x^2 - \frac{6}{3!}x^3$$

$$M_3(x) = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n$$

7) $f(x) = \sin 3x$ $f(0) = 0$

$f'(x) = 3\cos 3x$ $f'(0) = 3$

$f''(x) = -9\sin 3x$ $f''(0) = 0$

$f'''(x) = -27\cos 3x$ $f'''(0) = -27$

$$M_3(x) = 0 + 3x + \frac{0}{2!}x^2 - \frac{27}{3!}x^3$$

$$M_3(x) = 3x - \frac{27}{3!}x^3 + \frac{3^5}{5!}x^5 - \frac{3^7}{7!}x^7 + \cdots + \frac{(-1)^n (3x)^{2n+1}}{(2n+1)!}$$

8) $f(x) = (x+1)^3$ $f(0) = 1$

$f'(x) = 3(x+1)^2$ $f'(0) = 3$

$f''(x) = 6(x+1)$ $f''(0) = 6$

$f'''(x) = 6$ $f'''(0) = 6$

$$M_3(x) = 1 + 3x + \frac{6}{2!}x^2 + \frac{6}{3!}x^3$$

$$M_3(x) = 1 + 3x + 3x^2 + x^3$$