AP Calculus BC

WS 84 Series Convergence Review 1

1) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = S$

S diverges by p-series
$p = \frac{1}{2} < 1$

2) $\sum_{n=1}^{\infty} \frac{-5}{n} = S$

$p = 1$, S diverges by p-series

3) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} = S$

AST

$* \lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

$* \; 1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{2}$

$\sum \frac{1}{\sqrt{n}}$ diverges by p-seres

S converges conditionally

4) $\sum_{n=1}^{\infty} \frac{1}{2n^3} = S$

$p = 3 > 1$, S converges by p-series

5) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} = S$

$\int_2^{\infty} (\ln x)^{-2} \cdot \frac{1}{x}\,dx$

$\lim_{b\to\infty} \int_2^{b} \frac{1}{x} \cdot (\ln x)^{-2}\,dx$

$\lim_{b\to\infty} \left[-(\ln x)^{-1}\right]_2^{b}$

$\lim_{b\to\infty} \left[-\frac{1}{\ln b} + \frac{1}{\ln 2}\right] = \frac{1}{\ln 2}$

S converges by Integral Test

6) $\sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1} = S$

AST

$* \lim_{n\to\infty} \frac{3n^2}{n^3+1} = 0$

$* \; \frac{3}{2}, \frac{4}{3}, \frac{27}{28}, \frac{48}{65}$

$\sum \frac{3n^2}{n^3+1}$

LCT

$\sum \frac{1}{n}$ diverges by p-series

$\lim_{n\to\infty} \left[\frac{3n^2}{n^3+1} \cdot \frac{n}{1}\right] = 3 > 0$

S converges conditionally.

7) $\sum_{n=1}^{\infty} \frac{n+1}{n!} = S$

Ratio Test

$\lim_{n\to\infty} \left[\frac{n+2}{(n+1)!} \cdot \frac{n!}{n+1}\right]$

$\lim_{n\to\infty} \left[\frac{n+2}{(n+1)(n+1)}\right] = 0 < 1$

S converges by Ratio Test

8) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n!} = S$

Ratio Test

$\lim_{n\to\infty} \left|\left[\frac{(-3)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-3)^n}\right]\right|$

$\lim_{n\to\infty} \left|\frac{(-3)}{n+1}\right| = 0 < 1$

S converges.

9) $\sum_{n=1}^{\infty} \frac{2^n \cdot 3^n}{n^n} = \sum_{n=1}^{\infty} \frac{6^n}{n^n} = S$

Root Test

$\lim_{n\to\infty} \sqrt[n]{\frac{6^n}{n^n}} = \lim_{n\to\infty} \frac{6}{n} = 0 < 1$

S converges

10) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2+1}} = S$

$a_n = \frac{1}{n\sqrt{n^2+1}} \quad b_n = \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ converges by p-series

$\lim_{n\to\infty} \left[\frac{1}{n\sqrt{n^2+1}} \cdot n^2 \right] = 1 > 0$

S converges by LCT

11) $\sum_{n=1}^{\infty} \frac{(-1)^n (n^2+1)}{2n^2+n-1} = S$

$\lim_{n\to\infty} \frac{n^2+1}{2n^2+n-1} = \frac{1}{2} \neq 0$

S diverges by n^{th} term test

12) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}} = S$

DCT

$b_n = \frac{1}{n^{3/2}}$ $\quad \sum b_n$ converges by p-series

n	a_n	b_n
1	$1/\sqrt{6}$	1
2	$1/\sqrt{24}$	$1/\sqrt{8}$
3	$1/\sqrt{60}$	$1/\sqrt{27}$

$b_n > a_n$

By DCT, S converges