AP Calculus BC

WS 83 Absolute/Conditional Convergence

1) $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{10}\right)^n = S$

AST

$\lim_{n\to\infty}\left(\frac{1}{10}\right)^n = 0$

$\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}, \ldots$

$\sum\left(\frac{1}{10}\right)^n$

GST

$r<1$

S converges absolutely

2) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{1}{\sqrt{n}}\right) = S$

AST

$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n}}\right) = 0$

$1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{2}, \ldots$

$\sum\frac{1}{\sqrt{n}}$

p-series → diverges $p<1$

S is conditionally convergent

3) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1} = S$

AST

$\lim_{n\to\infty}\frac{n}{n^3+1} = 0$

$\frac{1}{2}, \frac{2}{9}, \frac{3}{28}, \ldots$

$\sum\frac{n}{n^3+1}$

LCT

$\sum_{n=1}^{\infty}\frac{1}{n^2}$ converges by p-series

$\lim_{n\to\infty}\left[\frac{n}{n^3+1}\cdot n^2\right] = 1 > 0$

converges

S is absolutely convergent

4) $\sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{n!}{2^n} = S$

AST

$\lim_{n\to\infty}\frac{n!}{2^n} = \infty$

S diverges

5) $\sum_{n=1}^{\infty}(-1)^n\cdot\frac{1}{n+3} = S$

$\lim_{n\to\infty}\frac{1}{n+3} = 0$

$\frac{1}{4}, \frac{1}{5}, \frac{1}{6}.$

$\sum\frac{1}{n+3}$

LCT

$\sum\frac{1}{n}$ diverges

$\lim_{n\to\infty}\left[\frac{1}{n+3}\cdot n\right] = 1 > 0$

$\sum\frac{1}{n+3}$ also diverges

S converges conditionally

6) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3+n}{5+n} = S$

$\lim_{n\to\infty}\frac{3+n}{5+n} = 1 \neq 0$

S is divergent by n^{th} term

7) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2} = S$

AST

$\lim_{n\to\infty}\frac{1+n}{n^2} = 0$

$\frac{1}{2}, \frac{3}{9}, \frac{5}{16}$

$\sum\frac{1+n}{n^2}$

LCT

$\sum\frac{1}{n}$ diverges

$\lim_{n\to\infty}\left[\frac{1+n}{n^2}\cdot n\right] = 1 > 0$

$\sum\frac{1+n}{n^2}$ also diverges

S Converges conditionally

8) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (n+1)^n}{(2n)^n} = S$

<u>AST</u>

$\displaystyle \lim_{n\to\infty} \left(\frac{n+1}{2n}\right)^n = 0$

$1, \frac{9}{16}, \frac{64}{6^3} \cdots$

$\displaystyle \sum \frac{(n+1)^n}{(2n)^n}$

<u>Root Test</u>

$\displaystyle \lim_{n\to\infty} \sqrt[n]{\frac{(n+1)^n}{(2n)^n}}$

$\displaystyle \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2} < 1$

converges

S converges absolutely

9) $\displaystyle\sum_{n=1}^{\infty} \frac{n^{10}}{10^n} = S$

<u>Root Test</u>

$\displaystyle \lim_{n\to\infty} \sqrt[n]{\frac{n^{10}}{10^n}} = \lim_{n\to\infty} \frac{\left(\sqrt[n]{n}\right)^{10}}{10}$

$= \frac{1}{10} < 1$

S converges

10) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3} = S$

<u>LCT</u>

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by p-series

$\displaystyle \lim_{n\to\infty} \left[\frac{\ln n}{n^3} \cdot n^2\right] = 0$

S converges

11) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)(n+2)}{n!} = S$

<u>Ratio Test</u>

$\displaystyle \lim_{n\to\infty} \left[\frac{(n+2)(n+3)}{(n+1)!} \cdot \frac{n!}{(n+1)\cdot(n+2)}\right]$

$\displaystyle \lim_{n\to\infty} \left[\frac{n+3}{(n+1)^2}\right] = 0 < 1$

S converges

12) $\displaystyle\sum_{n=1}^{\infty} \frac{n \cdot 2^n (n+1)!}{3^n\, n!} = S$

<u>Ratio Test</u>

$\displaystyle \lim_{n\to\infty} \left[\frac{(n+1)\cdot 2^{n+1}(n+2)!}{3^{n+1}(n+1)!} \cdot \frac{3^n n!}{n \cdot 2^n (n+1)!}\right]$

$\displaystyle \lim_{n\to\infty} \left[\frac{(n+1)\cdot 2^{n+1}(n+2)\cdot\cancel{(n+1)!}}{3^{n+1}\cancel{(n+1)!}} \cdot \frac{3^n \cancel{n!}}{n\, 2^n (n+1)\cdot\cancel{n!}}\right] = \frac{2}{3} > 0$

S converges