2007 AB5-BC5 (**4.6**)

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.	$2: \begin{cases} 1: \text{estimate} \\ 1: \text{conclusion with reason} \end{cases}$
(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$ $\left.\frac{dV}{dt}\right\|_{t=5} = 4\pi(30)^2 2 = 7200\pi \text{ ft}^3/\text{min}$	$3: \begin{cases} 2: \frac{dV}{dt} \\ 1: \text{answer} \end{cases}$
(c) $\int_0^{12} r'(t)\,dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$ $= 19.3$ ft $\int_0^{12} r'(t)\,dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.	$2: \begin{cases} 1: \text{approximation} \\ 1: \text{explanation} \end{cases}$
(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t)\,dt$.	1 : conclusion with reason
Units of ft^3/min in part (b) and ft in part (c)	1 : units in (b) and (c)

(a) $L'(5.5) \approx \dfrac{L(7)-L(4)}{7-4} = \dfrac{150-126}{3} = 8$ people per hour	2 : { 1 : estimate 1 : units
(b) The average number of people waiting in line during the first 4 hours is approximately $\frac{1}{4}\left(\frac{L(0)+L(1)}{2}(1-0)+\frac{L(1)+L(3)}{2}(3-1)+\frac{L(3)+L(4)}{2}(4-3)\right)$ $= 155.25$ people	2 : { 1 : trapezoidal sum 1 : answer
(c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.	3 : { 1 : considers change in sign of L' 1 : analysis 1 : conclusion
OR	OR
The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$. [*Note: There is a function* L *that satisfies the given conditions with* $L'(t) = 0$ *for exactly three values of* t.]	3 : { 1 : considers relative extrema of L on $(0, 9)$ 1 : analysis 1 : conclusion
(d) $\int_0^3 r(t)\,dt = 972.784$ There were approximately 973 tickets sold by 3 P.M.	2 : { 1 : integrand 1 : limits and answer

2010 AB2-BC2 (**3.92**)

(a) $E'(6) \approx \dfrac{E(7) - E(5)}{7 - 5} = 4$ hundred entries per hour	1 : answer
(b) $\dfrac{1}{8}\int_0^8 E(t)\,dt \approx$ $\dfrac{1}{8}\left(2\cdot\dfrac{E(0)+E(2)}{2} + 3\cdot\dfrac{E(2)+E(5)}{2} + 2\cdot\dfrac{E(5)+E(7)}{2} + 1\cdot\dfrac{E(7)+E(8)}{2}\right)$ $= 10.687$ or 10.688 $\dfrac{1}{8}\int_0^8 E(t)\,dt$ is the average number of hundreds of entries in the box between noon and 8 P.M.	3 : { 1 : trapezoidal sum; 1 : approximation; 1 : meaning }
(c) $23 - \int_8^{12} P(t)\,dt = 23 - 16 = 7$ hundred entries	2 : { 1 : integral; 1 : answer }
(d) $P'(t) = 0$ when $t = 9.183503$ and $t = 10.816497$.	3 : { 1 : considers $P'(t) = 0$; 1 : identifies candidates; 1 : answer with justification }

t	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time $t = 12$.

2013 AB3-BC3 (**5.55**)

(a) $C'(3.5) \approx \dfrac{C(4)-C(3)}{4-3} = \dfrac{12.8-11.2}{1} = 1.6$ ounces/min	$2: \begin{cases} 1: \text{approximation} \\ 1: \text{units} \end{cases}$
(b) C is differentiable $\Rightarrow$ C is continuous (on the closed interval) $\dfrac{C(4)-C(2)}{4-2} = \dfrac{12.8-8.8}{2} = 2$ Therefore, by the Mean Value Theorem, there is at least one time t, $2 < t < 4$, for which $C'(t) = 2$.	$2: \begin{cases} 1: \dfrac{C(4)-C(2)}{4-2} \\ 1: \text{conclusion, using MVT} \end{cases}$
(c) $\frac{1}{6}\int_0^6 C(t)\,dt \approx \frac{1}{6}[2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$ $= \frac{1}{6}(2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$ $= \frac{1}{6}(60.6) = 10.1$ ounces $\frac{1}{6}\int_0^6 C(t)\,dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \le t \le 6$ minutes.	$3: \begin{cases} 1: \text{midpoint sum} \\ 1: \text{approximation} \\ 1: \text{interpretation} \end{cases}$
(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$ $B'(5) = 6.4e^{-0.4(5)} = \dfrac{6.4}{e^2}$ ounces/min	$2: \begin{cases} 1: B'(t) \\ 1: B'(5) \end{cases}$