

Scoring Guidelines

2006 AB2-BC2 (5.93)

(a) $\int_0^{18} L(t) dt \approx 1658$ cars

$$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$$

(b) $L(t) = 150$ when $t = 12.42831, 16.12166$

Let $R = 12.42831$ and $S = 16.12166$

$L(t) \geq 150$ for t in the interval $[R, S]$

$$\frac{1}{S - R} \int_R^S L(t) dt = 199.426 \text{ cars per hour}$$

$$3 : \begin{cases} 1 : t\text{-interval when } L(t) \geq 150 \\ 1 : \text{average value integral} \\ 1 : \text{answer with units} \end{cases}$$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$$\int_{13}^{15} L(t) dt = 431.931 > 400$$

$$4 : \begin{cases} 1 : \text{considers 400 cars} \\ 1 : \text{valid interval } [h, h + 2] \\ 1 : \text{value of } \int_h^{h+2} L(t) dt \\ 1 : \text{answer and explanation} \end{cases}$$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \geq 200$ on that interval.

$L(t) \geq 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

OR

$$4 : \begin{cases} 1 : \text{considers 200 cars per hour} \\ 1 : \text{solves } L(t) \geq 200 \\ 1 : \text{discusses 2 hour interval} \\ 1 : \text{answer and explanation} \end{cases}$$

Yes, a traffic signal is required.

(a) $\int_0^7 f(t) dt \approx 8264$ gallons

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

(b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

$$2 : \begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0, 3$, and 7 .

$$5 : \begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$$

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

(a) $\int_0^6 f(t) dt = 142.274$ or 142.275 cubic feet

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

1 : answer

(c) $h(0) = 0$

For $0 < t \leq 6$, $h(t) = h(0) + \int_0^t g(s) ds = 0 + \int_0^t 0 ds = 0$.

For $6 < t \leq 7$, $h(t) = h(6) + \int_6^t g(s) ds = 0 + \int_6^t 125 ds = 125(t - 6)$.

For $7 < t \leq 9$, $h(t) = h(7) + \int_7^t g(s) ds = 125 + \int_7^t 108 ds = 125 + 108(t - 7)$.

$$\text{Thus, } h(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 6 \\ 125(t - 6) & \text{for } 6 < t \leq 7 \\ 125 + 108(t - 7) & \text{for } 7 < t \leq 9 \end{cases}$$

3 : $\begin{cases} 1 : h(t) \text{ for } 0 \leq t \leq 6 \\ 1 : h(t) \text{ for } 6 < t \leq 7 \\ 1 : h(t) \text{ for } 7 < t \leq 9 \end{cases}$

(d) Amount of snow is $\int_0^9 f(t) dt - h(9) = 26.334$ or 26.335 cubic feet.

3 : $\begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

$$(a) \quad H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$$

$$= \frac{52 - 60}{3} = -2.666 \text{ or } -2.667 \text{ degrees Celsius per minute}$$

1 : answer

(b) $\frac{1}{10} \int_0^{10} H(t) dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left(2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

3 : $\left\{ \begin{array}{l} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{array} \right.$

$$(c) \quad \int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : $\left\{ \begin{array}{l} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{array} \right.$

$$(d) \quad B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275; \quad H(10) - B(10) = 8.817$$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : $\left\{ \begin{array}{l} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{array} \right.$

Score _____