AP Calculus AB

Unit 3 Review

1) $f(x) = 2x^2+1$

$$f'(x) = \lim_{h\to 0} \frac{2(x+h)^2+1-(2x^2+1)}{h}$$

$$= \lim_{h\to 0} \frac{2(x^2+2xh+h^2)+1-2x^2-1}{h}$$

$$= \lim_{h\to 0} \frac{2x^2+4xh+2h^2+1-2x^2-1}{h}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2}{h}$$

$$= \lim_{h\to 0} (4x+2h)$$

$$\boxed{f'(x) = 4x}$$

2) $y = \dfrac{\boxed{2-x}}{3x+1}$

$$y' = \frac{(3x+1)\, d\boxed{2-x} - \boxed{2-x}\, d(3x+1)}{(3x+1)^2}$$

$$y' = \frac{(3x+1)(-1)-(2-x)(3)}{(3x+1)^2}$$

$$y' = \frac{-3x-1-6+3x}{(3x+1)^2}$$

$$y' = \frac{-7}{(3x+1)^2}$$

3) $y = \dfrac{2}{(5x+1)^3}$

$$y = 2(5x+1)^{-3}$$

$$y' = -6\boxed{5x+1}^{-4} d\boxed{5x+1}$$

$$y' = -6(5x+1)^{-4}(5)$$

$$y' = \frac{-30}{(5x+1)^4}$$

4) $f(x) = \dfrac{x}{\sqrt{1-x^2}}$

$$f'(x) = \frac{(1-x^2)^{1/2}\, d\boxed{x} - \boxed{x}\, d(1-x^2)^{1/2}}{\cancel{((1-x^2)^{1/2})^2}}$$

$$f'(x) = \frac{(1-x^2)^{1/2}\cdot(1) - x\left(\frac{1}{2}\boxed{1-x^2}^{-1/2} d\boxed{1-x^2}\right)}{1-x^2}$$

$$= \frac{(1-x^2)^{1/2} - \frac{1}{2}x(1-x^2)^{-1/2}(-2x)}{1-x^2}$$

$$= \frac{(1-x^2)^{1/2} + x^2(1-x^2)^{-1/2}}{1-x^2}$$

$$= \frac{(1-x^2)^{1/2} + \dfrac{x^2}{(1-x^2)^{1/2}}}{1-x^2}$$

$$= \frac{\dfrac{1-x^2+x^2}{(1-x^2)^{1/2}}}{1-x^2}$$

$$= \frac{1}{(1-x^2)^{3/2}}$$

5) $f(x) = (x^2-2)(x^{-1}+2)$

$$f'(x) = \boxed{x^2-2}\, d(x^{-1}+2) + (x^{-1}+2)\, d\boxed{x^2-2}$$

$$= (x^2-2)(-x^{-2}) + (x^{-1}+2)(2x)$$

$$= -1+2x^{-2}+2+4x$$

$$= 4x+2x^{-2}+1$$

6) $y = 3x^2 + \frac{2}{x} - \frac{5}{x^2}$

$y = 3x^2 + 2x^{-1} - 5x^{-2}$

$y' = 6x - 2x^{-2} - 10x^{-3}$

$y' = 6x - \frac{2}{x^2} - \frac{10}{x^3}$

7) $y = 2x^3 - 3x^2 - 12x + 20$

$y' = 6x^2 - 6x - 12 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$y(2) = 0 \quad y(-1) = 27$

$(2, 0) \quad (-1, 27)$

8) $f(x) = 2x^3 + x^2 - 1$

$f'(x) = 6x^2 + 2x$

$f'(1/2) = \frac{3}{2} + 1 = \frac{5}{2}$

slope of normal : $-\frac{2}{5}$

9) $y = \sqrt{x^3+1} = (x^3+1)^{1/2}$

point

$(2, 3)$

slope

$y' = \frac{1}{2}(x^3+1)^{-1/2} \cdot d(x^3+1)$

$y' = \frac{1}{2}(x^3+1)^{-1/2}(3x^2)$

$y'(2) = \frac{1}{2}\left(\frac{1}{3}\right)(12) = 2$

Tangent: $y - 3 =$

10) $y = \boxed{x^2+1}(x^3+1)$

$y' = \boxed{x^2+1}\, d\,(x^3+1) + (x^3+1)\, d\, \boxed{x^2+1}$

$y' = (x^2+1)(3x^2) + (x^3+1)(2x)$

$y'(-1) = (2)(3) + (0)$

$y'(-1) = 6$

11) $f(x) = x^2 + 2x - 3$

Tangent : $2x - y = 3$

$-y = -2x + 3$

$y = 2x - 3$

$f'(x) = 2x + 2$

$2x + 2 = 2$ ← slope

$x = 0$

$\boxed{(0, -3)}$

12) $f(x) = x^3 + 1$

Tangent : $y = 3x - 1$

$f'(x) = 3x^2$

$3x^2 = 3$ ← slope

$\boxed{x = \pm 1}$

13) $h(x) = f(x) + g(x)$

$h'(x) = f'(x) + g'(x)$

$h'(2) = f'(2) + g'(2)$

$= 1 + \frac{3}{2}$

$= \boxed{\frac{5}{2}}$

14) $d(x) = f(x) - g(x)$

$d'(x) = f'(x) - g'(x)$

$d'(3) = f'(3) - g'(3)$

$= 1 - \left(-\frac{1}{2}\right)$

$= \boxed{\frac{3}{2}}$

15) $p(x) = f(x) \cdot g(x)$

$p'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

$p'(3) = f(3) \cdot g'(3) + g(3) \cdot f'(3)$

$p'(3) = (3)(-\frac{1}{2}) + (5)(1)$

$= \boxed{\frac{7}{2}}$

16) $q(x) = \frac{g(x)}{f(x)}$

$q'(x) = \frac{f(x)\,g'(x) - g(x)\,f'(x)}{[f(x)]^2}$

$q'(3) = \frac{f(3)g'(3) - g(3)f'(3)}{[f(3)]^2}$

$q'(3) = \frac{(3)(-\frac{1}{2}) - (5)(1)}{[9]}$

$= \frac{-\frac{3}{2} - 5}{9} = \boxed{-\frac{13}{18}}$

17) $k(x) = (f(x))^2$

$k'(x) = 2\boxed{f(x)} \cdot d\boxed{f(x)}$

$= 2f(x) \cdot f'(x)$

$k'(1) = 2f(1) \cdot f'(1)$

$= 2(1)(1)$

$= \boxed{2}$

18) $c(x) = f(g(x))$

$c'(x) = f'(g(x)) \cdot g'(x)$

$c'(1) = f'(g(1)) \cdot g'(1)$

$= f'(2) \cdot (2)$

$= (1)(2)$

$= \boxed{2}$