

AP Calculus AB

Differentiability

- 1) a) $f(x)$ is continuous on $(-\infty, -1), (1, \infty)$
 b) $f(x)$ is differentiable on $(-\infty, -1), (-1, \infty)$
- 2) a) $f(x)$ is continuous on $(-\infty, \infty)$.
 b) $f(x)$ is differentiable on $(-\infty, -3), (-3, 3), (3, \infty)$.
- 3) a) $f(x)$ is continuous on $(-\infty, -2), (-2, 2), (2, \infty)$
 b) $f(x)$ is differentiable on $(-\infty, -2), (-2, 2), (2, \infty)$
- 4) a) $f(x)$ is cont. on $(-\infty, \infty)$
 b) $f(x)$ is diff. on $(-\infty, 3), (3, \infty)$
- 5) a) $f(x)$ is cont. on $[1, \infty)$
 b) $f(x)$ is diff. on $[1, \infty)$
- 6) a) $f(x)$ is cont on $(-\infty, 0), (0, \infty)$
 b) $f(x)$ is diff on $(-\infty, 0), (0, \infty)$

7)
$$f(x) = \begin{cases} 1, & x \leq 1 \\ x^2, & x > 1 \end{cases}$$
check continuity

check diff.

- I. $f(1) = 1$

- II. $\lim_{x \rightarrow 1^-} f(x) = 1 = \lim_{x \rightarrow 1^+} f(x) = 1$
 $\lim_{x \rightarrow 1} f(x) = 1$

Left
 $\frac{d}{dx} [1] = 0$

Right
 $\frac{d}{dx} [x^2] \Big|_{x=1}$
 $= 2x \Big|_{x=1}$
 $= 2$

- III. $f(1) = \lim_{x \rightarrow 1} f(x) = 1$

$0 \neq 2$

$f(x)$ is not differentiable at $x = 1$

$$f'(x) = \begin{cases} 0 & x < 1 \\ 2x & x > 1 \end{cases}$$

$$8) f(x) = \begin{cases} x^2 - 1, & x \leq 2 \\ 4x - 5, & x > 2 \end{cases}$$

continuity

I. $f(2) = 3$

II. $\lim_{x \rightarrow 2^-} f(x) = 3$ $\lim_{x \rightarrow 2^+} f(x) = 3$

$\lim_{x \rightarrow 2} f(x) = 3$

III. $f(2) = \lim_{x \rightarrow 2} f(x)$

Left $\frac{d}{dx} [x^2 - 1] \Big|_{x=2} = 2x \Big|_{x=2} = 4$

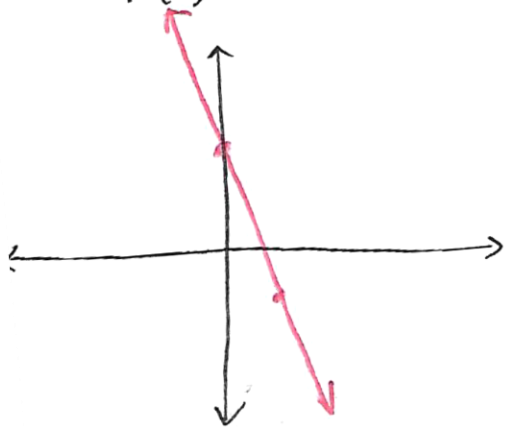
Right $\frac{d}{dx} [4x - 5] \Big|_{x=2} = 4$

$4 = 4$

$f(x)$ is diff. at $x=2$.

$$f'(x) = \begin{cases} 2x & x \leq 2 \\ 4 & x > 2 \end{cases}$$

9) $f(0) = 2$
 $f'(x) = -3$ for $-\infty < x < \infty$



10) $f(0) = 4$
 $f'(0)$ is und.
 $f'(x) < 0$ for $x < 0$
 $f'(x) > 0$ for $x > 0$

