1)

$f(x) = 2x+3$

$\lim_{h\to 0} \frac{2(x+h)+3-(2x+3)}{h}$

$\lim_{h\to 0} \frac{\cancel{2x}+2h+\cancel{3}-\cancel{2x}-\cancel{3}}{h}$

$\lim_{h\to 0} \frac{2h}{h}$

$\lim_{h\to 0} 2 = \boxed{2}$

2)

$f(x) = x^2-4x$

$\lim_{\Delta x\to 0} \frac{(x+\Delta x)^2-4(x+\Delta x)-(x^2-4x)}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\cancel{x^2}+2x\Delta x+(\Delta x)^2-\cancel{4x}-4\Delta x-\cancel{x^2}+\cancel{4x}}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{2x\Delta x+(\Delta x)^2-4\Delta x}{\Delta x}$

$\lim_{\Delta x\to 0} (2x+\Delta x-4)$

$= \boxed{2x-4}$

3) $f(x) = \frac{4}{x}$

$\lim_{\Delta x\to 0} \frac{\frac{4}{x+\Delta x}-\frac{4}{x}}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\frac{4x-4(x+\Delta x)}{x(x+\Delta x)}}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{\frac{\cancel{4x}-\cancel{4x}-4\Delta x}{x(x+\Delta x)}}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{-4\Delta x}{x(x+\Delta x)} \cdot \frac{1}{\Delta x}$

$\lim_{\Delta x\to 0} \frac{-4}{x(x+\Delta x)} = \boxed{-\frac{4}{x^2}}$

4)

$f(x) = \sqrt{x}$

$\lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$

$\lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}$

$\lim_{h\to 0} \frac{\cancel{x}+h-\cancel{x}}{h(\sqrt{x+h}+\sqrt{x})}$

$\lim_{h\to 0} \frac{h}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}}$

$= \boxed{\frac{1}{2\sqrt{x}}}$

5) $\lim_{x\to 2} f(x) = 2$

6) $\lim_{x\to -3} f(x)$ DNE

7) $\lim_{x\to c} f(x)$ exists for $(-\infty,-3)\cup(-3,\infty)$

8) $\lim_{x \to 1} \frac{x^2+3x+2}{x-1}$

$= \frac{6}{0}$

$\boxed{\text{DNE}}$

9) $\lim_{x \to 5} \frac{x-5}{x+1}$

$= \frac{0}{6}$

$= \boxed{0}$

10) $\lim_{x \to 2} \frac{x-3}{x^2-25}$

$= \frac{-1}{-21} = \boxed{\frac{1}{21}}$

11) $\lim_{x \to \frac{\pi}{6}} (\sin^2 x)$

$= \boxed{\frac{1}{4}}$

12) $\lim_{x \to -1} f(x)$

$\lim_{x \to -1} 1 = 1 \qquad \lim_{x \to -1} (x^2+2x+2) = 1$

By The Squeeze Theorem, $\lim_{x \to -1} f(x) = 1$

13) $\lim_{x \to 1} f(x)$

$\lim_{x \to 1} (-3\cos \pi x) = 3 \qquad \lim_{x \to 1} (x^3+2) = 3$

By Squeeze Th. $\lim_{x \to 1} f(x) = 3$

14) $\lim_{x \to 0} g(x)$

$\lim_{x \to 0} (2-x^2) = 2 \qquad \lim_{x \to 0} (2\cos x) = 2$

By Squeeze Th., $\lim_{x \to 0} g(x) = 2$

15) $\lim_{x \to 4} \frac{f(x)-5}{x-2} = 1$

$\frac{\lim_{x \to 4} f(x) - 5}{4-2} = 1$

$\lim_{x \to 4} f(x) - 5 = 2$

$\boxed{\lim_{x \to 4} f(x) = 7}$