WS 10

Evaluating Limits
Indeterminate Forms

1) $\lim_{x \to 1} (12x^3 + x^2 - 1) = \boxed{12}$

2) $\lim_{x \to 5} \frac{x+1}{x+2} = \boxed{\frac{6}{7}}$

3) $\lim_{x \to 4} \frac{x^2 + 5x + 4}{x+2} = \boxed{\frac{40}{6}}$

4) $\lim_{x \to 0} \frac{x^2 - 2x}{x}$

$\lim_{x \to 0} (x^2 - 2x) = 0 \quad \lim_{x \to 0} x = 0$

$\lim_{x \to 0} \frac{x(x-2)}{x}$

$\lim_{x \to 0} (x-2) = \boxed{-2}$

5) $\lim_{x \to 4} \frac{4-x}{2-\sqrt{x}}$

$\lim_{x \to 4} (4-x) = 0 \quad \lim_{x \to 4} (2-\sqrt{x}) = 0$

$\lim_{x \to 4} \frac{(4-x)}{(2-\sqrt{x})} \cdot \frac{(2+\sqrt{x})}{(2+\sqrt{x})}$

$\lim_{x \to 4} \frac{(4-x)(2+\sqrt{x})}{4-x}$

$\lim_{x \to 4} (2+\sqrt{x}) = \boxed{4}$

6) $\lim_{x \to \frac{\pi}{4}} (\sin^2 x + \cos^2 x)$

$\left[\lim_{x \to \frac{\pi}{4}} \sin x\right]^2 + \left[\lim_{x \to \frac{\pi}{4}} \cos x\right]^2$

$\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2} + \frac{1}{2} = \boxed{1}$

7) $\lim_{x \to -1} \frac{2x^2 - x - 3}{x+1}$

$\lim_{x \to -1} (2x^2 - x - 3) = 0 \quad \lim_{x \to -1} (x+1) = 0$

$\lim_{x \to -1} \frac{(2x-3)(x+1)}{x+1}$

$\lim_{x \to -1} (2x-3) = \boxed{-5}$

8) $\lim_{x \to 3} \frac{\sqrt{x+13} - 4}{x-3}$

$\lim_{x \to 3} (\sqrt{x+13} - 4) = 0 \quad \lim_{x \to 3} (x-3) = 0$

$\lim_{x \to 3} \frac{(\sqrt{x+13} - 4)}{(x-3)} \frac{(\sqrt{x+13} + 4)}{(\sqrt{x+13} + 4)}$

$\lim_{x \to 3} \frac{x-3}{(x-3)(\sqrt{x+13} + 4)} = \lim_{x \to 3} \frac{1}{\sqrt{x+13} + 4}$

$= \boxed{\frac{1}{8}}$

9) $\lim_{x \to -3} \frac{2x+1}{x+3}$ DNE

$\to \frac{-5}{0}$

10) $\lim_{y \to -8} \frac{y^2 + 9y + 8}{y^2 - 64}$

$\lim_{y \to -8} (y^2 + 9y + 8) = 0 \quad \lim_{y \to -8} (y^2 - 64) = 0$

$\lim_{y \to -8} \frac{(y+8)(y+1)}{(y+8)(y-8)} = \lim_{y \to -8} \frac{(y+1)}{y-8} = \frac{7}{16}$

11) $\lim_{x \to 10} \frac{x^2 - 100}{x-10}$

$\lim_{x \to 10} (x^2 - 100) = 0 \quad \lim_{x \to 10} (x-10) = 0$

$\lim_{x \to 10} \frac{(x-10)(x+10)}{x-10} = \lim_{x \to 10} (x+10) = 20$

12) $\lim_{h\to 0} \frac{(9+h)^2-81}{h}$

$\lim_{h\to 0}[(9+h)^2-81]=0 \quad \lim_{h\to 0} h=0$

$\lim_{h\to 0} \frac{81+18h+h^2-81}{h}$

$\lim_{h\to 0} \frac{18h+h^2}{h} = \lim_{h\to 0}(18+h)=18$

13) $f(x)=\begin{cases} 2x+1, & x<1 \\ x-3, & x\geq 1 \end{cases}$

$\lim_{x\to 1^-}(2x+1)=3 \neq \lim_{x\to 1^+}(x-3)=-2$

$\lim_{x\to 1} f(x)$ DNE

14) $\lim_{x\to 3} 4[f(x)-2g(x)]$

$4\left[\lim_{x\to 3} f(x) - 2\lim_{x\to 3} g(x)\right]$

$4[-5 \ -2(2)] = \boxed{-36}$

15) $f(x)=\begin{cases} x^2-ax & x\leq 2 \\ 3x+6 & x>2 \end{cases}$

$\lim_{x\to 2^-}(x^2-ax) = \lim_{x\to 2^+}(3x+6)$

$4-2a = 12$

$-2a = 8$

$\boxed{a=-4}$

16) $f(x)=\begin{cases} x^2-4x & x<-1 \\ ax^3-2 & x>-1 \end{cases}$

$\lim_{x\to -1^-}(x^2-4x) = \lim_{x\to -1^+}(ax^3-2)$

$5 = -a-2$

$\boxed{a=-7}$

17) $\lim_{x\to 7} \frac{f(x)+9}{x-7} = 5$

$\lim_{x\to 7}(f(x)+9)=0$

$\lim_{x\to 7} f(x) = -9$

18) $\lim_{x\to 4} \frac{f(x)-7}{x-4} = 6$

$\lim_{x\to 4}(f(x)-7)=0$

$\lim_{x\to 4} f(x) = 7.$