AP Calculus AB – Unit 2 Outline – Limits and Continuity

Friday 8/25	**Today's Topic**: **Introduction to Limits:** Finding Limits Graphically and Numerically

Define: Limit – The value that a function **approaches** as the input approaches a value from both sides.

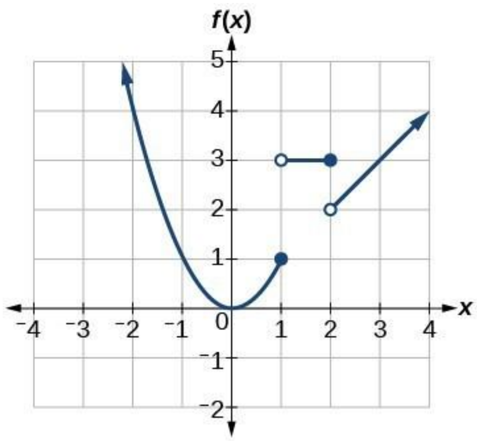
In-class examples:

Ex. 1 For each of the following, what is $f(x)$ approaching as x approaches a value of 2?

a) $f(x) = x - 1$ b) $f(x) = \dfrac{x^2 - 3x + 2}{x - 2}$

Ex. 2 For $f(x) = \dfrac{|x|}{x}$, what is $f(x)$ approaching as x approaches a value of 0?

Use the graph of $f(x)$ shown below to find each value.



Ex. 3

a) $\lim_{x \to 0^-} f(x)$ b) $\lim_{x \to 0^+} f(x)$ c) $\lim_{x \to 0} f(x)$ d) $f(0)$

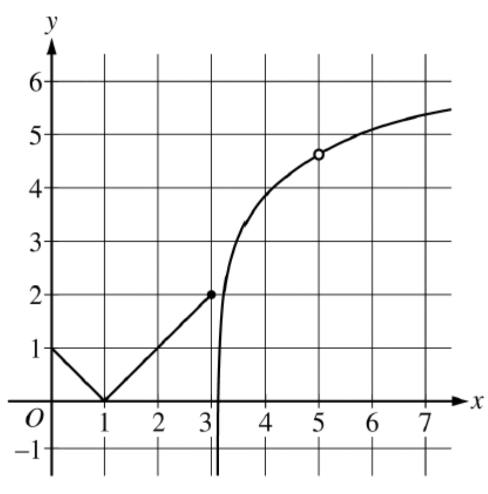
Ex. 4

a) $\lim_{x \to 1^-} f(x)$ b) $\lim_{x \to 1^+} f(x)$ c) $\lim_{x \to 1} f(x)$ d) $f(1)$

Ex. 5

a) $\lim_{x \to -2} f(x)$ b) $\lim_{x \to 2} f(x)$ c) $\lim_{x \to 3} f(x)$ d) $\lim_{x \to -\infty} f(x)$

AP Multiple Choice



Graph of f

The graph of a function f is shown above. Which of the following limits does not exist?

(A) $\lim_{x \to 1^-} f(x)$ (B) $\lim_{x \to 1} f(x)$ (C) $\lim_{x \to 3^-} f(x)$ (D) $\lim_{x \to 3} f(x)$ (E) $\lim_{x \to 5} f(x)$

Homework: Worksheet 7

Monday 8/28	**Today's Topic**: Failing Limits; Limit Laws

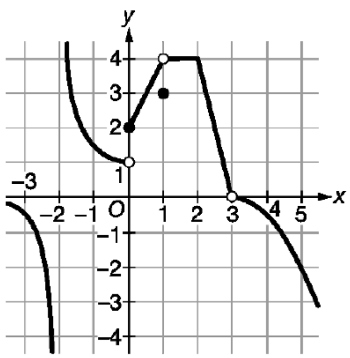
In-class examples:

Ex. 1 Identify the three ways in which a limit will fail to exist. Include a graph for each.

Ex. 2 Assume that $\lim_{x\to 4} f(x) = -1$, $\lim_{x\to 4} g(x) = 3$, and $\lim_{x\to 4} h(x) = 2$. Find

a. $\lim_{x\to 4}\left(2g(x)+3f(x)\right)$ b. $\lim_{x\to 4}\sqrt{h(x)+\left[g(x)\right]^2}$ c. $\lim_{x\to 4}\dfrac{g(x)-3}{f(x)+x}$

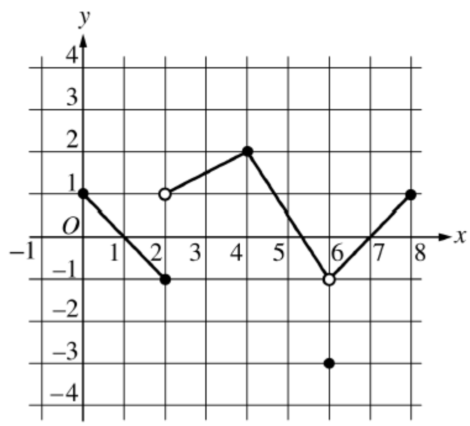
Ex. 3 Given the graph of $f(x)$ and a table for selected values of $g(x)$ below, estimate $\lim_{x\to 1}\left(3g(x)-2f(x)\right)$



Graph of f

x	0.8	0.9	0.09	0.009	1.001	1.01	1.1	1.2
$g(x)$	4.16	4.59	4.960	4.996	5.004	5.040	5.39	5.76

AP Multiple Choice



The figure above shows the graph of the function f. Which of the following statements are true?

I. $\lim_{x\to 2^-} f(x) = f(2)$

II. $\lim_{x\to 6^-} f(x) = \lim_{x\to 6^+} f(x)$

III. $\lim_{x\to 6} f(x) = f(6)$

(A) II only

(B) III only

(C) I and II only

(D) II and III only

(E) I, II, and III

Homework: Worksheet 8

Tuesday 8/29	**Today's Topic**: Algebraic Methods for finding Limits

In-class examples:

Ex. 1 Evaluate $\lim_{x\to 2}\left(4x^2+3\right)$ using limit laws.

Ex. 2 Evaluate $\lim_{x\to 0}\left(\cos x-4\right)$ using direct substitution.

Ex. 3 Evaluate the following limits:

a) $\lim_{x\to 0}\frac{x^2-3x+2}{x+1}$ b) $\lim_{x\to \pi} x\cos x$ c) $\lim_{x\to 2}\frac{x^2-4}{x+2}$ d) $\lim_{x\to 3}\frac{x+6}{x-3}$

Ex. 4 For $f\left(x\right)=\begin{cases}\frac{1}{2}x+1, & x\le 2\\ 3-x, & x>2\end{cases}$, evaluate $\lim_{x\to 2} f\left(x\right)$.

Ex. 5 For $f\left(x\right)=\begin{cases}x^2-5, & x<-3\\ 0 & x=-3\\ 8-x, & x>-3\end{cases}$, evaluate $\lim_{x\to -3} f\left(x\right)$.

AP Multiple Choice

If $f(x)=\begin{cases}\ln x & \text{for } 0<x\le 2\\ x^2\ln 2 & \text{for } 2<x\le 4,\end{cases}$ then $\lim_{x\to 2} f(x)$ is

(A) $\ln 2$ (B) $\ln 8$ (C) $\ln 16$ (D) 4 (E) nonexistent

Homework: Worksheet 9

Wednesday 8/30	**Today's Topic**: Algebraic Methods for Finding Limits; Indeterminate Forms

In-class examples:

Ex.1 Evaluate the following limits:

a) $\lim_{x\to 3}\frac{x-3}{x^2-9}$ b) $\lim_{x\to 2}\frac{x^2-5x+6}{x^2-7x+10}$

c) $\lim_{x\to 3}\frac{x-3}{\sqrt{x+1}-2}$ d) $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$

AP Multiple Choice:

$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$ is

(A) $-\frac{1}{4}$ (B) 0 (C) 1 (D) $\frac{5}{4}$ (E) nonexistent

Homework: Worksheet 10

Thursday 8/31	**Today's Topic**: Limits involving the Difference Quotient; The Squeeze Theorem

Warm-Up: Evaluate $\dfrac{f(x+h)-f(x)}{h}$ for a) $f(x)=4x-1$ b) $f(x)=x^2-2x+1$

In-class examples:

Ex. 1 Evaluate $\lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$ for a) $f(x)=3x+2$ b) $f(x)=x^2+4x+5$

The Squeeze Theorem

Ex. 2 If $-3\cos(\pi x)\le f(x)\le x^3+2$, evaluate $\lim\limits_{x\to 1} f(x)$. Justify your answer.

AP Multiple Choice

If $a \neq 0$, then $\lim\limits_{x\to a}\dfrac{x^2-a^2}{x^4-a^4}$ is

(A) $\dfrac{1}{a^2}$ (B) $\dfrac{1}{2a^2}$ (C) $\dfrac{1}{6a^2}$ (D) 0 (E) nonexistent

Homework: Worksheet 11

Friday 9/1	**Today's Topic**: Limits Quiz #1

In-class examples: None

Homework: None

Tuesday 9/5	**Today's Topic**: Limits at Infinity (Principle of Dominance); End Behavior; Definition of a Horizontal Asymptote.

In-class examples

Ex. 1 Find the limit: $\lim\limits_{x\to\infty}\dfrac{2x-1}{x+1}$

Ex. 2 Find the limit: $\lim\limits_{x\to\infty}\left(5-\dfrac{2}{x^2}\right)$

Ex. 3 Find the limit: $\lim\limits_{x\to\infty}\dfrac{3x^3+2x^2-5}{3x^2-4x+1}$

Ex. 4 Find the limit: $\lim\limits_{x\to\infty}\dfrac{\sin x}{3x^2+1}$

Ex. 5 Find the limit: $\lim\limits_{x\to\infty}\dfrac{e^x}{3x^{20}+1}$

Ex. 6 Find the limit: $\lim\limits_{x\to\infty}\dfrac{3x-2}{\sqrt{2x^2+1}}$

Ex. 7 Find the limit: $\lim\limits_{x\to-\infty}\dfrac{\ln x}{e^x}$

Ex. 8 Find the limit: $\lim\limits_{x\to\infty} x\cos\dfrac{1}{x}$

AP Multiple Choice

$\lim\limits_{x\to\infty}\dfrac{\sqrt{9x^4+1}}{4x^2+3}$ is

(A) $\dfrac{1}{3}$ (B) $\dfrac{3}{4}$ (C) $\dfrac{3}{2}$ (D) $\dfrac{9}{4}$ (E) infinite

Homework: Worksheet 12

Wednesday 9/6	**Today's Topic**: Special Trig Limits

In-class examples:

Ex. 1 Use a graphing calculator to determine: $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$ **Ex. 2** $\lim\limits_{x\to 0}\dfrac{\sin 3x}{x}$

Ex. 3 Use a graphing calculator to determine: $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}$ **Ex. 4** $\lim\limits_{x\to 0}\dfrac{1-\cos 7x}{x}$

Homework: Find each limit:

1. $\lim\limits_{x\to 0}\dfrac{\sin \frac{1}{2}x}{x}$	2. $\lim\limits_{x\to 0} x\sec x$	3. $\lim\limits_{x\to 0}\dfrac{\sin x}{2x}$	4. $\lim\limits_{x\to 0}\dfrac{3\sin x}{x}$
5. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{5x}$	6. $\lim\limits_{x\to 0}\dfrac{\sin^2 2x}{3x^2}$	7. $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$	8. $\lim\limits_{x\to 0}\dfrac{1-\cos(2x)}{2x}$
9. $\lim\limits_{x\to 0}\dfrac{x+2}{\cos x}$	10. $\lim\limits_{x\to \frac{\pi}{4}} \tan x$	11. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{1-\cos x}$ (Hint: Use an identity)	12. $\lim\limits_{x\to 0}\dfrac{3x}{\sin x}$

Thursday 9/7	**Today's Topic**: 3 Part Definition of Continuity

In-class examples:

Ex. 1 Show that $f(x)=x^2+3x+2$ is continuous at $x=1$ using the 3-part definition of continuity.

Ex. 2 Determine if $f(x)=\dfrac{1}{x-2}$ is continuous at $x=2$ using the 3-part definition of continuity.

Ex. 3 For $f(x)=\begin{cases}\sqrt{x+1}, & 0\le x\le 3\\ 5-x, & 3<x\le 5\end{cases}$, determine if $f(x)$ is continuous at $x=3$.

AP Multiple Choice

Let f be the function given by $f(x)=\dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only

Homework: Worksheet 13

Friday 9/8	**Today's Topic:** Removable discontinuities

In-class examples:

Ex. 1 Sketch the graph of any function *f* such that, $\lim_{x\to 2} f(x)=1$ and $f(2)=5$.

Ex. 2

a) Determine the *x*-coordinates of any discontinuities on the graph of $f(x)=\dfrac{x-3}{x^2-9}$.

b) Identify the discontinuities as either **infinite** or **removable.**

c) Evaluate the limit at each **removable** discontinuity.

d) State the interval(s) on which $f(x)$ is continuous.

e) Write an extended function of $g(x)$ that is continuous.

Ex. 3

a) Determine the *x*-coordinates of any discontinuities on the graph of $g(x)=\dfrac{x^2-3x+2}{x^2-4}$.

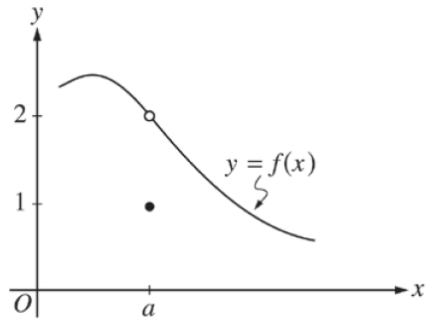
b) Identify the discontinuities as either **infinite** or **removable**.

c) Evaluate the limit at each **removable** discontinuity.

d) State the interval(s) on which $f(x)$ is continuous.

e) Write an extended function of $g(x)$ that is continuous.

AP Multiple Choice



The graph of a function f is shown in the figure above. Which of the following statements is true?

(A) $f(a) = 2$

(B) f is continuous at $x = a$.

(C) $\lim_{x\to a} f(x) = 1$

(D) $\lim_{x\to a} f(x) = 2$

(E) $\lim_{x\to a} f(x)$ does not exist.

Homework: Worksheet 14

Monday 9/11	**Today's Topic**: Continuity; Infinite Discontinuities

In-class Examples:

Ex. 1 Sketch the graph of any function *f* such that, $\lim_{x\to 2^-} f(x) = \infty$ and $\lim_{x\to 2^+} f(x) = -\infty$.

Ex. 2

Recall Example 2 from Yesterday:

f) Identify any vertical asymptotes on the graph of $f(x)$. Use limits to describe the behavior of $f(x)$ on the left and right of the vertical asymptote.

Ex. 3

Recall Example 3 from Yesterday:

f) Identify any vertical asymptotes on the graph of $f(x)$. Use limits to describe the behavior of $f(x)$ on the left and right of the vertical asymptote.

AP Multiple Choice

The vertical line $x = 2$ is an asymptote for the graph of the function *f*. Which of the following statements must be false?

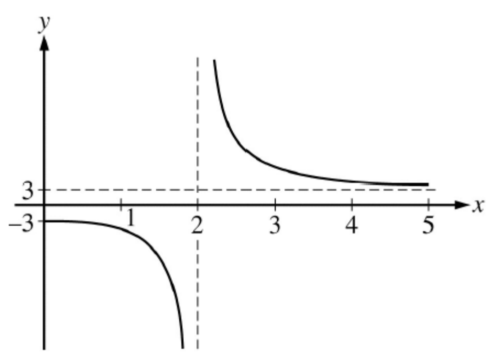
(A) $\lim_{x\to 2} f(x) = 0$

(B) $\lim_{x\to 2} f(x) = -\infty$

(C) $\lim_{x\to 2} f(x) = \infty$

(D) $\lim_{x\to \infty} f(x) = 2$

(E) $\lim_{x\to \infty} f(x) = \infty$



The function *f* is given by $f(x) = \dfrac{ax^2 + 12}{x^2 + b}$. The figure above shows a portion of the graph of *f*. Which of the following could be the values of the constants a and b?

(A) $a = -3,\ b = 2$

(B) $a = 2,\ b = -3$

(C) $a = 2,\ b = -2$

(D) $a = 3,\ b = -4$

(E) $a = 3,\ b = 4$

Homework: Worksheet 15

Tuesday 9/12	**Today's Topic**: Continuity; Jump Discontinuities; Review

In-class examples:

1) Sketch the graph of any function *f* such that, $f(3)=\lim_{x\to 3^+} f(x)=1$ and $\lim_{x\to 3^-} f(x)=0$.

Determine whether *f* is continuous at *c*: $f(x)=\begin{cases} x^2 & x<1 \\ 3x+2 & x\geq 1 \end{cases}$, $c=1$.

2) Determine the numbers at which the following function is continuous: $f(x)=\begin{cases} x^2 & \text{if } x\leq 0 \\ x+1 & \text{if } 0<x<2 \\ 5-x & \text{if } 2\leq x\leq 5 \end{cases}$

AP Multiple Choice

$$f(x)=\begin{cases} x^2-3x+9 & \text{for } x\leq 2 \\ kx+1 & \text{for } x>2 \end{cases}$$

The function f is defined above. For what value of k, if any, is f continuous at $x=2$?

(A) 1

(B) 2

(C) 3

(D) 7

(E) No value of k will make f continuous at $x=2$.

Homework: Worksheet 16

Wednesday 9/13	**Today's Topic**: Intermediate Value Theorem

In-Class Examples

Ex. 1 Use the Intermediate Value Theorem to show that the polynomial function $f(x)=x^3+2x-1$ has a zero in the interval $[0,1]$.

Ex. 2 If $f(x)=x^2+10\sin x$, show that there is a number *c* such that $f(c)=800$ in $[0,30]$.

AP Multiple Choice

Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2)=10$ and $f(4)=20$. Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f(x)=13$ has at least one solution in the open interval $(2, 4)$.

(B) $f(3)=15$

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f'(x)=5$ has at least one solution in the open interval $(2, 4)$.

(E) $f'(x)>0$ for all x in the open interval $(2, 4)$.

Homework: Worksheet 17

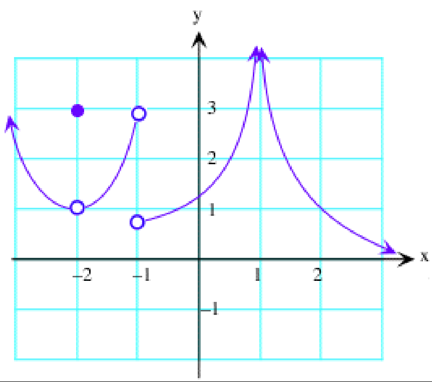
Thursday 9/14	**Today's Topic**: Limits/Continuity Review

In-class examples

Ex. 1 Let $f(x)=\begin{cases} x^2-a^2x, & x<2 \\ 4-2x^2, & x\geq 2 \end{cases}$. Find all values of *a* that make *f* continuous at $x=2$.

Ex. 2 Let $f(x)=\begin{cases} x^2\sin(\pi x), & x<2 \\ x^2-cx+18, & x\geq 2 \end{cases}$. Find all values of *a* that make *f* continuous at $x=2$.

Ex. 3



The graph of $f(x)$ is shown at left.

a) Determine the domain of $f(x)$.

b) Determine the values of *c* for which $\lim_{x\to c} f(x)$ exists.

c) Determine the values of *x* for which $f(x)$ is continuous.

Homework: Worksheet 18

Friday 9/15	**Today's Topic**: **Unit 2 Test - Limits/Continuity – Good Luck!!**

- Good luck on today's exam

Homework: None