# Geometry

These curriculum maps are designed to address Common Core State Standards (CCSS) Mathematics and Literacy outcomes. The overarching focus for all curriculum maps is building students' content knowledge focusing on their math practice abilities and literacy skills. Each unit provides several weeks of instruction. Each unit also includes various assessments. Taken as a whole, this curriculum map is designed to give teachers recommendations and some concrete strategies to address the shifts required by CCSS.

## Instructional Shifts in Mathematics

| | |
|---|---|
| **Focus:**<br><br>Focus strongly where the Standards focus | **Focus requires that we significantly narrow and deepen the scope of content in each grade so that students experience concepts at a deeper level.**<br>• Instruction engages students through cross-curricular concepts and application. Each unit focuses on implementation of the Math Practices in conjunction with math content.<br>• Effective instruction is framed by performance tasks that engage students and promote inquiry. The tasks are sequenced around a topic leading to the big idea and essential questions in order to provide a clear and explicit purpose for instruction. |
| **Coherence:**<br><br>Think across grades, and link to major topics within grades | **Coherence in our instruction supports students to make connections within and across grade levels.**<br>• Problems and activities connect clusters and domains through the art of questioning.<br>• A purposeful sequence of lessons build meaning by moving from concrete to abstract, with new learning built upon prior knowledge and connections made to previous learning.<br>• Coherence promotes mathematical sense making. It is critical to think across grades and examine the progressions in the standards to ensure the development of major topics over time. The emphasis on problem solving, reasoning and proof, communication, representation, and connections require students to build comprehension of mathematical concepts, procedural fluency, and productive disposition. |
| **Rigor:**<br><br>In major topics, pursue conceptual understanding, procedural skills and fluency, and application | **Rigor helps students to read various depths of knowledge by balancing conceptual understanding, procedural skills and fluency, and real-world applications with equal intensity.**<br>• Conceptual understanding underpins fluency; fluency is practiced in contextual applications; and applications build conceptual understanding.<br>• These elements may be explicitly addressed separately or at other times combined. Students demonstrate deep conceptual understanding of core math concepts by applying them in new situations, as well as writing and speaking about their understanding. Students will make meaning of content outside of math by applying math concepts to real-world situations.<br>• Each unit contains a balance of challenging, multiple-step problems to teach new mathematics, and exercises to practice mathematical skills |

# 8 Standards for Mathematical Practice

**The Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. They describe how students should learn the content standards, helping them to build agency in math and become college and career ready. **The Standards for Mathematical Practice are interwoven into every unit. Individual lessons may focus on one or more of the Math Practices, but every unit must include all eight.**

| | |
|---|---|
| **1. Make sense of problems and persevere in solving them** | Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches. |
| **2. Reason Abstractly and quantitatively** | Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects. |
| **3. Construct viable arguments and critique the reasoning of others** | Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. |

| | |
|---|---|
| **4. Model with mathematics** | Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose. |
| **5. Use appropriate tools strategically** | Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts. |
| **6. Attend to precision** | Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions. |
| **7. Look for and make use of structure** | Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as $2 \times 7$ and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$. |
| **8. Look for and express regularity in repeated reasoning** | Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. |

# English Language Development Standards

**The California English Language Development Standards** (CA ELD Standards) describe the key knowledge, skills, and abilities in core areas of English language development that students learning English as a new language need in order to access, engage with, and achieve in grade-level academic content, with particular alignment to the key knowledge, skills, and abilities for achieving college- and career-readiness. English Learners must have full access to high quality English language arts, mathematics, science, and social studies content, as well as other subjects, at the same time as they are progressing through the ELD level continuum. The CA ELD Standards are intended to support this dual endeavor by providing fewer, clearer, and higher standards. **The ELD Standards are interwoven into every unit.**

| **Interacting in Meaningful Ways** |
|---|
| **A. Collaborative** (engagement in dialogue with others) |
| 1. Exchanging information/ideas via oral communication and conversations |
| |
| **B. Interpretive** (comprehension and analysis of written and spoken texts) |
| 5. Listening actively and asking/answering questions about what was heard |
| 8. Analyzing how writers use vocabulary and other language resources |
| |
| **C. Productive** (creation of oral presentations and written texts) |
| 9. Expressing information and ideas in oral presentations |
| 11. Supporting opinions or justifying arguments and evaluating others' opinions or arguments |

## How to Read this Document

- The purpose of this document is to provide an overview of the progression of units of study within a particular grade level and subject describing what students will achieve by the end of the year. The work of **Big Ideas and Essential Questions** is to provide an overarching understanding of the mathematics structure that builds a foundation to support the rigor of subsequent grade levels. The **Performance Task** will assess student learning via complex mathematical situations. Each unit incorporates components of the **SAUSD Theoretical Framework** and the philosophy **of Quality Teaching for English Learners (QTEL).** Each of the math units of study highlights the Common Core instructional shifts for mathematics of focus, coherence, and rigor.

- The **8 Standards for Mathematical Practice** are the key shifts in the pedagogy of the classroom. These 8 practices are to be interwoven throughout every lesson and taken into consideration during planning. These, along with the **ELD Standards,** are to be foundational to daily practice.

- First, read the **Framework Description/Rationale** paragraph, as well as the **Common Core State Standards.** This describes the purpose for the unit and the connections with previous and subsequent units.

- The units show the progression of units drawn from various domains.

- The timeline tells the length of each unit and when each unit should begin and end.

# SAUSD Scope and Sequence for Geometry:

| Unit 1 | Unit 2 | Unit 3 | Unit 4 |
|---|---|---|---|
| 9/1/15 – 10/16/15<br>7 Weeks | 10/19/15 – 11/13/15<br>4 Weeks | 11/16/15 –12/18/15<br>4 Weeks | 1/4/16 – 1/22/16<br>3 Weeks |
| Constructions<br>Transformations | Congruence<br>(intro to Proofs) | Similarity | Trig and Right Triangles |

******SEMESTER******

| Unit 5 | Unit 6 | Unit 7 | Unit 8 | Enrichment |
|---|---|---|---|---|
| 2/1/16 – 3/11/16<br>6 Weeks | 3/14/16 – 4/1/16<br>3 Weeks | 4/11/16 – 4/29/16<br>3 Weeks | 5/2/16 – 5/27/16<br>4 Weeks | 6/30/16 – 6/16/16<br>3 Weeks |
| Area & Perimeter/ Expressing Geometric Properties With Equations | Circles (Arcs and Segments) | Geometric Measurement & Dimension | Probability<br>&<br>Statistics | Enrichment |

## Geometry Overview:

The fundamental purpose of the Geometry course is to introduce students to formal geometric proof and the study of plane figures, culminating in the study of right triangle trigonometry and circles. They begin to prove results about the geometry of the plane formally, by using previously defined terms and notions. Similarity is explored in greater detail, with an emphasis on discovering trigonometric relationships and solving problems with right triangles. The correspondence between the plane and the Cartesian coordinate system is explored when students connect algebraic concepts with geometric ones. Students explore probability concepts and use probability in real-world situations. The major mathematical ideas in the Geometry course include geometric transformations, proving geometric theorems, congruence and similarity, analytic geometry, right-triangle trigonometry, and probability.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). In the higher mathematics courses, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. In the standards for grades seven and eight, students began to see two-dimensional shapes as part of a generic plane (the Euclidean Plane) and began to explore transformations of this plane as a way to determine whether two shapes are congruent or similar. In the Geometry course, these notions are formalized and students use transformations to prove geometric theorems. The definition of congruence in terms of rigid motions provides a broad understanding of this notion, and students explore the consequences of this definition in terms of congruence criteria and proofs of geometric theorems.

Students investigate triangles and decide when they are similar; with this newfound knowledge and their prior understanding of proportional relationships, they define trigonometric ratios and solve problems using right triangles. They investigate circles and prove theorems about them. Connecting to their prior experience with the coordinate plane, they prove geometric theorems using coordinates and describe shapes with equations. Students extend their knowledge of area and volume formulas to those for circles, cylinders and other rounded shapes. Finally, continuing the development of statistics and probability, students investigate probability concepts in precise terms, including the independence of events and conditional probability.

(From the CA Mathematics Framework for Geometry)

# SAUSD Curriculum Map 2015-2016: Geometry

## Unit 1: **Constructions & Transformations**

*(7 weeks: 9/1 – 10/16)*

| **Big Idea** | Two figures are congruent if, through a sequence of rigid motions and/or constructions, one shape maps ***exactly*** onto another. |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • What are rigid motions and how can they be defined?<br>• What is congruence and how does it relate to rigid motions?<br>• What are some real-life applications of congruence? | Rigid Motion C2 2014 p.6-7<br>Transformer C2 2013 p.2-3<br>Flip Sliding Away C2 2012 p.2-3<br>Parallels C2 2001 p.3-4<br>A 30º Angle C2 2011 p.10-11 | Do the Tessellation and Teacher's Notes |

| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| Definitions of Key Vocabulary:<br>• Angles<br>• Circle<br>• Perpendicular/Parallel Lines<br>• Line Segment<br>Key Topics:<br>• Understand undefined notion of point, line and distance along a line<br>• Understand congruence in terms of rigid motions<br>• Make formal geometric constructions with a variety of tools and methods.<br>• Construct an equilateral triangle, square and regular hexagon inscribed in a circle.<br>• Represent Transformations in the plane.<br>• Describe transformations (reflection, rotation and translation) that carry a figure onto itself.<br>• Develop definitions (informal and precise) of reflection, rotation and translation.<br>• Draw the transformed figure using patty paper, geometric software, graph paper, etc.<br>• Specify a sequence of transformations that carry a given figure onto another.<br>• Predict the effect of a given rigid motion on a given figure.<br>• Decide if two figures are congruent through a sequence of rigid motions. | **Experiment with transformations in the plane**<br>**G-CO 1.** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.<br><br>**G-CO 2**. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).<br><br>**G-CO 3**. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.<br><br>**G-CO 4.** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.<br><br>**G-CO 5**. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.<br><br>**G- CO**<br>**Understand Congruence in terms of rigid motions**<br>**G-CO 6**. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.<br><br>**G-CO 7**. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | **Essential Resources:**<br>• **CCSS Geometry Framework pgs. 7, 9**<br>**Instructional Resources**<br>• **SAUSD Unit of Study:** Transformations Triangle Congruence<br>• **Text:** Ch. 4, section 1, 2, 3, 4 & 5 (Intro only); Ch. 12, section 1, 2, 3, 4; Constructions are introduced throughout text.<br>• EngageNY Module 1: Lessons 1-27, 31, 32<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.G-CO: Congruence" standards 1-8 and 12-13)<br>• Arizona College and Career Ready Standards (Constructions)<br>• Louisiana Dept. of Ed. Geometry – Year at a Glance (Transformation and Congruence)<br>• UCI NSF Focus Program<br>• Illustrative Math |

| | | |
|---|---|---|
| • Explain how the criteria for triangle congruence (SSS, SAS, ASA) follow from the definition of congruence in terms of rigid motions.<br>• Construct congruent segments and angles.<br>• Bisect segments and angles.<br>• Construct perpendicular and parallel lines.<br>• Construct regular polygons inscribed in a circle. | **G-CO 8**. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.<br><br>**G-CO**<br>**Make geometric constructions**<br>**G-CO 12**. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*<br><br>**G-CO 13**. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | Activities: Click on the hyperlinks labeled with "G-CO"<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard) |

# Unit 1: **Constructions & Transformations**

## *(Instructional Support & Strategies)*

### Framework Description/Rationale

Students will a) make geometric constructions (formalize and explain processes) using a variety of tools and methods, and b) experiment with transformations in the plane. Through this unit, the concept of rigid motions will be used to help students discover the definition of congruence (See CA Geometry Framework pgs. 7, 9 for more details).

**Example:** *Defining Rotations.* Mrs. B wants to help her class understand the following definition of a *rotation:*

> A *rotation* about a point $P$ through angle $\alpha$ is a transformation $A \longmapsto A'$ such that (1) if point $A$ is different from $P$, then $PA = PA'$ and the measure of $\angle APA' = \alpha$; and (2) if point $A$ is the same as point $P$, then $A' = A$.

She gives her students a handout with several geometric shapes on it and a point $P$ indicated on the page. In pairs, students are to copy the shapes onto a transparency sheet and rotate them through various angles about $P$. Students then transfer the rotated shapes back onto the original page, and measure various lenghts and angles as indicated in the definition.

While justfiying that the properties of the definition hold for the shapes she has given them, the students also make some observations about the effects of a rotation on the entire plane, for instance that:

- Rotations preserve lengths.
- Rotations preserve angle measures.
- Rotations preserve parallelism.

Later, Mrs. B plans to allow students to explore more rotations on dynamic geometry software, asking them to create a geometric shape and rotate it by various angles about various points $P$, both part of the object and not.

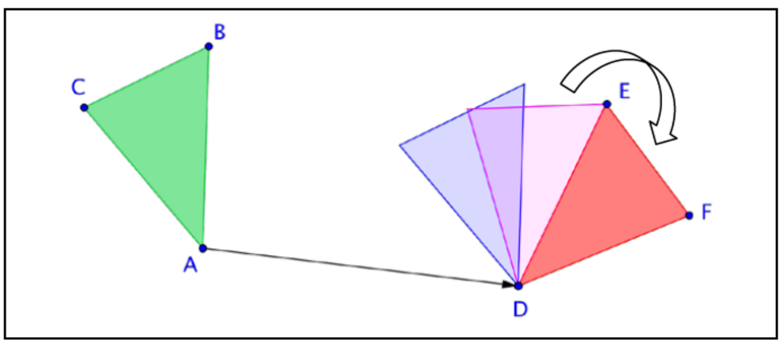



Figure 2: Illustration of the reasoning that corresponding parts being congruent implies triangle congruence, in which point *A* is translated to *D*, the resulting image of Δ*ABC* is rotated so as to place *B* onto *E*, and finally, the image is then reflected along line segment *DE* to match point *C* to *F*.

| Academic Language Support | Instructional Tool/Strategy Examples | Preparing the Learner |
|---|---|---|
| Key Terms for Word Bank:<br>• Equilateral Triangle<br>• Segment<br>• Arc Mark<br>• Angle<br>• Midpoint<br>• Degree<br>• Zero and Straight Angle<br>• Right Angle<br>• Perpendicular Lines<br>• Equidistant<br>• Perpendicular Bisector<br>• Medians<br>• Straight Angle | • Use patty paper to visualize changes occurring to a figure during transformations<br>• Implement Geometry software such as Geometer's Sketchpad, Geogebra<br>• Use of a graphing calculator app for solving triangles, coordinate graphing<br>• Have students try exploratory group work including strategies such as jigsaw groups, gallery walks, think-pair-share, collaborative conversations, etc.<br>• Use sets of applicable problems, students find patterns, form hypotheses, and eventually prove their hypotheses<br>• Use flowcharts, thinking maps, and other | Basic Understanding of:<br>• Reflection<br>• Rotation<br>• Translation<br>• Complementary<br>• Supplementary<br>• Vertical<br>• Adjacent<br>*(With the new standards these items are now a large focus in earlier grades; therefore the Geometry standards don't* |

| | | |
|---|---|---|
| • Vertical Angles<br>• Transversal<br>• Alternate Interior Angles<br>• Corresponding Angles<br>• Isosceles Triangle<br>• Angles of a Triangle<br>• Exterior Angles of a Triangle<br>• Rigid Motions<br>• Reflection<br>• Rotation<br>• Translation<br>• Compositions of Transformations<br>• Center/Angle of Rotation<br>• Rotational Symmetry<br>• Line of Reflection<br>• Pre-Image/Image<br>• Parallelogram<br>• Quadrilaterals | graphic organizers to promote connections between concepts (e.g. use a tree map to list the various types of angle relationships and their attributes, then create a double-bubble map comparing and contrasting them)<br>• Use of manipulatives such as spaghetti to show the Triangle Inequality theorem<br>• Use of paper triangles to show the Triangle Sum Theorem<br>• Use of written instructions, diagrams, and pair work for constructions to encourage independence and student agency | *directly address them. However, they are essential for the understanding of this unit, so for this transition time, it is important to address them).* |
| <u>Teacher Notes:</u> | | |

# Unit 2: **Congruence and Introduction to Proofs**

*(4 Weeks: 10/19 – 11/13)*

| **Big Idea** | Two figures are congruent if, through a sequence of rigid motions, one shape maps ***exactly*** onto another. |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • How can theorems help prove figures congruent?<br>• Which theorems exist for triangles and quadrilaterals? | Congruence C2 2000 p.6<br>Company Logo C2 2012 p.8-9<br>Dean's Triangles C2 2013 p.6-7<br>Wendy's Puzzle C2 2013 p.10-11<br>Writing Desk C2 2001 p.2<br>Pentagon C2 2009 p.84-85<br>Quadrilaterals C2 2005 p.14 | Between the Lines and Teacher's Notes |

| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| Write formal proofs:<br>• Congruent Triangles<br>• Vertical Angles are congruent<br>• Alternate Interior and Corresponding Angles are congruent<br>• Perpendicular Bisector and Equi-distance<br>• Triangle Sum Theorem<br>• Base angles of an Isosceles Triangle are Congruent<br>• Triangle Mid-Segment Theorem<br>• Pythagorean Theorem<br>• Medians of Triangles<br>• Parallelograms and properties | **G-CO: Congruence**<br>**Prove geometric theorems**<br>**G-CO 9**. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*<br><br>**G-CO 10.** Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*<br><br>**G-CO 11**. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* | **Essential Resources:**<br>• **CCSS Geometry Framework** pgs. 11<br><br>**Instructional Resources**<br>• **Resource Packet**<br>• EngageNY Module 1 Lessons 28, 29, 30, 33<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.G-CO: Congruence" standards 9-11; and the standards labeled H.G-SRT: Similarity, right triangles, and trigonometry 4-5)<br>• Illustrative Math Activities: Click on the hyperlinks labeled with "G-CO" or "G-SRT"<br>• Arizona College and Career Ready Standards (Puzzle Proof, Proofs involving Quadrilaterals)<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard) |

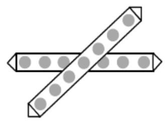
# Unit 2: Congruence and Introduction to Proofs

## *(Support & Strategies)*

### Framework Description/Rationale

The students will prove geometric theorems about lines and angles, triangles and parallelograms. Through this unit, students will be introduced to a formal method of proof (See CA **Geometry Framework** pg. 11). The idea of transformations with rigid motions will be the focus strategy for proving congruence. Below as an example pulled from the Framework:

**Example.** *The Kite Factory*. The hypothetical situation of a kite factory is presented to students, wherein kite engineers wish to know how the shape of a kite affects how it flies (e.g. the lengths of the rods, where they are attached, the angle at which



the rods are attached, etc.). In this activity, students are given pieces of cardstock of various lengths, hole-punched at regular intervals so they can be attached in different places. These two "rods" form the frame for a kite at the kite factory. By changing the angle at which the sticks are held, and the places where they are attached, students discover different properties of quadrilaterals.

Students are challenged to make conjectures and use precise language to describe their findings about which diagonals result in which quadrilaterals. They can discover properties unique to certain quadrilaterals, such as the fact that diagonals that are perpendicular bisectors of each other imply the quadrilateral is a rhombus. See videos of this lesson being implemented in a high school classroom at *insidemathematics.org*.

| Academic Language Support | Instructional Tool/Strategy Examples | Preparing the Learner |
|---|---|---|
| Key Terms for Word Bank:<br>• Congruence<br>• Arc marks/tick marks<br>• Included Angle/Side<br>• Non-included Side<br>• Shared Side<br>• Vertical Angles<br>• Alternate Interior Angles<br>• Parallel/Perpendicular Lines<br>• Transversal<br>• Corresponding Parts of Congruent Triangles<br>• SSS, SAS, ASA, AAS<br>• Quadrilaterals<br>• Rectangles<br>• Square<br>• Rhombus<br>• Kite<br>• Consecutive Angles<br>• Opposite Angles<br>• Mid-segment<br>• Median<br>• Perpendicular Bisector<br>• Exterior Angle<br>• Remote Interior Angles<br>• Pythagorean Theorem<br>• Midsegment<br>• Medians | • Provide opportunities for writing proofs in 2-column form, flowchart form, and paragraph form to prepare students for higher math<br>• Use sets of applicable problems, students find patterns, form hypotheses, and eventually prove their hypotheses<br>• Use patty paper to visualize changes occurring to a figure during transformations<br>• Implement Geometry software such as Geometer's Sketchpad, Geogebra<br>• Have students try exploratory group work including strategies such as jigsaw groups, gallery walks, think-pair-share, collaborative conversations, etc.<br>• Use flowcharts, thinking maps, and other graphic organizers to promote connections between concepts (e.g. use a flow map to help students understand of proving step-by-step)<br>• Use of written instructions, diagrams, and pair work for constructions to encourage independence and student agency | *<u>Pre-requisite Skills: (can be reinforced throughout unit):</u>*<br><br>Basic Theorems:<br>• Vertical Angles are congruent<br>• Angles formed when two Parallel Lines are cut by a Transversal<br>• Angles formed when two Perpendicular Lines Meet<br>• Triangle Sum Theorem: The angles of a triangle add to 180 degrees |

**<u>Teacher Notes:</u>**

# Unit 3: **Similarity**
## *(4 Weeks: 11/16 – 12/18)*

| **Big Idea** | Similar figures map one shape proportionally onto another through non-rigid motions. |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • What is a non-rigid motion and how can it be defined?<br>• How can two figures be proven similar, both informally (hands-on) and formally (theorems)?<br>• What are some real-life applications of similarity? | Parallelogram C2 2010 p.8-9<br>Pentagon C2 2008 p.66-67<br>Hopewell Geometry C2 2006 p.24-25<br>Garden Chair C2 2003 p.7-8 | Shape of Things and Teacher's Notes<br>Measuring Mammals and Teacher's Notes |

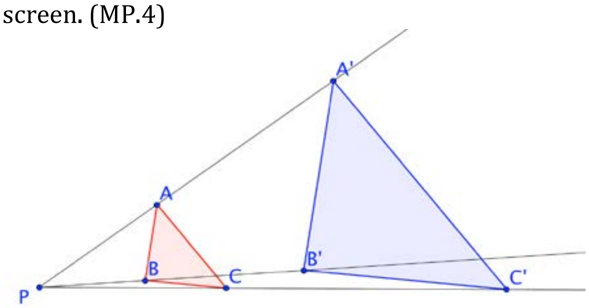
| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| Understand similarity in terms of similarity transformations:<br>• Understand and verify properties of Dilations<br>• Apply and understand Scale Factor<br>• Understand proportionality of corresponding pairs and angles<br>• Establish and apply AA Similarity Theorem, and given two figures decide if they are similar<br><br>Prove theorems involving similarity:<br>• Prove theorems about triangles<br>• Explain the meaning of similarity for triangles<br>• Apply similarity to prove Pythagorean Theorem<br>• Understand the Proportionality Theorem and its Converse | **G-SRT: Similarity, Right Triangles, and Trigonometry**<br>**Understand similarity in terms of similarity transformations**<br>**G-SRT 1.** Verify experimentally the properties of dilations given by a center and a scale factor:<br>a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.<br>b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.<br><br>**G-SRT 2**. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.<br><br>**G-SRT 3**. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.<br><br>**G-SRT: Similarity, Right Triangles, and Trigonometry**<br>**Prove theorems involving similarity**<br>**G-SRT 4**. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*<br><br>**G-SRT 5.** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | **Essential Resources:**<br>• **CCSS Geometry Framework** pg. 12<br><br>**Instructional Resources**<br>• EngageNY Module 2 Lessons 1-18<br>• Text: Ch. 7, Sections 1, 2, 3, 4, 5, 6<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.G-SRT: Similarity, right triangles, and trigonometry" standards 1-5)<br>• Illustrative Math Activities: Scroll down to find the links to the "SRT" Standards<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard) |

# Unit 3: **Similarity** *(Instructional Support & Strategies)*

## Framework Description/Rationale

Students will use similarity transformations to extend their understanding of the relationship between corresponding parts of figures. Through this unit, students will be introduced to the effects of dilations (non-rigid motion) to understand and prove similarity (See Geometry Framework pg. 12).

**Example Pulled from the Framework:**
**Experimenting with dilations.** Students are given opportunities to experiment with dilations and determine how they affect planar objects. Students first make sense of the definition of a *dilation of scale factor* $k>0$ *with center* $P$ as the transformation that moves a point $A$ along the ray $PA$ to a new point $A'$, so that $|PA'|=k\cdot|PA|$. For example, students apply the dilation of scale factor 2.5 with center P to the points A, B, and C illustrated using a ruler. Once they've done so, they consider the two triangles $\Delta ABC$ and $\Delta A'B'C'$. What they discover is that the lengths of the corresponding sides of the triangles have the same ratio as dictated by the scale factor. (G-SRT.2)
Students learn that parallel lines are taken to parallel lines by dilations; thus corresponding segments of $\Delta ABC$ and $\Delta A'B'C'$ are parallel. After students have proved results about parallel lines intersected by a transversal, they can deduce that the angles of the triangles are congruent. Through experimentation, they see that the congruence of corresponding angles is a necessary and sufficient condition for the triangles to be similar, leading to the *AA* criterion for triangle similarity. (G.SRT.3.) For a simple investigation, students can observe how the distance at which a projector is placed from a screen affects the size of the image on the screen. (MP.4)




| Academic Language Support | Instructional Tool/Strategy Examples | Preparing the Learner |
|---|---|---|
| Key Terms for Word Bank:<br>• Similarity<br>• Ratio<br>• Proportion<br>• Scale Factor<br>• Corresponding Parts<br>• Cross-Products or Product of the Means is equal to the Product of the Extremes<br>• Means vs. Extremes<br>• AA, SSS, SAS Similarity | • Alternate assessment with a project (mini-building or mini-mall)<br>• Use perspective drawings to verify properties of dilations, real-life applications (measure objects on paper and then measure on screen projector to confirm similarity)<br>• Use Projection measurements to decide if two figures are similar (similar to above)<br>• Use real-life application to explain the meaning of similarity of triangles (e.g. "Mini-Me" & "Dr. Evil" from *Austin Powers*)<br>• Use Measuring with Protractors, AngLegs (segments that can be snapped together) to help establish the AA criterion for two triangles to be similar<br>• Use constructions to prove theorems about triangles<br>• Use a Shadow or Mirror Activity to solve problems and prove relationships in geometric figures | *__Pre-requisite Skill:__*<br>• Solving Proportions<br>• Compute Scale Factor |

**__Teacher Notes:__**

# SAUSD Curriculum Map 2015-2016: Geometry

## Unit 4: **Trigonometry and Right Triangles**
*(3 Weeks: 1/4-1/22)*

| **Big Idea** | Trigonometric Ratios and the Pythagorean Theorem can be used to solve right triangles. |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • What are the Trigonometric Ratios and when can each be used?<br>• How can the Trigonometric Ratios be applied to real-life situations?<br>• How is Similarity related to the Trigonometric Ratios? | Ramps C2 2005 p.17<br>Angry Birds C3 2014 p.6-7<br>Rectangles and Squares C2 2003 p.11-12<br>Square Peg C2 2002 p.6-7 | Measuring Mammals and Teacher's Notes<br>Shape of Things and Teacher's Notes |

| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| Define trigonometric ratios and solve problems involving right triangles:<br>• Understand and apply Side Ratios in Right Triangles<br>• Know Definitions of Trigonometric Ratios<br>• Understand Relationship between sine and cosine<br>• Solve Right Triangles (real-life applications)<br>• Understand Special Right Triangles | **G-SRT: Similarity, Right Triangles, and Trigonometry**<br>**Define trigonometric ratios and solve problems involving right triangles**<br>**G-SRT 6.** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.<br><br>**G-SRT 7.** Explain and use the relationship between the sine and cosine of complementary angles.<br><br>**G-SRT 8**. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.<br><br>**G-SRT *8.1*** Derive and use the trigonometric ratios for Special Right Triangles (30-60-90 and 45-45-90 degree triangles) CA | **Essential Resources:**<br>• **CCSS Geometry Framework** pg. 13-14<br><br>**Instructional Resources**<br>• EngageNY Module 2 Lessons 21-32<br>• **Text:** Ch. 8, sections 1, 2, 3, 4, 5<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.G-SRT: Similarity, right triangles, and trigonometry" standards 6-8)<br>• Illustrative Math Activities: Scroll down to find the links to the "SRT" Standards<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard) |

# Unit 4: Trigonometry and Right Triangles

## *(Support & Strategies)*

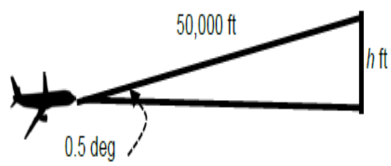
**Framework Description/Rationale**

Students will a) define trigonometric ratios and solve problems involving right triangles and b) derive and use the trigonometric ratios for special right triangles (30°, 60°, 90 °and 45°, 45°, 90°) CA. Through this unit, students will be introduced to Trigonometry as a method for finding unknown sides and angles (See **Geometry Framework** pg. 13-14 for more details).

**Example:** *Using Trigonometric Relationships.* Planes that fly at high speed and low elevations often have onboard radar systems to detect possible obstacles in the path of the plane. The radar can determine the range of an obstacle and the angle of elevation to the top of the obstacle. Suppose that the radar detects a tower that is 50,000 feet away, with an angle of elevation of 0.5 degree. By how many feet must the plane rise in order to pass above the tower?

*Solution:* The sketch of the situation below shows that there is a right triangle with hypotenuse 50,000 (ft) and smallest angle 0.5 (degree). To find the side opposite this angle, which represents the minimum height the plane should rise, we use $\sin 0.5° = h/50{,}000$, so that

$h = (50{,}000 \text{ ft}) \sin 0.5° \approx 436.33$ ft.




| Academic Language Support | Instructional Tool/Strategy Examples | Preparing the Learner |
|---|---|---|
| Key Terms for Word Bank:<br>• Sine<br>• Cosine<br>• Tangent<br>• Trigonometric Ratios<br>• Proportion<br>• Opposite Side<br>• Adjacent Side<br>• Hypotenuse<br>• Right Angle<br>• Reference Angle<br>• Pythagorean Theorem<br>• Radicals/Square Roots<br>• Common Right Triangles (3-4-5, 5-12-13, 8-15-17, 7-24-25)<br>• Special Right Triangles (30-60-90, 45-45-90 degree triangles) | • Use definitions to understand that by similarity, side ratios in right triangles are properties of the angles (this is how we define sine (x), cos (x), tan (x).: soh-cah-toa; make foldables (student examples)<br>• Use Foldables to explain and use the relationship between the sine and cosine of complementary angles | • How to simplify a radical<br>• How to solve a proportion |

**Teacher Notes:**

# Unit 5: **Area & Perimeter-Part A**
# **Expressing Geometric Properties with Equations- Part B**
## *(6 Weeks: 2/1 – 3/11)*

| **Big Idea** | Connecting algebra and geometry through coordinates |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • How can coordinates and the coordinate plane be used to prove theorems algebraically? | Parallel and Perpendicular G8 2014 p.10-11<br>Sloping Squares C2 2007 p.44-45<br>Shaded Shapes C2 2002 p.4-5<br>Pentagons C2 2004 p.39-40 | Lyle's Triangles and Teacher's Notes |

| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| • Explain, solve, and apply formulas for area and perimeter<br>• Explore areas in the coordinate plane<br>• Investigate the relationships between areas of trapezoids and rectangles<br>• Explore the Pythagorean Theorem and its relationship to the distance formula<br>• Derive Equation of a circle (includes use of Complete the Square), given the center and radius using the Pythagorean Theorem<br>• Derive the Equation of a parabola given a focus and directix<br>• Prove simple geometric theorems algebraically using coordinates<br>• Prove slope criteria for parallel/perpendicular lines<br>• Use coordinates to compute perimeters of polygons and areas of triangles and rectangles using the distance formula. | **G-GPE: Expressing Geometric Properties with Equations**<br>**Translate between the geometric description and the equation for a conic section.**<br>**G-GPE 1**. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.<br><br>**G-GPE 2.** Derive the equation of a parabola given a focus and directrix.<br><br>**Use coordinates to prove simple geometric theorems algebraically**<br>**G-GPE 4.** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*<br><br>**G-GPE 5**. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).<br><br>**G-GPE 6**. Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | **Essential Resources:**<br>• **CCSS Geometry Framework** pg. 18<br><br>**Instructional Resources**<br>• **Text:** Ch. 3, section 3, 4, 5, 6; Ch. 1, section 6; Ch. 9, section 4<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.G-GPE: Expressing Geometric Properties with Equations" standards 1-2 and 4-7)<br>• Illustrative Math Activities: Scroll down to find the links to the "G-GPE" Standards<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard) |

# Unit 5: **Area & Perimeter/Expressing Geometric Properties with Equations** *(Support & Strategies)*

| Framework Description/Rationale |
|---|
| Students will a) translate between the geometric description and equation for a conic section, and b) use coordinates to prove simple geometric theorems. Throughout this unit, students will use coordinate geometry to prove theorems algebraically (See CA **Geometry Framework** pg. 18). |

| Academic Language Support | Instructional Tool/Strategy Examples | Preparing the Learner |
|---|---|---|
| Key Terms for Word Bank:<br>• Slope<br>• Parallel/Perpendicular<br>• Coordinates<br>• Coordinate Geometry<br>• Midpoint Formula<br>• Distance Formula<br>• Parabola<br>• Quadrilaterals<br>• Inscribed/Circumscribed Triangles and Quadrilaterals | • Example of activity to help students derive the equation of a circle:<br>• Students construct a circle with a center at (0,0) and a radius of 5 units. A point on the circle may be (5,0). A second point may be(0,5). Students construct third side of triangle by connecting (0,5) and (5,0). The resulting triangle is a 45-45-90 right triangle and students may then use the Pythagorean Theorem to verify the hypotenuse measures 5 radical 2 units. | **Text:** Ch. 3, section 3, 4, 5, 6 (using coordinates to prove theorems algebraically); Ch. 11, section 7; Ch. 5, section 7 |

**Teacher Notes:**

# Unit 6: **Circles**

*(3 Weeks: 3/14 – 4/1)*

| **Big Idea** | Prove that all circles are similar and that relationships exist between angles formed by radii, chords, secants and tangents. |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • What patterns do you notice between angles formed by radii, chords, secants and tangents?<br>• What patterns do you notice about segment lengths involving chords, secants and tangents? | Circle and Squares C2 2008 p.74<br>Tammy's Circle Design C2 2013 p.4-5<br>Circles in Triangles C2 2007 p.51-52<br>The Arrow Store C2 2014 p.10-11<br>The Dolphin Tank C2 2014 p.4-5<br>Lampshade C2 2003 p.9-10<br>Jeff's Circles and Squares C2 2012 p.4-5<br>Pipes C2 2005 p.3 | Circular Reasoning and Teacher's Notes |

| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| Understand and apply theorems about circles:<br>• Prove all circles are similar<br>• Identify and describe Circle and Angle Relationships: inscribed angles, central angles, angles formed by chords, tangents and secants<br>• Understand Segment Relationships: Intersecting Chords, tangents and secants<br>• Prove properties for a quadrilateral inscribed in a circle<br><br>Find arc lengths and areas of sectors of circles:<br>• Understand the length of arc is proportional to the radius<br>• Understand area of a sector<br>• Define Radian measure<br>• Convert between degrees and radians | **G- C: Circles**<br>**Understand and apply theorems about circles**<br>**G- C 1**. Prove that all circles are similar.<br><br>**G- C 2**. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*<br><br>**G- C 3**. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.<br><br>**Find arc lengths and areas of sectors of circles**<br>**G- C 5**. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. | **Essential Resources:**<br>• **CCSS Geometry Framework** pg. 14-16<br><br>**Instructional Resources**<br>• **Text:** Ch. 11, sections 1, 2, 3, 4, 5, 6<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.G-C: Circles" standards 1-3, and 5)<br>• Illustrative Math Activities: Scroll down to find the links to the "G-C" Standards<br>• ***Irvine Math Project***<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard) |

# Unit 6: **Circles** *(Instructional Support & Strategies)*

## Framework Description/Rationale

Students extend their understanding of the usefulness of similarity transformations through investigating circles. For instance, students can reason that any two circles are similar by describing precisely how to transform one into the other, as the example illustrates with two specific circles. Students continue investigating properties of circles and relationships among angles, radii and chords (see CA **Geometry Framework** pgs. 14-16 for further details and examples).

**Example.** Students can show that the two circles $C$ and $D$ given by the equations below are similar.

$$C: (x-1)^2 + (y-4)^2 = 9$$
$$D: (x+2)^2 + (y-1)^2 = 25$$

**Solution.** Since the centers of the circles are $(1, 4)$ and $(-2, 1)$, respectively, we first translate the center of circle $C$ to the center of circle $D$ using the translation $T(x, y) = (x-3, y-3)$. Finally, since the radius of circle $C$ is 3 and the radius of circle $D$ is 5, we dilate from the point $(-2, 1)$ by a scale factor of $5/3$.

| Academic Language Support | Instructional Tool/Strategy Examples | Preparing the Learner |
|---|---|---|
| Key Terms for Word Bank:<br>• Circle<br>• Radius/Radii<br>• Chords<br>• Secants<br>• Tangents<br>• Central Angle<br>• Major Arc<br>• Minor Arc<br>• Semi-circle<br>• Intercepted Arc<br>• Inscribed Angles<br>• Circumscribed Triangles<br>• Area/Circumference<br>• Arclength<br>• Radian Measure<br>• Sector | Example of an activity students to compare circles:<br>• Students build circle one with center at (-3, -5) and a radius of 4 units. Students build circle two with a center at (4,5) and a radius of 8 units. Students then describe how the two circles late to each other through transformations and expansion/dilation.<br><br>Example of an activity students to measure inscribed circles:<br>• Students construct a circle, and then construct the circle's diameter. Finally, students construct an inscribed angle using the diameter end points s two of the three vertices. Students then measure the inscribed angle 90 degrees to confirm it is half of the circle or 180 degrees. | • Area and Circumference of a Circle<br>• Concept of π |

**Teacher Notes:**

# Unit 7: **Geometric Measurement and Dimension**

*(3 Weeks: 4/11 – 4/29)*

| **Big Idea** | Shapes & Solids: Two- and three-dimensional objects with or without curved surfaces can be described, classified, and analyzed by their attributes. |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • In what ways do we use cones, cylinders, spheres, right rectangular prisms, triangular prisms in real-life?<br>• How do I find the surface area and volume of a three dimensional figure?<br>• How do surface volume and area compare to each other? | Glasses C2 2008 p.62-63<br>Filling a Swimming Pool C2 2014 p.8-9<br>Cubic Design C2 2014 p.2-3<br>Insane Propane C2 2007 p.40-41<br>Barbeque C2 2013 p.8-9<br>At the Gym C2 2004 p.16-17 | Turn Up the Volume and Teacher's Notes |

| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| • Explain, solve , and apply formulas for area of circles, volume of a cylinder, pyramid, cone, and sphere<br>• Give an informal argument for the formulas for the circumference or a circle, area of a circle, volume of a cylinder, pyramid and cone.<br>• Understand and apply scale factors<br>• Identify the shapes, cross-sections, and objects by rotation<br>• Visualize and identify the relationships between two-dimensional and three-dimensional objects<br>• Identify three-dimensional objects generated by rotations of two-dimensional objects<br>• Know the effect of a scale factor k greater than zero on length, area and volume<br>• Use dissection arguments, Cavalieri's Principle, and informal limit arguments.<br>• Apply concepts of density<br>• Verify in a triangle, angle and side relationships<br>• Integrate Modeling Tasks<br>• Use geometric shapes to describe objects<br>• Apply concepts of density in modeling situations<br>• Apply geometric methods to solve design problems | **G-GMD: Geometric Measurement and Dimension**<br>**Explain volume formulas and use them to solve problems**<br>**G-GMD 1.** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*<br><br>**G-GMD 3.** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.<br><br>**Visualize relationships between two-dimensional and three- dimensional objects**<br>**G-GMD 4.** Identify the shapes of two-dimensional cross-sections of three- dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.<br><br>**G-GMD 5.** Know that the effect of a scale factor k greater than zero on length, area, and volume is to multiply each by k, k^2, and k^3 respectively; determine length, area and volume measures using scale factors.<br><br>**G-MG 1.** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★<br><br>**G-MG 2.** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot)★<br><br>**G-MG 3.** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★ | **Essential Resources:**<br>• **CCSS Geometry Framework** pg. 19-21<br><br>**Instructional Resources**<br>• **SAUSD Unit of Study** : Area and Perimeter *(to introduce concepts)*<br>• EngageNY Module 3 Lessons 1-13<br>• **Text** pp. 651-739<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.G-GMD: Geometric Measurement and Dimension" standards 1 and 3-5; and "H.G-MG: Modeling with Geometry" standards 1-3)<br>• Illustrative Math Activities: Click on the hyperlinks labeled with "G-GMD"<br>• CPM Resource<br>• Geogebra (free downloadable interactive resource)<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard)<br>• Lesson Activity Ideas from: http://sausdmath.pbworks.com/w/browse/#view=ViewAllObjects:<br>• Pistons and displacement in a combustion engine.<br>• Is King Kong possible? How are surface area, weight and volume affected by enlargement or reduction due to scale factor?<br>• How large a giant would fit to the world's largest shoe?<br>• How much coke would it take to fill the giant coke bottle on display in Las Vegas?<br>• Prisms, cylinders, cones, spheres<br>• Scale factors (length, area, volume, ratios)<br>• IMP Good, better, best container |

# Unit 7: **Geometric Measurement and Dimension**
## *(Support & Strategies)*

### Framework Description/Rationale

The ability to visualize two- and three-dimensional shapes is a useful skill. This group of standards addresses that skill and includes understanding and using volume and area formulas for curved objects. Students also have the opportunity to make use of the notion of a limiting process, an idea that plays a large role in calculus and advanced mathematics courses, when they investigate the formula for the area of a circle. By experimenting with grids of finer and finer mesh, they can repeatedly approximate the area of a unit circle, and thereby get a better and better approximation for the irrational number. They also dissect shapes and make arguments based on the dissections. For instance, a cube can be dissected into three congruent pyramids, as shown in figure 3, which can lend weight to the formula that the volume of a pyramid o$\frac{1}{3}Bh$

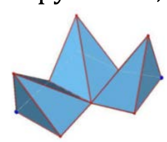


Figure 3: Three congruent pyramids that make a cube.

(Park City Mathematics Institute 2013)

The set of modeling standards are rich with opportunities for students to apply modeling with geometric concepts. See example below:

**Example (Illustrated Mathematics 2013).** *Ice Cream Cone.* You have been hired by the owner of a local ice cream parlor to assist in his company's new venture. The company will soon sell its ice cream cones in the freezer section of local grocery stores. The manufacturing process requires that the ice cream cone be wrapped in a cone-shaped paper wrapper with a flat circular disc covering the top. The company wants to minimize the amount of paper that is wasted in the process of wrapping the cones. Use a real ice cream cone or the dimensions of a real ice cream cone to complete the following tasks.

a. Sketch a wrapper like the one described above, using the actual size of your cone. Ignore any overlap required for assembly.

b. Use your sketch to help you develop an equation the owner can use to calculate the surface area of a wrapper (including the lid) for another cone given its base had a radius of length, $r$, and a slant height, $s$.

c. Using measurements of the radius of the base and slant height of your cone, and your equation from the previous step, find the surface area of your cone.

d. The company has a large rectangular piece of paper that measures 100 cm by 150 cm. Estimate the maximum number of complete wrappers sized to fit your cone that could be cut from this one piece of paper. Explain your estimate.

Solutions can be found at *illustrativemath.org*

(See CA **Geometry Framework** pg. 19-21 for more information)

| Academic Language Support | Instructional Tool/Strategy Examples | Preparing the Learner |
|---|---|---|
| Key Terms for Word Bank:<br>• Circle<br>• Circumference<br>• Area<br>• Cylinder<br>• Volume<br>• Surface area<br>• Pyramid<br>• Cone<br>• Cavalieri's principle<br>• Sphere | ● Use 2D and 3D manipulatives such as cones, rubber bands, etc. to help students conceptualize various geometric concepts such as scale factor, area, volume, etc.<br>● Compose and decompose shapes (using styrofoam, fruit/veggies, etc.)<br>● Decompose a cube into three congruent pyramids<br>● Find the cross sections of various foods<br>● Use grids, mesh and/or graphing paper to estimate and justify area formulas | Topics:<br>• 2-D formulas review<br>• Practice using an area model<br>• Pythagorean Theorem<br>• Pythagorean Triples<br>• Special right triangles<br>• Show many right triangles where students can derive 3rd side mentally |

| | | |
|---|---|---|
| • Cross section<br>• 2-d<br>• 3-d<br>• Big B (base area)<br>• Scale factor<br>• <u>Strategy Examples:</u><br>• Thinking Maps:<br>• 2-D area and perimeter review<br>• 3-D Lateral Area, Surface Area and Volume formulas | ● Use Geometric software (Sketch pad, Geogebra) to show various geometric concepts such as scale factor, area, volume, etc.<br>● Implement Pair Share, Gallery Walks,<br>● Use of realia to work through modeling tasks (such as ice cream cones and surface area wrappers and various household items)<br>● Use of measuring devices such as rulers, tape measures, measuring cups, etc. for modeling tasks<br>● Use various viscous fluids for density (oil, water, etc.) | |
| <u>Teacher Notes:</u> | | |

# Unit 8: **Statistics and Probability** *(4 Weeks: 5/2 – 5/27)*

| **Big Idea** | Chance: The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events. |
|---|---|

| **Essential Questions** | **Performance Task** | **Problem of the Month** |
|---|---|---|
| • How is geometric probability different than theoretical probability?<br>• How can we represent the probability of an event using geometric representations and the concept of length or area?<br>• How do you calculate geometric probability? | Discs C2 2002 p.1<br>Math Team C3 2013 p.6-7<br>Dropping Cups C2 2000 p.1<br>A Random Choice C3 2012 p.4-5<br>Marble Game G8 2009 p.68-69<br>Flora, Freddie, Future G8 2008 p.60-61<br>Playoff Party G7 2014 p.10-11 | Fair Games<br>Teacher's Notes |

| **Unit Topics/Concepts** | **Content Standards** | **Resources** |
|---|---|---|
| • Know how to collect the data and analyze that data in order to make predictions based on the subject of probability of events<br>• Describe Events and Sample Spaces<br>• Understand what makes two events Independent and understand Dependent and Mutually Exclusive Events<br>• Be able to put into their own words what independent and conditional probability is and state how to use the data or knowledge in everyday life.<br>• Construct and Interpret two-way frequency tables of data and decide if events are independent and approximate conditional probabilities<br>• Use their knowledge of conditional probability of A given B as a fraction to come out with possible outcomes for each event.<br>• Interpret whether two events are conditional on another event and how an event can depend on the other<br>• Understand Geometric Probability Models<br>• Understand how different events relate to each other (outcomes, unions, intersections, or complements of other events)<br>• Apply the addition rule, P(A or B)= P(A)+P(B)-P(A and B), and be able to interpret what their answers mean in regards of the model. | **S-CP: Conditional Probability and the rules of Probability**<br>**Understand independence and conditional probability and use them to interpret data**<br>**S-CP 1.** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").<br>**S-CP 2.**Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.<br>**S-CP 3.** Understand the conditional probability of *A* given *B* as P(*A* and *B*)/*P*(*B*), and interpret independence of A and B as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.<br>**S-CP 4.** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your class will favor science given that the student is a boy. Do the same for other subjects and compare the results.<br>**S-CP 5.** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of being unemployed if you are female with the chance of being female if you are unemployed.<br><br>**Use the rules of probability to compute probabilities of compound events in a uniform probability model**<br>**S-CP 6.** Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.<br>**S-CP 7.** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model. | **Essential Resources:**<br>• **CCSS Geometry Framework** pg. 21-25<br><br>**Instructional Resources**<br>• CPM Statistics and Probability Unit: Student Version and Teacher Version<br>• Text pp. 628-638, Conditional probability, pp 62-80<br>• Mathematics Assessment Project Formative Assessment Lessons (scroll down to find the standards labeled "H.S-CP: Conditional Probability and the Rules of Probability" standards 1-7).<br>• Illustrative Math Activities: Scroll down to find the links to the "S-CP" Standards<br>• Dan Meyer 3-act videos (list and interactive link to Dan Meyer's videos by standard) |

# Unit 8: Statistics and Probability
## *(Instructional Support & Strategies)*

| **Framework Description/Rationale** |
|---|
| In this unit students will understand independence and conditional probability and use them to interpret data. They will also use the rules of probability to compute probabilities of compound events in a uniform probability model and use probability to evaluate outcomes of decisions<br>(See CA **Geometry Framework** pg. 21-25 for more information) |

| **Academic Language Support** | **Instructional Tool/Strategy Examples** | **Preparing the Learner** |
|---|---|---|
| Key Terms for Word Bank:<br>• Experiment<br>• Outcome<br>• Sample space<br>• Event<br>• Probability<br>• Fair<br>• Theoretical probability<br>• Complement of an event<br>• Geometric probability<br>• Reasonable answer<br>• Most likely<br>• Least likely<br>• Spinner | • Use sporting events that display similar data, but don't correlate together. Use examples and non-examples for students to create their own rules behind independent events.<br>• Use analogies and comparisons to real world examples<br>• Use surveys generated by students and collect data to analyze and make outcome predictions based on their gathered data<br>• Incorporate Think Pair Share activities, where students generate their definitions and have group discussions on their own examples.<br>• Have students critique the predictions made by others to explore which prediction is correct for the data collected<br>• Use probability games.<br>• Have students can come up with their own scenario to make fair decisions<br>• Analyze a store and come up with predictions of what the store should sell and how much they should sell.<br>• Real life topics to discuss data/take surveys, etc:<br>• Weather (hurricanes, earthquakes, sunny, rain, ect) Gas Prices, Political Events, Speed limit vs. accident rate<br>• Batting Averages, Errors, Pitching, different divisions<br>• 3 Figure Skating Pairs, blood types, kidney matches, bone marrow<br>• Preferred subject, favorite colors, music, etc.<br>• Student generate other example topics<br>• Students are given tables, charts, and graphs with three different prediction outcomes given, with only one right, and students will need to discuss which one is correct, and why<br>• Incorporate spinner, dice, cards, etc. | |

| Teacher Notes: |
|---|
| |